

## CUMULATIVE HISTERETIC ABSORBED ENERGY

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### ABSTRACT :

Cumulative energy balance, during the earthquake action, defines the change of energies: potential, kinetic, dumping and hysteretic energy from the beginning to the end of earthquake action.

Cumulative hysteretic energy at the end of earthquake time acting defines level of energy absorption done by structure.

It presents the level of cumulative structural damage after the earthquake.

Paper is based on large number of non-linear structural response analysis to various earthquakes with different intensities and dominant frequencies as input load.

In all of them structure were modelled as SDOF system.

Results of provided analysis are structure response parameters: ductility, hysteretic energy and number of plastic excursions.

All of them define post-elastic state of the structure.

**KEYWORDS:** Absorption, Hysteretic Energy, Spectrum, Damage ratio

## 1 INTRODUCTION

Large number of non-linear structural response analysis for various earthquakes, with different intensities and dominant frequencies as input load, was done. In all of them structures were modelled as SDOF system with defined damping, elastic stiffness ( $K_e$ ), yield base shear ( $S_y$ ) and post-elastic stiffness ( $K_2$ ). For all SDOF systems presented in this paper were assumed constant damping, defined with 5% of critical, and constant weight  $G=1000$  kN. Variation of the elastic stiffness was in function of the basic period of the system representing real regular structure. Elastic stiffness was modified in such manner to realise change in basic period in steps of 0,1 s., from 0,05s to 5,0s. Yield base shear, which define yield point and the end of elastic stiffness, were modified in three levels: 0,1G represented structures with low elastic seismic resistance, 0,3G represented structures with high elastic seismic resistance and 0,6G represented structures with very high elastic seismic resistance, almost absolutely elastic in conditions caused by most frequently happened earthquakes. Post-elastic stiffness, which represents residual stiffness after yield point had reached, was modelled as percent of initial elastic stiffness. Variation of that parameter was done, also, in three steps:  $K_2=0,00$ , without residual stiffness, which represents structures designed with small level of absorption capacity,  $K_2=0,4K_e$ , which represents structures designed with significant level of absorption capacity and  $K_2=0,6K_e$ , which represents structures designed with huge level of absorption capacity. For example mark SY01K204Ke (figures 2. and 3.) defines structure with small yield base shear, equal 10% of structures weight, and with significant level of absorption capacity e.g. designed by actual seismic codes. In such manner classifications contents nine different structures. Input seismic loads were ground accelerations time-histories of real earthquakes. Intention was to choose earthquakes with maximum ground accelerations near 0,1g, 0,2g and 0,35g that present earthquake levels defined by codes (VII, VIII and IX zones of intensities). Chosen input loads were San Fernando with  $a_{g,max} = 0,13g$ , Impalvally with  $a_{g,max} = 0,21g$  and Northridge1 with  $a_{g,max} = 0,35g$ . As extremely rare and catastrophic earthquake was choose Northridge2 time-history with a  $g_{g,max} = 0,61g$  which is the same earthquake recorded on different location. For all models and for all described time-histories as input load, using program NONLIN (1), the non-linear structural response time history analyses were done. Results of provided analysis were structure response parameters as time-history displacements, ductility, hysteretic energy and number of plastic excursions. All of them define post-elastic state of the structure. This paper gives results of cumulative energy balance analysis resulted by provided calculations.

Energy absorption by structure, during the earthquake, depends of elastic seismic resistance point and level of reached displacements. Absolute value of hysteretic energy  $E_y$  or its percent in energy balance, at the end of earthquake, is quite good quality measure of damage level of system and can be used in quantification of damage. It is acceptable, in same way, response parameter for damage ratio equation.

## 2 HYSTERETIC ENERGY ( $E_y$ ) and DAMAGE RATIO (DR) (2)

Level of structural damage (damage ratio DR) can be described as the combination of the structure response parameters:

1. Displacement ductility (D) defines the measure of post-elastic region.
2. Maximum base shear force ( $BS$ )<sub>max</sub> and maximum top displacement  $u$ <sub>max</sub> define residual stiffness ( $K'$ ) of the structure at the end of the earthquake.
3. Number of yield excursions ( $N_y$ ) and hysteretic energy ( $E_y$ ) define post-elastic cyclic nature of damage ratio developing.

First two parameters define damage mechanism under monotonic load and third one take in account cyclic failure. Damage coefficient ratio (DR) is defined as linear combination of these two groups, as linear combination of plastic deformations, stiffness degradation and energy dissipation of structure at the end of earthquake time:

$$DR = \frac{1}{30} \left[ D + \Delta K + \sqrt[3]{(N_y E_y / W)} \right] \quad (2.1)$$

where:

- $D = u^{\max} / u^y$  is displacement ductility demand
- $\Delta K = K_{el} / K'$  is relative degradation of stiffness at the end of earthquake

$K_{el} = (BS)_y / u^y$  is initial structure stiffness

$K' = (BS)_{max} / u^{max}$  is residual secant stiffness of structure after the earthquake

- $(N_Y)$  is number of yield excursions reached during the earthquake
- $E_Y / W$  is hysteretic energy per unit of structure mass, dissipated during the earthquake.

In such approach value of damage ratio can be used to declare decrease of residual seismic yield resistance (figure 1.), as follow:

$$S_Y^{RESIDUAL} = S_Y^{INITIAL} \cdot \sqrt{(1 - DR)} \quad (2.2)$$

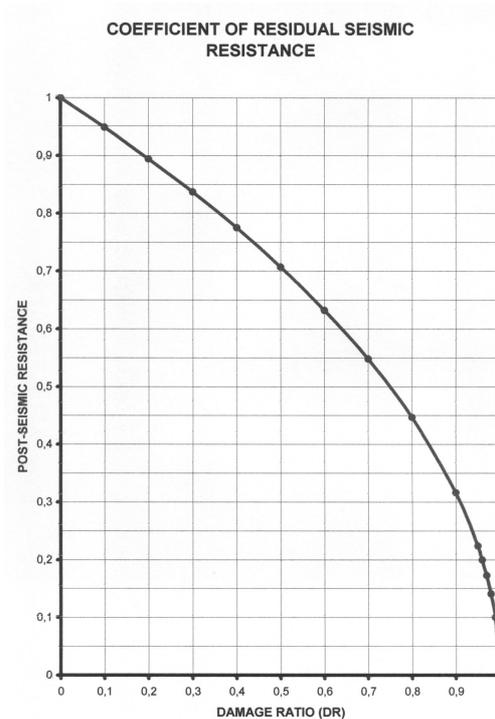


Figure 1. Decrease of initial seismic resistance as a function of damage ratio (DR)

The connection of damage ratio values (DR) with the values of damage level identification (S), defined in the Croatian codes for post disasters damage assessment is presented in table 2.1:

Table 2.1 Physical interpretation of damage coefficient

Damage ratio (DR)	structural damage description	Possibilities of reparation	Code damage level (S) (1 <sup>o</sup> to 6 <sup>o</sup> )
0 < DR < 0, 3	insignificant	Repairable	1 <sup>o</sup> - 2 <sup>o</sup>
0,3 < DR < 0, 5	moderate	Repairable	3 <sup>o</sup>
0,5 < DR < 0, 8	severe	Repairable	4 <sup>o</sup>
0,8 < DR < 1,0	heavy	Repairable	5 <sup>o</sup>
1,0 < DR	extremely high level or collapse	non-repairable	6 <sup>o</sup>

### 3 HYSTERETIC ENERGY SPECTRUM FUNCTIONS

For nine types of, in introduction, described structure spectrum functions of cumulative absorbed hysteretic energy, at the end of three chosen earthquakes (0,21g, 0,35g and extremely 0,61g) are presented in figure 2. Energy is presented as cumulative value of work divided by structure weight. Work is the product of base shear and displacement in time.

Percent of absorbed hysteretic energy in energy cumulative balance is presented in figure 3.

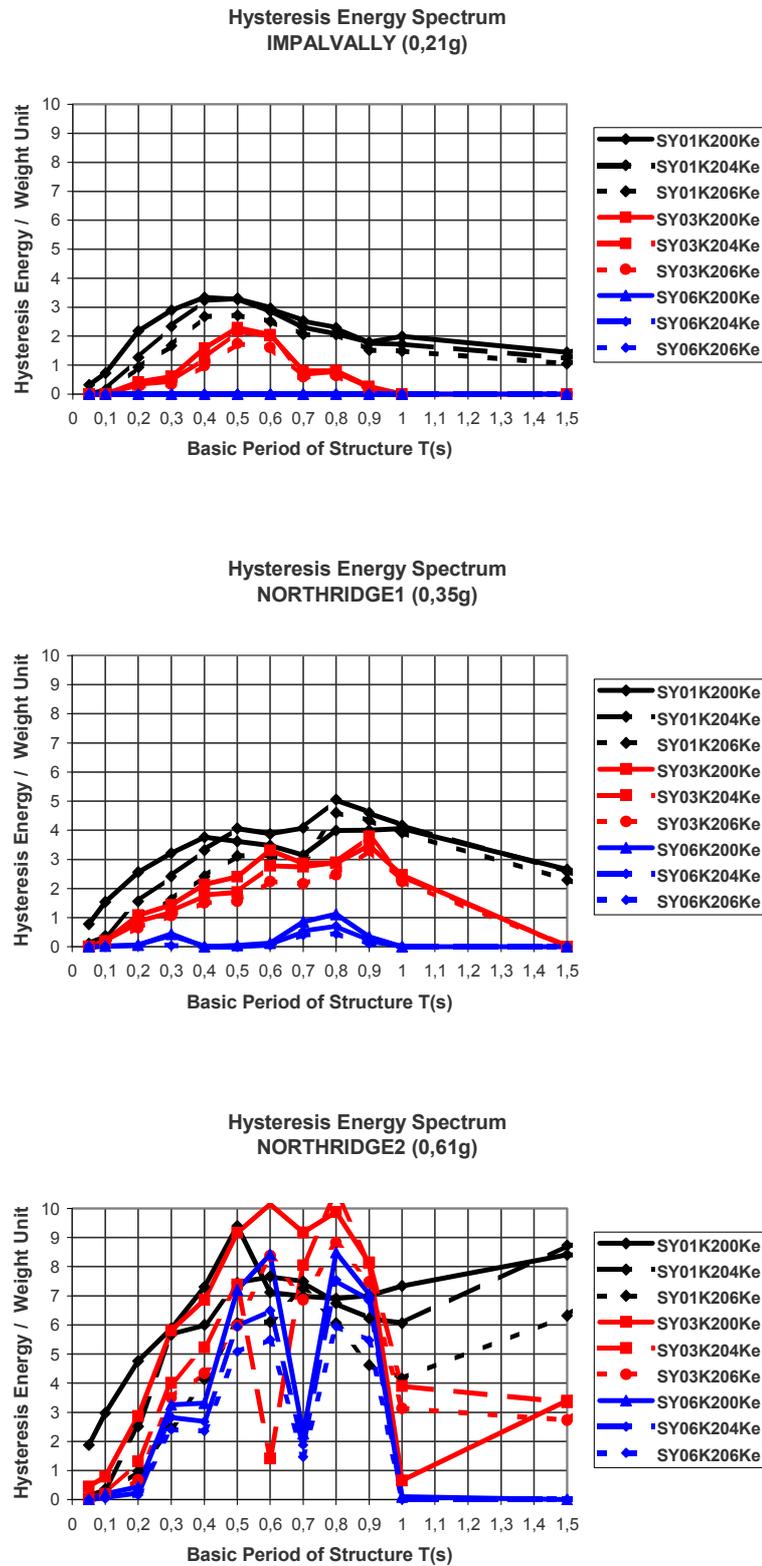


Figure 2: structure spectrum functions of cumulative absorbed hysteretic energy

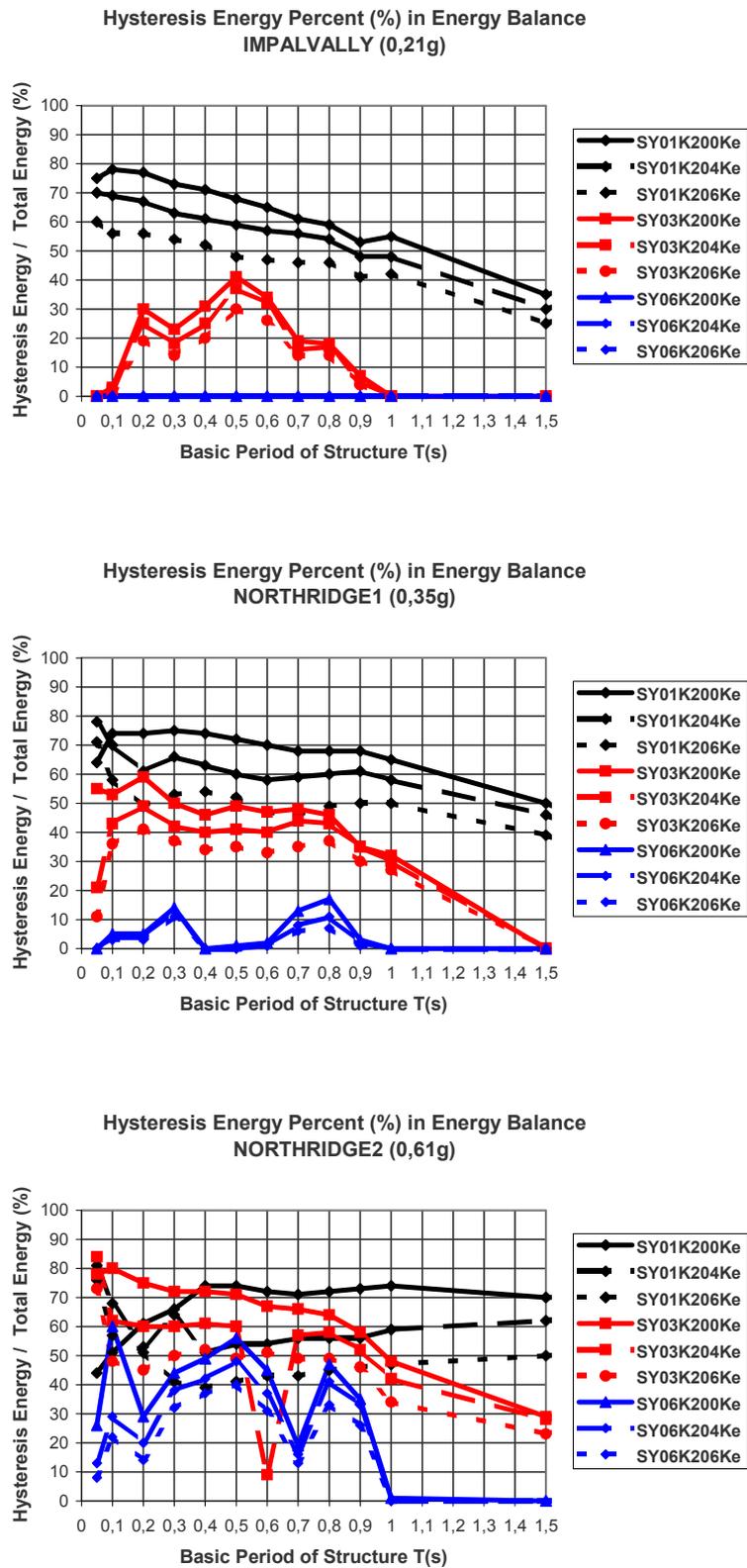


Figure 3: Percent of absorbed hysteric energy in energy cumulative balance

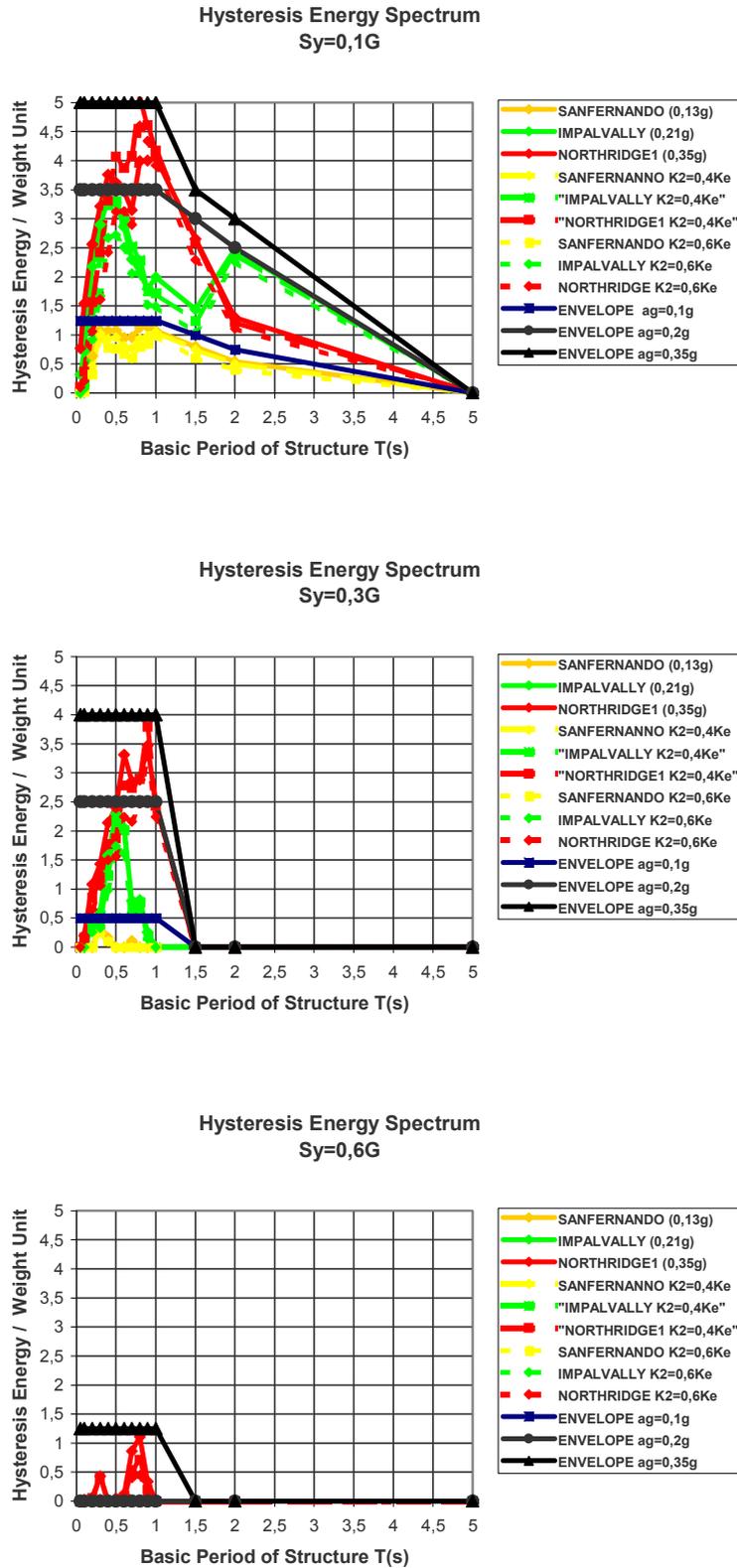


Figure 4: Hysteretic spectrum functions for three different levels of yield base shear

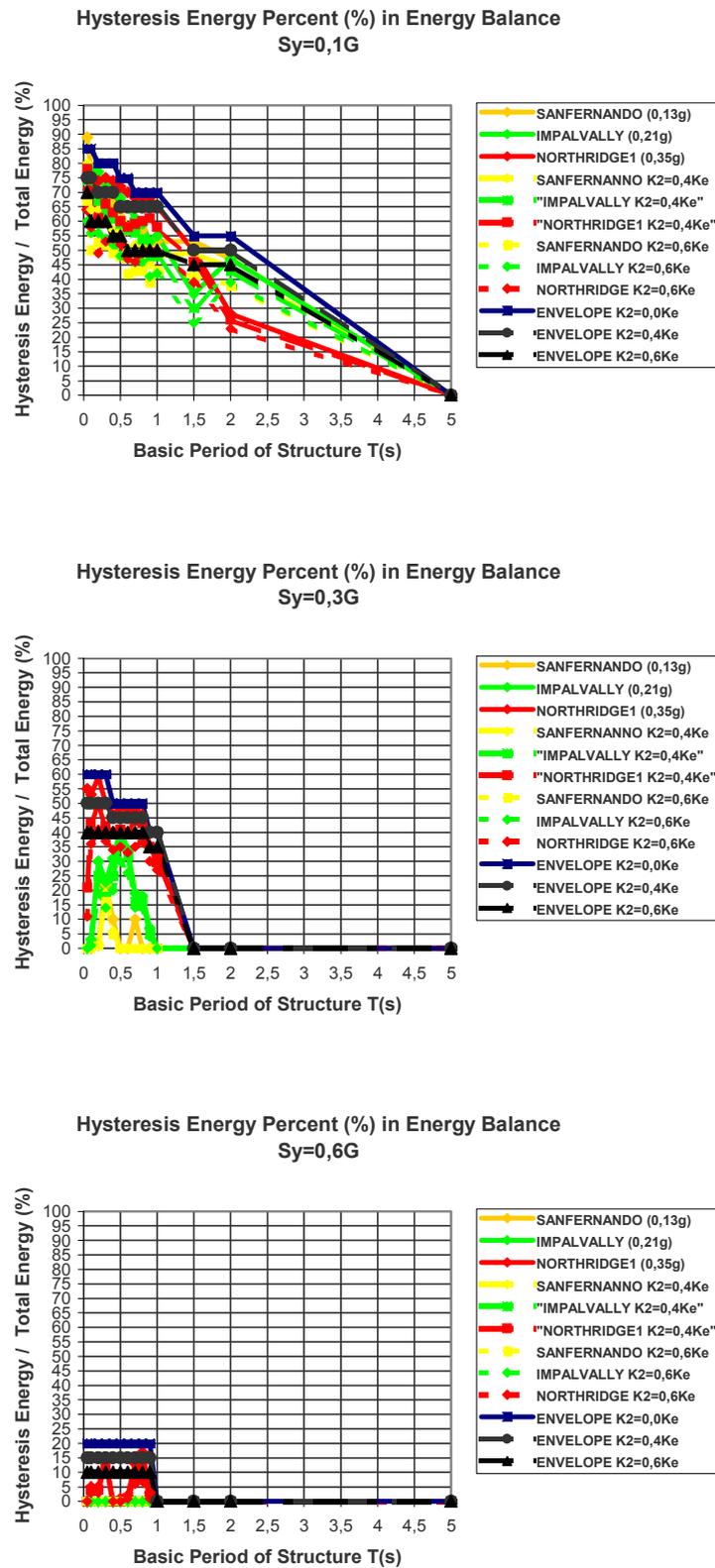


Figure 5: Spectrum functions of percent of hysteretic energy of earthquake input energy

Hysteretic spectrum functions for three different levels of yield base shear, which define elastic seismic resistance, are present in figure 4. Functions are given for earthquakes with maximum ground accelerations near 0,1g, 0,2g and 0,4g that present earthquake levels defined by codes (VII, VIII and IX zones of intensities), and for all three chosen post elastic stiffness. Yellow lines define response for San Fernando  $a_{gmax}=0,13g$ , green lines defines response for Impalvally with  $a_{gmax}=0,21g$  and red lines response for Northridge1 with  $a_{gmax}=0,35g$ . Black lines represent envelope curves for all three levels of earthquake maximum ground accelerations (0,1g, 0,21g and 0,35g). Based on envelope curves it is possible to quantificate level of absorbed hysteretic energy per weight unit of structure. Quantification is significant for structures with usual basic period of structures less than 1 second and is present in following table:

Table 3.1: Hysteretic energy per weight unit (Eh / G) in function of earthquake input

	SY=0,1G	SY=0,3G	SY=0,6G
$a_g = 0,35$ (IX)	5,0	4,0	1,5
$a_g = 0,2g$ (VIII)	3,5	2,5	0
$a_g = 0,1g$ (VII)	1,5	0,5	0

Figure 5. Presents spectrum functions of percent of hysteretic absorbed cumulative energy versus cumulative earthquake input energy. Black lines represent envelope curves for all three levels of post-elastic stiffness and for all three-earthquake maximum ground accelerations (0,1g, 0,21g and 0,35g). Quantification depends more on basic period of structures. For rigid structures percent of hysteretic absorbed cumulative energy versus cumulative earthquake input energy in following table:

Table 3.2: Hysteretic energy / Total energy (Eh / Ei (%)) in function of post-elastic stiffness K2.

	SY=0,1G	SY=0,3G	SY=0,6G
K2 = 0,00	90	60	20
K2 = 0,4Ke	70	50	15
K2 = 0,60Ke	60	40	10

#### 4 CONCLUSIONS

It is obviously that the earthquake with maximum ground acceleration less than elastic seismic resistance of structure produces hysteretic absorption e.g. structural damage. Reason is dynamic amplification.

Influence of post-elastic residual stiffness of structure is proportional with damage level e.g. hysteretic absorbed energy. In case of extremely rare and catastrophic earthquake absolute absorbed hysteretic energy is proportional with elastic seismic resistance. Percent of absorbed hysteric energy in energy cumulative balance strongly depends to post elastic residual stiffness. Higher level of residual stiffness will decrease level of damage. That means that stiffness of the structure of structural element after initial damage should be saved as much as it is possible. It has to be reached by design measures. Hysteretic energy per weight unit coefficient depends on relation between elastic seismic resistance of structure and ground motion acceleration e.g. earthquake intensity. If ground motion acceleration value is higher then elastic seismic resistance coefficient is from 3,5 to 5,0. In other case its value is maximum 2,5 and it is consequence of dynamic amplification.

Influence of post-elastic stiffness on decrease of hysteretic energy e.g. damage level is significant. For structures with low elastic seismic resistance increase of post-elastic stiffness decrease damage level about 30% and for structures with high elastic seismic resistance about 50%.

#### REFERENCES

NONLIN, Nonlinear Dynamic Time History Analysis of Single Degree of Freedom Systems, developed by Finley A. Charney.

Morić D, Hadzima M, Ivanušić D. (2003), Seismic Damage Model for Regular Structures, *International Journal for Engineering Modelling*, 14, 1-4, 29-44.