A DESIGN METHOD FOR STEEL FRAMES EQUIPPED WITH BUCKLING RESTRAINED BRACES

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ABSTRACT :

In this paper a design method for buckling-restrained braced steel frames is examined. The proposed method aims to obtain an uniform yielding of all Buckling Restrained Braces (BRBs), thus avoiding concentration of plastic deformation at some storey level. This is a critical point in the design since BRBs usually have very low hardening in post-elastic response, especially if they are used in steel frames with pinned joints. An optimal distribution of column stiffness and of brace stiffness can be preliminarily defined from the free vibration equations of an equivalent shear-deformable cantilever. This continuum model permits to obtain closed form solutions that can be adopted for the design of structures with regular mass distribution over the height. A similar procedure is directly applied to discrete models. An optimal solution for the equivalent SDOF system is achieved by using force-based or displacement based design approaches. Being the proposed method based on the first vibration mode, dynamic analysis are performed and discussed in order to evaluate the reduction of performance in the nonlinear range deriving from higher vibration modes.

KEYWORDS: Buckling restrained braces, braced steel frames, design method.

1. INTRODUCTION

Buckling-restrained braced frames were introduced to enhance the compressive capacity of braces while not affecting their stronger tensile capacity, hence producing a symmetric hysteretic response (Uang and Nakashima 2004). A buckling restrained brace (BRB) usually consists of a core steel brace encased in a steel tube filled with concrete or grout. BRBs can constitute the whole diagonal brace of the bracing system or they can be placed in series with an over-strengthened brace that remains elastic. BRBs have been used extensively for seismic applications in Japan and United States due to their simple and efficient behavior, as testified by several applications (Uang and Nakashima 2004, Whittaker and Constantinou 2004), and by the inclusion in code recommendations such as the AISC seismic provisions (American Institute of Steel Construction 2005). The introduction of BRB members undoubtedly represents a major advancement compared to conventional braces in terms of cyclic inelastic deformation capacity and reduction of design forces. However, buckling restrained braced frames may undergo large inelastic storey drifts without the ability to distribute the ductility demand over the height of multi-storey structures, due to possible localizations of inelastic deformations. This latter aspect deserves particular attention due to the bracing system characteristics, i.e., statically determinate structural configuration and limited BRB hardening, especially in steel frames with beams connected to columns by means of pinned joints as often happens in Europe. As a result, the frame global ductility is strongly dependent on the distribution of BRB strength and stiffness at each storey level.

In this paper a single degree of freedom (SDOF) based design method for steel frames with BRBs is examined. The bracing system is modeled as a continuum cantilever beam where BRBs are associated to the shear stiffness and columns are associated to the flexural stiffness. This continuum model allows a more clear identification of the parameters influencing the structural behavior and a more simple definition of the design procedure as compared to discrete models. Closed form solutions can be obtained for cantilever beams with uniformly distributed mass, resulting in simple analytical expressions that can be adopted for the design of structures with regular mass distribution over the height. An optimal solution for the SDOF system is achieved by using force-base or displacement based design approaches (Faifar 1999), similarly to other methods based on the same type of approach (Della Corte 2006). However, the actual dynamic nonlinear response can significantly be influenced by higher vibration modes and by deformation localization at some floor levels. Thus preliminary results of numerical simulated response analyses are illustrated in order to highlight advantages and limitations of the presented design procedure paying particular attention to the differences between static analysis based on the first vibration mode and nonlinear dynamic analysis.

2. PROPOSED DESIGN METHOD

2.1. Dynamics of shear-deformable beams

The bracing system behavior may be described by a continuous model consisting in a cantilever beam including flexural and shear deformability. The balance equations and relevant boundary conditions of this continuous model are (Humar 2002):

$$\mu\ddot{u} - \left[A(u'+\psi)\right]' = q \qquad A(u'+\psi)\big|_L = 0 \qquad u\big|_0 = 0 \tag{2.1}$$

$$I_\mu\ddot{\psi} - (J\psi')' + A(u'+\psi) = 0 \qquad J\psi'\big|_L = 0 \qquad \psi\big|_0 = 0 \tag{2.2}$$

where the functions $u(\zeta;t),\psi(\zeta;t):[0,L]\times[0,\infty)\mapsto\Re$ are the cross section transverse displacements and rotations respectively, L is the total height, $q(\zeta;t)$ is the time dependent load distribution, $A(\zeta)$ and $J(\zeta)$ are the distribution of flexural and shear stiffness respectively, $\mu(\zeta)$ is the mass per unit length, $I_\mu(\zeta)$ is the rotation inertia, primes denote differentiation with respect to ζ and superposed dots denote differentiation with respect to time t. The strain field is described by the shear deformation $\gamma = u'+\psi$ and by the curvature $\theta = \psi'$. The solution is defined once the initial conditions are assigned. In the sequel the rotation inertia is neglected.

The proposed design method aims at finding a stiffness distribution giving uniform shear deformation $\overline{\gamma}$ and uniform curvature $\overline{\theta}$ along the cantilever in the first vibration mode. The shear deformation is related to the deformation of the diagonal braces while the curvature is related to the deformation of the columns. Thus the uniform strain condition leads to simultaneous diagonal braces yielding if the response is dominated by the first vibration mode and an adequate over-strength is guaranteed for the columns. Starting from the assumed uniform strain field, it is possible to evaluate the functions $v(\zeta)$ and $\varphi(\zeta)$ describing the transverse displacement and rotation of the beam in the first vibration mode of circular frequency ω, by using the beam compatibility equations and the relevant kinematic boundary conditions:

$$v'+\varphi = \overline{\gamma} \qquad v_i\big|_0 = 0 \tag{2.3}$$

$$\varphi' = \overline{\theta} \qquad \varphi_i\big|_0 = 0 \tag{2.4}$$

The displacement and rotation functions describing the first vibration mode are obtained by integration:

$$v = \overline{\gamma}\zeta - \frac{1}{2}\overline{\theta}\zeta^2 \tag{2.5}$$

$$\varphi = \overline{\theta}\zeta \tag{2.6}$$

From the modal shape it is possible to compute the coefficient m^* (i.e., the mass of the equivalent SDOF system) and the modal participation factor Γ, for displacement v normalized with respect to the top end value:

$$m^* = \frac{1}{v\big|_L}\int_0^L \mu(\zeta)v(\zeta)d\zeta \qquad \Gamma = \frac{v\big|_L^2 m^*}{\int_0^L \mu(\zeta)v^2(\zeta)d\zeta} \tag{2.7, 2.8}$$

Thus the stiffness distributions $A(\zeta)$ and $J(\zeta)$ are determined from the differential balance equations of the first vibration mode (inverse problem) deduced substituting Eqns. 2.3-2.4 and the relations $u = ve^{i\omega t}, \psi = \varphi e^{i\omega t}$ in Eqns. 2.1-2.2:

$$-\mu\omega^2 v - A'\bar{\gamma} = 0 \qquad A\big|_L = 0 \tag{2.9}$$

$$-J'\bar{\theta} + A\bar{\gamma} = 0 \qquad J\big|_L = 0 \tag{2.10}$$

The solution of this homogeneous differential system is defined except for a scale factor assumed to be the shear stiffness of the base section $A_0 = A(0)$. The problem is solved taking as unknowns the functions $a(\zeta) = A(\zeta)/A_0$ and $j(\zeta) = J(\zeta)/A_0$. The function *a* is evaluated by integrating Eqn. 2.9; afterward the function *j* is obtained from (2.10). In the case of uniform mass distribution the following analytical expressions are obtained:

$$a = \frac{\left(1-(\zeta/L)^2\right) - \frac{\beta}{3}\left(1-(\zeta/L)^3\right)}{1-\frac{\beta}{3}} \tag{2.11}$$

$$j = \frac{L^2}{\frac{\beta}{2}\left(1-\frac{\beta}{3}\right)}\left[\left(-\frac{1}{3}+\frac{1}{2}\zeta/L-\frac{1}{6}(\zeta/L)^3\right) - \beta\left(-\frac{1}{8}+\frac{1}{6}\zeta/L-\frac{1}{24}(\zeta/L)^4\right)\right] \tag{2.12}$$

where a dimensionless parameter $\beta = \bar{\theta}L/\bar{\gamma}$ is introduced. Trends of shear and flexural stiffness distributions along the cantilever length are depicted in Figure 1, for different values of β spanning from 0.5 (small rotational deformation with respect to shear deformation) to 1.5 (large rotational deformation with respect to shear deformation).

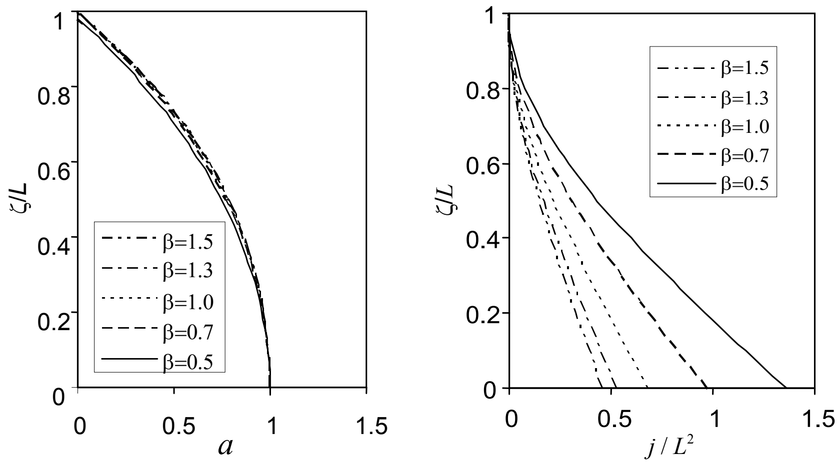

Figure 1 Trends of shear and flexural stiffness distributions along the cantilever.

It should be noted that, differently from the flexural stiffness distribution, the shear stiffness distribution does not strongly depends on parameter β. The circular frequency (ω) is defined except for a scale factor and can be given as a function of the circular frequency $\omega_{(1)}$ corresponding to a unitary base shear stiffness ($A_0 = 1$). From the above analytical expressions of displacements and stiffness, the parameter $\omega_{(1)}$ can be derived:

$$\omega^2 = A_0 \frac{\overline{\theta}^2 \int_0^L j d\zeta + \overline{\gamma}^2 \int_0^L a d\zeta}{\int_0^L \mu v^2 d\zeta} = A_0 \omega_{(1)}^2 \tag{2.13}$$

2.2. Base shear stiffness design

It is assumed that: (i) the cantilever has a linear elastic – perfect plastic shear behavior; (ii) the yield shear $V_y = A\overline{\gamma}$ is attained for the shear deformation $\overline{\gamma}$; (iii) the ultimate shear strain is given by $\gamma_u = \mu_d \overline{\gamma}$ where μ_d is the shear design ductility. The bracing system ductility relevant to the first vibration mode (μ_s) can be deduced by subdividing the displacement of the cantilever top end (point of control) in the flexural contribution $v_c|_L = -\overline{\theta} L^2 / 2$ and in the shear contribution $v_d|_L = \overline{\gamma} L$ (Eqn. 2.5) and can be put in the form:

$$\mu_s = \frac{v_c|_L + \mu_d v_d|_L}{v_c|_L + v_d|_L} \tag{2.14}$$

The base shear stiffness A_0 is designed comparing the capacity of the bracing system to the seismic demand. The capacity (C) may be given by the maximum acceleration achievable in the equivalent simple oscillator and it is directly proportional to the base section shear stiffness A_0:

$$C(A_0) = \frac{A_0 \overline{\gamma}}{m^* \Gamma} \tag{2.15}$$

The demand (D) is given by the inelastic design response spectra (Fajfar 1999). In the force-based design approach, the inelastic spectrum in term of pseudo acceleration $S_a(\omega, \mu_s)$ for the simple oscillator with ductility μ_s and circular frequency ω is taken into account and it depends on A_0, since $\omega = \sqrt{A_0}\,\omega_{(1)}$:

$$D(A_0) = S_a\left(\sqrt{A_0}\,\omega_{(1)}, \mu_s\right) \tag{2.16}$$

Thus A_0 can be deduced by the equality:

$$C(A_0) = D(A_0) \tag{2.17}$$

Otherwise, in the deformation-based deign approach, it is possible to evaluate the capacity by means of the maximum displacement $C = v|_L$ (scale factor independent) and the demand from the displacement design spectra $D(A_0) = S_d\left(\sqrt{A_0}\,\omega_{(1)}, \mu_s\right)$ dependent upon A_0 by means of the circular frequency. In this case the two approaches are equivalent and give the same results. Alternately to these analytical approaches, the capacity spectrum method (Fajfar 1999) may be applied, which consists of comparing, by means of a graphical procedure, the capacity of the structure (evaluated by means of a push-over analysis) with the seismic demand (given by inelastic demand spectra in the ADRS format).

3. APPLICATION TO THE DESIGN OF V-BRACING SYSYEMS

3.1 V-bracing systems with concentrated mass

The same procedure may be adopted also in the case of bracing systems with masses concentrated at floor levels (discrete systems), once the typology of bracing system is chosen. For example, in the case of a V-bracing system with base b, inter-storey height h and number of floors p, the modal shape is known once the strains of the diagonal braces ($\bar{\varepsilon}_d$) and the columns ($\bar{\varepsilon}_c$) are assigned. The following values of the storey shear deformation γ_s and the floor curvature θ_s may be obtained

$$\gamma_s = 2\bar{\varepsilon}_d L_d^2 / bh \tag{3.1}$$
$$\theta_s = -2\bar{\varepsilon}_c / b \tag{3.2}$$

where $L_d = \sqrt{h^2 + (b/2)^2}$, as shown in Figure 2.

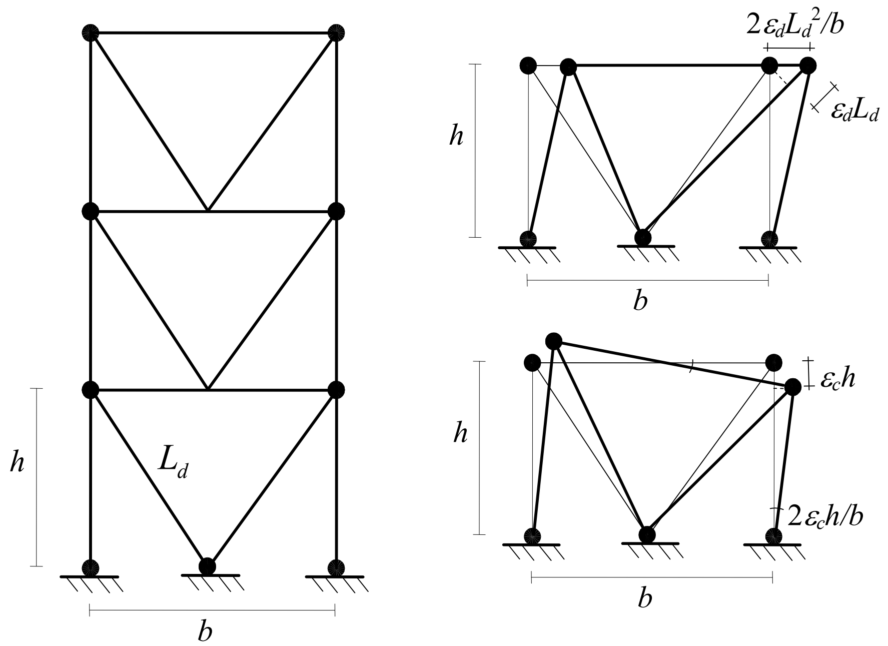

Figure 2 V-bracing system: geometry (a); floor shear deformation and floor curvature (b).

The storey displacements v_{di} and v_{ci} due to the diagonal brace deformation and to the column deformation respectively are given by

$$v_{di} = i\gamma_s h \tag{3.3}$$
$$v_{ci} = v_{ci-1} - i\theta_s h^2 \tag{3.4}$$

where $i = 1\ldots p$, whereas the storey rotations are

$$\varphi_i = i\theta_s h \tag{3.5}$$

As previously showed for the continuous case, the normalized shear and flexural stiffness a_i and j_i may be determined, by imposing at each floor the translational and rotational equilibrium equations. The subsequent recursive equations are obtained:

$$a_i = a_{i+1} + m_i v_i / \lambda \tag{3.6}$$
$$j_i = j_{i+1} + a_i \gamma_s h / \theta_s \tag{3.7}$$

where $\lambda = \sum_k m_k v_k$ and a_1=1.0 j_1=1.0. From Eqns. 23 and 24 the areas a_{di} of the diagonal braces and the areas a_{ci} of the columns, normalized with respect to the parameter A_0, are obtained:

$$a_{di} = 2a_i L_d^3 / Ehb^2 \tag{3.8}$$

$$a_{ci} = j_i / 2Eb^2 \tag{3.9}$$

The circular frequency $\omega_{(1)}$, the modal participation factor Γ and the factor m^* are:

$$\omega_{(1)}^2 = \frac{\bar{\theta}^2 h \sum_k j_k + \bar{\gamma}^2 h \sum_k a_k}{\sum_k m_k v_k^2} \tag{3.10}$$

$$\Gamma = \frac{v_p^2 m^*}{\sum_k m_k v_k^2} \tag{3.11}$$

$$m^* = \frac{1}{v_p} \sum_k m_k v_k \tag{3.12}$$

Approximate results may also be obtained by using shear-deformable beam equations illustrated in the previous section, by posing $\bar{\gamma} = \gamma_b, \bar{\theta} = \theta_b$ and $\mu = \sum_k m_k \big/ L$ (where $L = ph$). Finally the same procedures shown in section 2.2 may be used to evaluate the base shear stiffness A_0.

3.2 Numerical example

In this section the design procedure is applied to a case study consisting of a four-storey steel frame with pinned joints and including a V-bracing system as only seismic resistant component. The seismic floor mass estimated from vertical live and super-dead loads are 135 kNs^2/m at the top floor and 170 kNs^2/m at the first three levels (corresponding an steel office building with floor dimensions of approximately 20 m x 30 m and two bracing systems for each direction). Steel S275 is assumed for diagonal braces (BRB devices) and columns of the bracing system. Consequently the assumed design strain of the diagonal braces is $\bar{\varepsilon}_d = \sigma_y \big/ E = 275/210000 =$ 0.00131, whereas for the columns a lower value equal to $\bar{\varepsilon}_c = \bar{\varepsilon}_d \big/ \rho = 0.00131/4 = 0.0003275$ is chosen. The constant ρ was assumed equal to 4 in order to guarantee column resistance and to avoid column buckling under vertical and seismic loads (by considering an adequate over-strength with respect to the diagonal braces). The floor displacements are reported in Table 3.1. In the same table are also reported the values obtained for a_i and j_i. The following parameters may be calculated: $\omega_{(1)} = 0.0116$, $\Gamma = 1.4$ and $m^* = 365$ kNs^2/m.

The elastic spectrum given by the Italian seismic code (OPCM3431 2005) for ground types B,C,E with a peak ground acceleration equal to $a_g = 0.35g$ is considered. The inelastic design pseudo-acceleration spectrum is obtained by reducing the elastic spectrum by a factor R_μ according to the following equation (Fajfar 1999):

$$R_\mu = (\mu_s - 1)\frac{T}{T_c} + 1 \qquad \text{if} \quad T < T_c \tag{3.13a}$$

$$R_\mu = \mu_s \qquad \text{if} \quad T > T_c \tag{3.13b}$$

with T_c is defined in OPCM 3431 (2005). A design ductility equal to $\mu_d = 5$ is considered for the diagonal brace, considered made by a BRB device and an elastic connection brace. Larger values of μ_d lead to excessively deformable bracing systems. A structural ductility $\mu_s = 3.8$ is obtained by applying Eqn. 2.14. By considering the force-based approach, a base shear stiffness A_0 equal to $2.256 \cdot 10^5\, kN$ is obtained and the values of areas for the diagonal braces (A_{di}) and columns (A_{ci}) are reported in Table 3.1. In the same table the values of stiffness (K_{di}) and yielding force (F_{yi}) of diagonal braces are also reported.

Table 3.1 Modal shape and design results

floor	v_{ci} (*mm*)	v_{di} (*mm*)	v_i (*mm*)	a_i -	j_i -	A_{di} (cm^2)	A_{ci} (cm^2)	K_{di} (*kN/mm*)	F_{yi} (*kN*)
1	1.7	10.1	11.8	1	28.4	20	24	96.3	552
2	5.1	20.1	25.2	0.904	82.1	18	71	87.1	499
3	10.2	30.2	40.4	0.698	151.6	14	130	67.3	386
4	17.0	40.3	57.2	0.370	228.4	7	196	35.6	204

Both nonlinear static analysis and nonlinear dynamic analysis (time-history) of the designed bracing system were performed with the finite element structural analysis program SAP 2000 (advanced version 10.1.1). The nonlinear static push-over analysis, carried out by considering a force distribution correspondent to the first vibration mode of the system, showed that: (i) all diagonal braces reach yielding and ultimate displacements for the same load multiplier; (ii) the displacement of the point of control (last floor) coincides with the analytically predicted displacement $v_c\big|_L + \mu_d v_d\big|_L$ and (iii) the maximum acceleration that the bracing system can withstand is the design acceleration of 0.35g, as shown in Figure 3, where the push over curve (capacity curve) is plotted together with the design spectrum (demand curve) in the ADSR plane and their intersection point is the performance point of the structure.

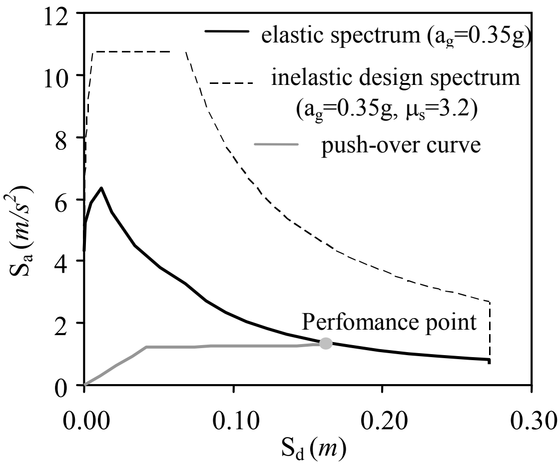

Figure 3 Static push-over analysis results

The results obtained by the nonlinear static analysis are compared with those obtained from nonlinear dynamic analysis of the system subjected to seven artificial accelerograms compatible with the elastic pseudo-acceleration spectrum used in the design procedure. The Bouc-Wen material model with low hardening stiffness (post yield stiffness ratio = 0.02) was adopted for the diagonal braces. This comparison is illustrated in order to evaluate the influence of higher vibration modes in the seismic response of the structure. Figure 4 depicts the distributions at each level of the diagonal brace maximum displacements (average values of maximum displacements obtained in the seven nonlinear dynamic analyses) for different values of the peak ground acceleration, from 0.1g to 0.4375g (which is the design peak ground acceleration given by 0.35gS = 0.4375g, where S is the soil factor and its value is 1.25 for ground types B,C,E). It is evident form the figure that all diagonal braces attain the yield displacement (i.e., $d_y = \varepsilon_d L_d$ = 5.7 *mm*), but the first and the last floors show larger plastic deformations, that exceed the ultimate displacement (i.e., $d_u = \mu_d \varepsilon_d L_d$ = 28.5 *mm*). In the analyzed case, diagonal braces should possess a ductility μ_d at least equal to 6.5, which is 1.3 times the value assumed for the bracing system design.

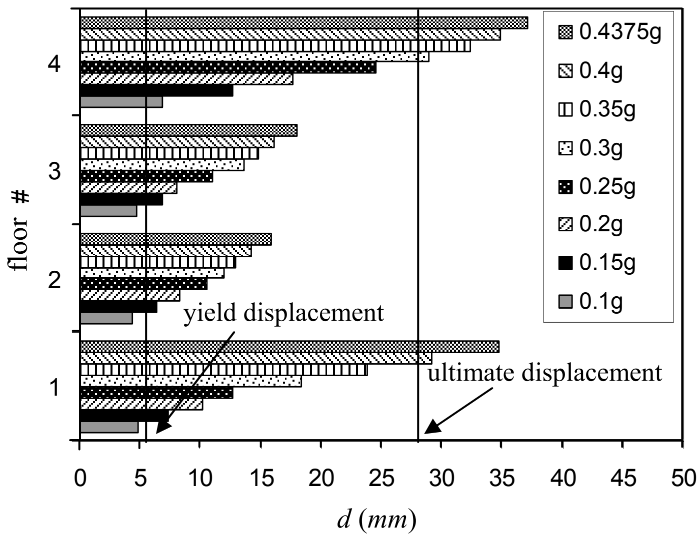

Figure 4 Distribution of BRB maximum displacements for different values of BRB ductility

4. CONCLUSION

In this paper a design method for steel frames equipped with BRBs is illustrated. The proposed method aims to obtain an uniform yielding of all BRBs, thus avoiding concentration of plastic deformation at some storey level. This is a critical point in the design since BRBs usually have a very low hardening in post-elastic response. An optimal distribution of column stiffness and of brace stiffness and strength is defined from the free vibration equations of a shear-deformable cantilever. Closed form solutions, useful in preliminary design, are given. Being the proposed method based on the first vibration mode, dynamic analysis results are discussed to evaluate the reduction of performance in the nonlinear range deriving from higher vibration modes. A critical situation in which beams are pinned to columns is considered to highlight the risk of strain localization. In all numerical simulations, BRBs (with ductility spanning from 5 to 15) widely exploited the plastic range and these preliminary investigations furnished satisfactory results.

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