SEISMIC RESPONSE REDUCTION OF STRUCTURES USING OPTIMUM ABSORBER PARAMETERS

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The effectiveness of the Tuned Mass Dampers (TMD) depends on the proper tuning of the characteristics of TMD to that of the structure and structures with different time period respond to same earthquake in different manner. From the literature it can be seen that no procedure has been carried out to optimize the mass ratio (μ) and the procedures adopted to optimize frequency ratio (α) and damping ratio of the TMD (γ) for the given μ, uses values of μ which may be too high for practical implementations. Hence this paper presents a procedure for optimization of parameters, μ, α and γ of TMD subjected to seismic excitation. The procedure has been validated using 20 earthquake records using the lumped- spring mass system.

Keywords: Seismic Excitation, Vibration Control, TMD, Optimization

1. INTRODUCTION:

Recent devastating earthquakes around the world have underscored the tremendous importance of understanding the way in which civil engineering structures respond during such dynamic events. Today, one of the main challenges in structural engineering is to develop innovative design concepts to protect civil structures, including their material contents and human occupants from hazards like wind and earthquakes.

The traditional approach to seismic hazard mitigation is to design structures with sufficient strength capacity and the ability to deform in a ductile manner. Alternately, newer concepts of structural control, including both passive and active control systems have been growing in acceptance and may preclude the necessity of allowing for inelastic deformations in the structural system. Of all the control devices passive control systems in the form of TMD's, base isolation and frictional dampers have been implemented in many building across the world.

Tuned Mass Dampers (TMD) comprising of mass-spring-dashpot system has been recognized as one of the attractive methods of passive vibration control device The additional mass increases the fundamental period of the structure so that it responds less dramatically to lateral seismic acceleration. TMD has been implemented in many structures. including, among others, the City Corp Center in New York, the John Hancock Building in Boston, the CN Tower TV antenna in Toronto, Canada, the Sydney Tower in Australia, the Yanbu Cement Plant in Saudi Arabia, the Bin Quasim Thermal Power Station in Pakistan, the Yaratsu Bridge and the Fukuoka Tower, both in Japan (Protective Systems Research Group 2005).

TMD can be categorized as Passive, Active and Hybrid systems based on the mode of application. A passive TMD or tuned vibration absorber is basically an energy dissipation device that in its simplest form consists of a mass that is attached to a structure (primary system) with spring and damper elements. Passive TMD can only be tuned to a single structural frequency. The feature that impresses the use of passive TMD is that, no external power or sophisticated hardware is required for its operation. The active-mass-damper (AMD), is similar to the tuned mass damper, since it also uses a mass-spring-damper system. It does, however, include an actuator that is used to position the mass at each instant, to increase the amount of damping achieved and the operational frequency range of the device. An HMD is a combination of a passive TMD and an active control actuator. The ability of this device to reduce structural responses relies mainly on the natural motion of the TMD. The forces from the control actuator are employed to increase the efficiency of the HMD. The energy and forces required to operate a typical HMD are far less than those associated with a fully Active Mass Damper (AMD) system of comparable performance. Design constraints, such as severe space limitations, can preclude the use of an HMD system. Such is the case in the AMD system.

The effectiveness of the TMD depends on the proper tuning of the characteristics of TMD to that of the structure. The modern concept of TMD for structural applications has its root in dynamic vibration absorbers studied as early as 1909 by Frahm. Parametric optimization was first carried out by Den Hartog (1956) using sinusoidal excitation. Empirical formulae have been suggested by the author for optimizing frequency ratio (α), damping ratio of the TMD (γ_d) and dynamic amplification factor (R). Subsequent to that, many researchers have developed procedures for the optimization of various parameters. For the classical single DOF main system with attached absorber, Jacquot and Hoppe (1973) optimized the main mass response with respect to damping ratio (γ_d) for specified values of damping ratio of the main structure (γ_m), mass ratio (μ) and frequency ratio (α).

Warburton (1982) has developed formulae for optimized absorber parameters for various type of excitation. The author optimized the main mass responses with respect to γ_d and α for specified values of γ_m and μ. R. Villaverede (1985) found that TMD's performed best when the first two complex modes of vibration of the combined structure and damper have approximately the same damping ratios as the average of the damping ratios of the structure and the TMD. To achieve this the TMD should be in resonance with the main structure and its damping ratio be $\xi = \beta + \phi\sqrt{\mu}$, where β is the damping ratio of the structure, μ is ratio of the mass of the absorber to the generalized mass of the structure in the given mode of vibration (usually the fundamental mode) and Φ is the amplitude of the mode shape of the TMD location. Lee (1990) has carried out the optimization of mass ratio. Optimal absorber mass ratios have been suggested to minimize the combined weight of the main system and attached absorber, for an applied Gaussian white noise base acceleration. The effective main system response is optimized with respect to γ_d, α and effective mass ratio (μ_{eff}) for specified values of γ_m and location of absorber mass. However, optimum mass ratio (μ_{opt}) takes values much larger than those considered practical hence does not appear useful in practice. The optimum parameters of TMD that result in considerable reduction in the response of structures to seismic loading was presented by Fahim Sadek, et.al (1995). The criterion used to obtain the optimum parameters is to select, for a given mass ratio, the frequency and damping ratios that would result in equal and large modal damping in the first two modes of vibrations. A genetic algorithm based integrated approach to optimize the total cost of building and TMD subjected to seismic excitation has been proposed by Anupam S. Ahlawat, et al (2004).Two examples, a three- and a nine-story building subjected to design seismic excitations have been studied. The study concludes that for 3-story example problem, building alone without any TMD is an optimal solution while for 9-story, a building with a TMD is the best design. Julio C. Miranda (2005) presents a study of TMDs using a classical modal analysis approximation to the response of a two degree-of-freedom mechanical system, formulated on the basis of its modal kinetic and strain energy. P.Lukkunaprasit and A.Wantikorkul (2001) have analytically shown that TMD can be employed to effectively reduce the hysteretic energy absorption demand for buildings in the range of 1.8 - 2.8s.

The effectiveness of the Tuned Mass Dampers (TMD) depends on the proper tuning of the characteristics of TMD to that of the structure and structures with different time period respond to same earthquake in different manner. From the literature presented above it can be seen that no procedure has been carried out to optimize the mass ratio (μ) and the procedures adopted to optimize frequency ratio (α) and damping ratio of the TMD (γ) for the given μ, uses values of μ which may be too high for practical implementations. Hence this paper describes a procedure for optimization of parameters, μ, α and γ of TMD subjected to seismic excitation. The procedure has been validated using 20 earthquake records using the lumped- spring mass system. The chapter ends with a comparison done with the previous work on optimization.

2 OPTIMIZATION PROCEDURE:

A TMD consists of a mass mounted on a structure via a spring system and a viscous damper, preferably in a location where the structure's deflections are greatest. The spring and mass are "tuned" so as to have a natural frequency close to that of the primary structure. When properly tuned, the TMD mass oscillates in the opposite direction from the primary structure. The motion of the mass relative to the main structure can be very large when the system is properly tuned and this provides an opportunity to dissipate a substantial amount of energy in the damper linking the mass to the main structure. The effectiveness of a TMD depends of three variables namely; the mass ratio μ (i.e., ratio of TMD mass to modal mass of the structure), the damping ratio γ of the TMD itself and the frequency or tuning ratio α.

2.1 Objective

The objective function of optimization is to reduce the peak response of structures with time period varying from 0.1 to 3 sec in increments of 0.01, subjected to seismic excitation. The parameters are optimized for structures damping ratio, ξ of 0%, 2% and 5%. Structures with time period ranging from 0.1 to 3 sec only are considered in this study since the prime importance is to reduce vibration in low to medium rise buildings and structures of this category fall in this range. Beyond this range the TMD may not be effective in resisting earthquake forces, since the wind forces play a major role in the design.

2.1 Parameters

To increase the effectiveness of the TMD the following parameters are optimized:

i. Mass ratio (μ) ie, ratio of mass of TMD (m) to the mass of the structure (M).Thus,

$$\mu = m/M \qquad (1)$$

for optimization, μ is varied from 0.1 to 1.5 % in increments of 0.1%.

ii. Frequency ratio (α) ie, fundamental frequency of the TMD (ω_t) to that of the structure (ω_0). Thus,

$$\alpha = \omega_t / \omega_o \qquad (2)$$

α is varied from 0.90 to 1.10 in increments of 0.01.

iii. Damping ratio of the TMD (γ) varied from 1% to 30% in increments of 1%. γ is given by,

$$\gamma = c_c / 2m\, \omega_t \qquad (3)$$

where,

c_c is the damping constant of the TMD.

2.2 Procedure

The procedure aims at optimizing the parameters of the first mode of vibration of the structure. In order to optimize the parameters of TMD, the structure was idealized as lumped spring–mass-damper, single degree of freedom system (SDOF) as shown in Figure 3.1. The mass of the SDOF was considered as the unit modal mass of the structure. The mass was assumed arbitrarily for each time period (T) of the structure and the stiffness, k_s of the system is calculated based on the time period as given below,

$$k_s = \omega_o^2 M \qquad (4)$$

where, ω_0 is the fundamental frequency of the structure which in terms of T is given as,

$$\omega_0 = 2\pi / T \qquad (5)$$

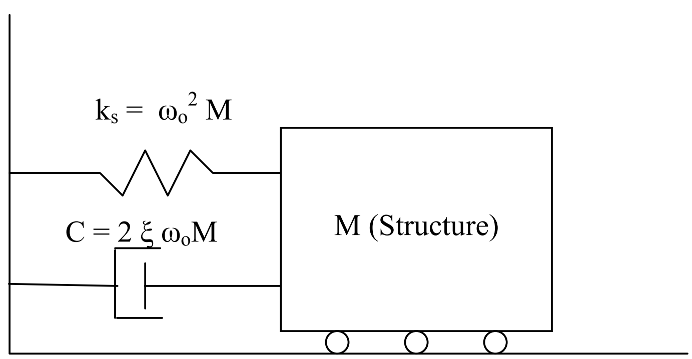

Figure 1 Lumped spring–mass–damper SDOF

The TMD was attached to the structure as spring-mass-damper system and the SDOF structure becomes 2DOF system as shown in Figure 3.2. The parameters of the TMD are found as follows,

i. Mass of TMD

$$m = \mu M \qquad (6)$$

ii. Stiffness of TMD (k_t)

$$K_t = \omega_o^2 \alpha^2 m \quad (7)$$

iii. Damping of the TMD (c_c)

$$c_c = 2 \gamma \omega_o \alpha m \quad (8)$$

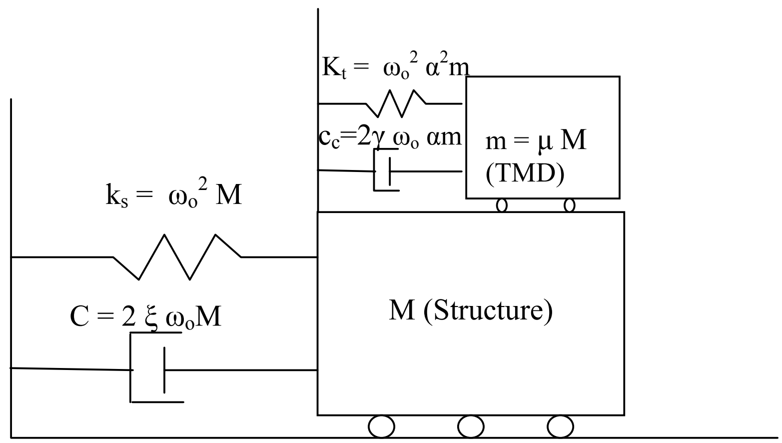

Figure 2 Tuned Mass Damper Mounted on the Structure

For given ξ and T of the structure to reduce the peak response of the structure subjected to seismic forces is the objective. To achieve this time history analysis was carried out using four earthquake records as shown in Table 1, the time history records are shown in Figure 3. Newmark's time integration technique was utilized to do time history analysis. Using LabVIEW a program was developed for the same

Table 1 Time History Records Summary

S.No	Earthquake	Name	Magnitude	PGA (g)	Duration in sec
1	Imperial Valley	e0	7.1	0.3	31.18
2	Kobe	k2	6.9	0.821	48
3	Northridge	n0	6.7	0.311	40
4	Loma Prieta	l0	6.9	0.537	39.905

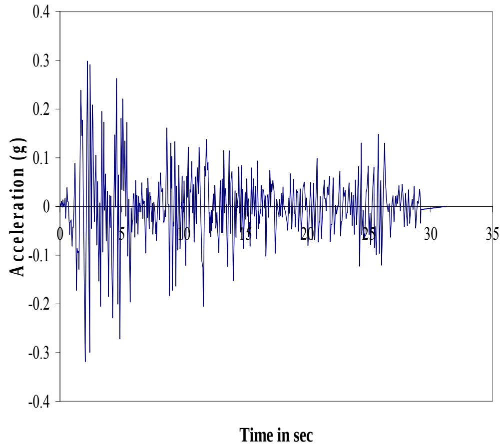

Figure 3a Elcentro

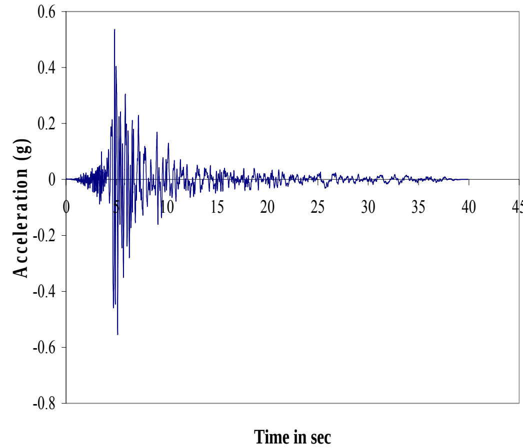

Figure 3b Loma Prieta

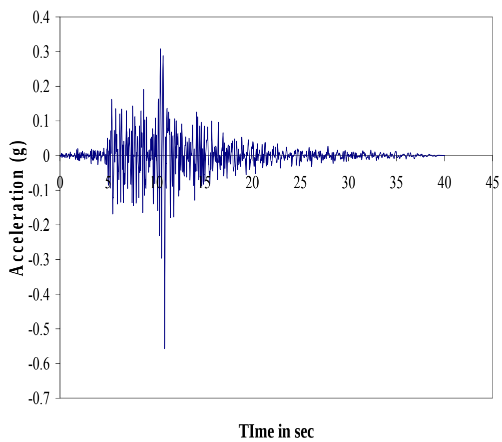

Figure 3c Northridge

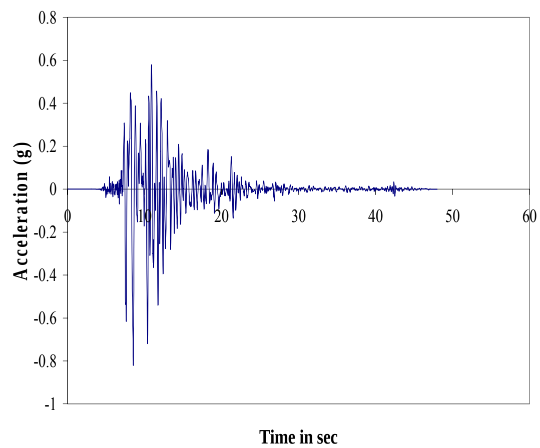

Figure 3d Kobe

Figure 3 Time History Records

To start with the analysis ξ was kept as 0%. The idealized SDOF structure., ie, without incorporating TMD in the structure, time history analysis was carried out with T of the structure varying from 0.1 to 3 in increments of 0.1. The analysis was repeated for all the four earthquake records as mentioned in Table 3.1 for each increment of T. The peak response of the SDOF system when subjected to the earthquake forces was noted.

The TMD was then attached to the structure as SDOF system. The structure now becomes 2DOF system. Time history analysis is again carried out for each T in increments of 0.1. In the analysis for each T, μ was varied from 0.1 to 1.5% in increments of 0.1%, α was varied from 0.9 to 1.1 in increments of 0.01and γ varied from 1 to 30% in increments of 1% and for each increment of μ, α and γ time history analysis was carried out using the all the four earthquake data.

The displacement-time history data was obtained for each earthquake loading and each increment of the parametric values. The parameters were then optimized by considering the quantity of reduction in displacement. The optimized values were arrived at for the parameters which provide maximum reduction in displacement. For each earthquake loading there was one optimized value. Hence there were four optimized values for μ, α and γ. The four values were then averaged to get single optimized parameter. The optimized parameters were then plotted with T on x axis and the parameter on y axis. The procedure was then repeated with ξ as 2% and 5% and the optimized parameters are obtained.

2.3. Results:

Time history analysis was carried out to optimize the parameters of TMD namely μ, α and γ using four earthquake records and the results are presented under the following two cases,

i. For ξ=0% ,2% ,5% and T varying from 0.1 to 3, μ_{opt} and α_{opt} for each increment of T, without considering damping of TMD (γ = 0%) ie., undamped TMD.

ii. For ξ=0% ,2% ,5% and T varying from 0.1 to 3, μ_{opt}, α_{opt} and $\gamma_{opt,}$ ie., damping of TMD considered for each increment of T (damped TMD).

The procedure was then repeated with ξ as 2% and 5% and the optimized parameters are obtained. The results are presented in the Figure 4 to Figure 8. Figures 4 and 5 presents optimized mass ratio and frequency ratio for undamped TMD. Figures 6,7 and 8 presents optimized values of mass ratio, frequency ratio and damping ratio of TMD for structures incorporating damping in TMD

2.4 Inferences

The following inferences are made based on the optimization results.

i. Structures with different time period requires unique value of mass ratio, frequency ratio and damping ratio of TMD.

ii. The Figure 4 presents the mass ratio for structures with time period varying from 0.1 to 3 sec with undamped TMD. It can be seen that increasing the damping in the structure increases the mass ratio except in the following regions, 1.38 to 1.4 sec, 1.79 to 2sec, 2.27 to 2.54 sec and 2.7 to 2.75sec where as the damping in structure increases the mass ratio decreases.

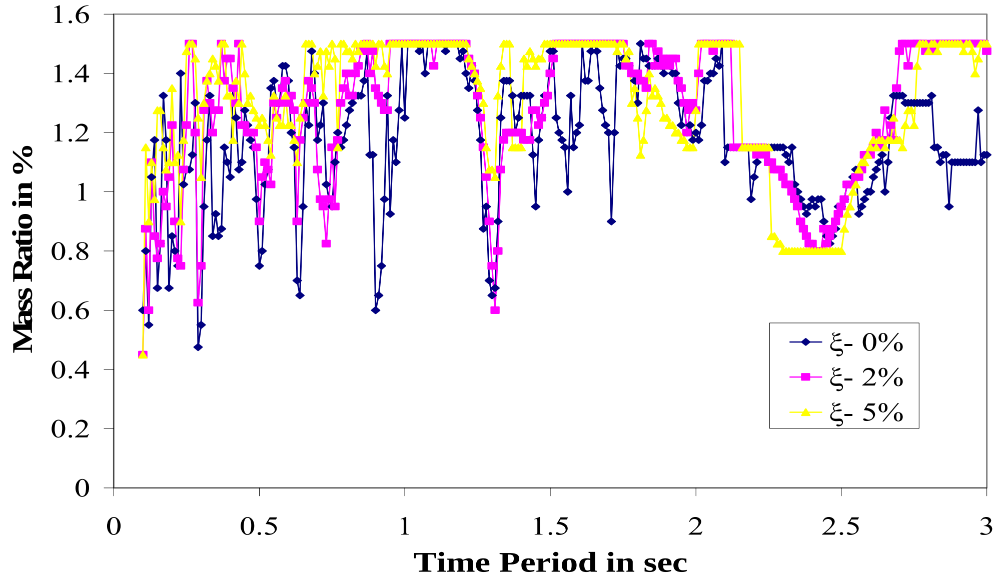

Figure 4 Optimized Values of Mass Ratio with undamped TMD

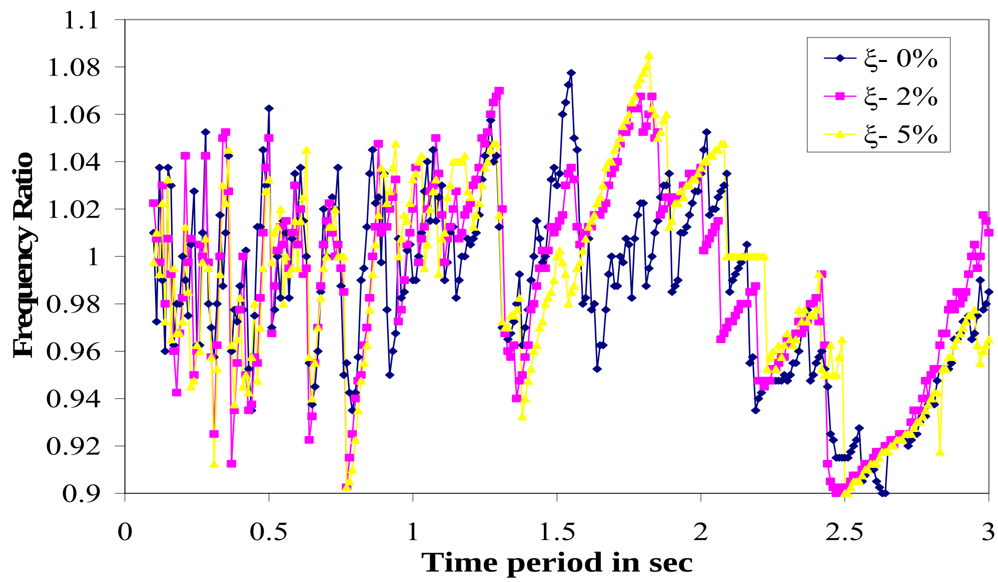

Figure 5 Optimized Values of Frequency Ratio with undamped TMD

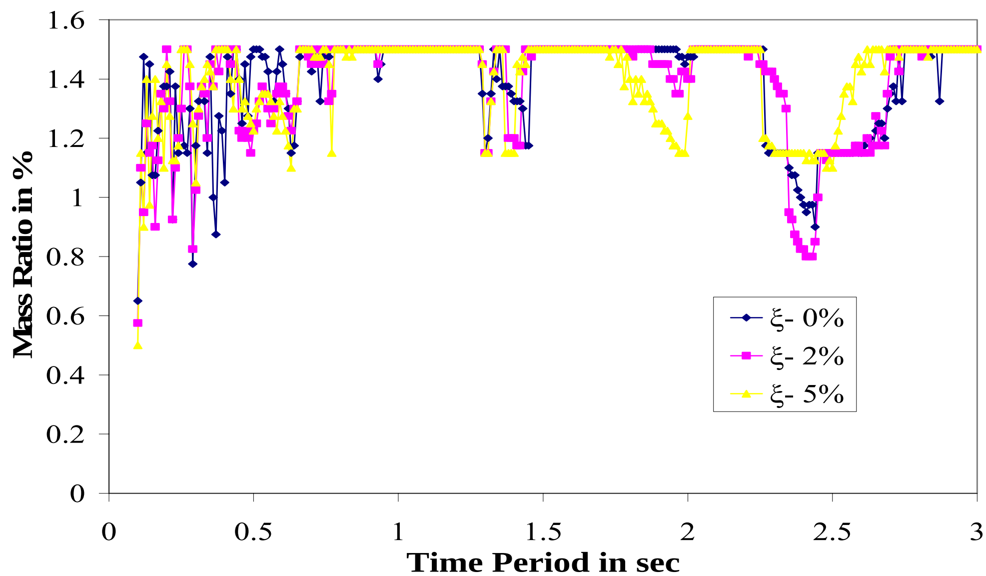

Figure 6 Optimized Values of Mass Ratio with damped TMD

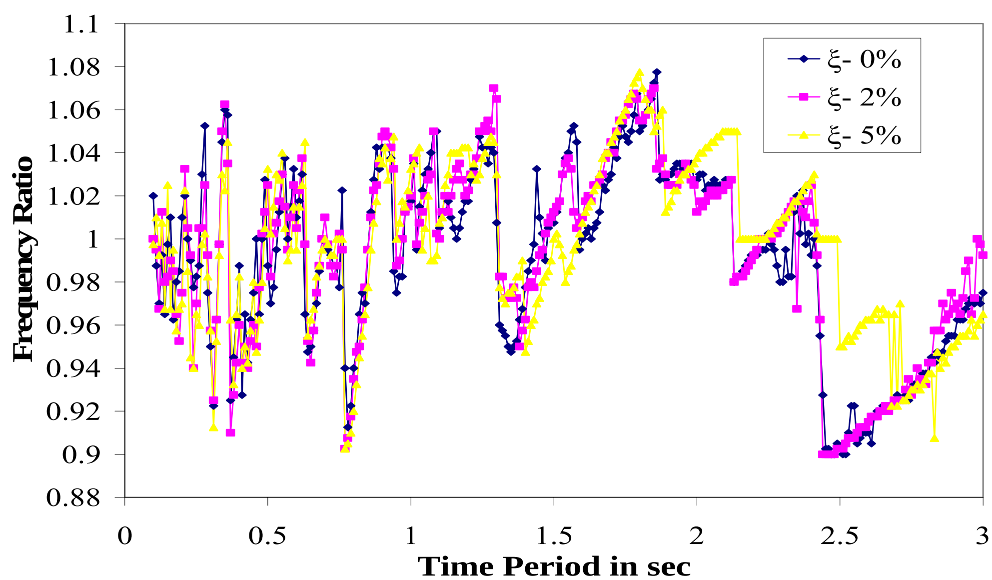

Figure 7 Optimized Values of Frequency Ratio with damped TMD

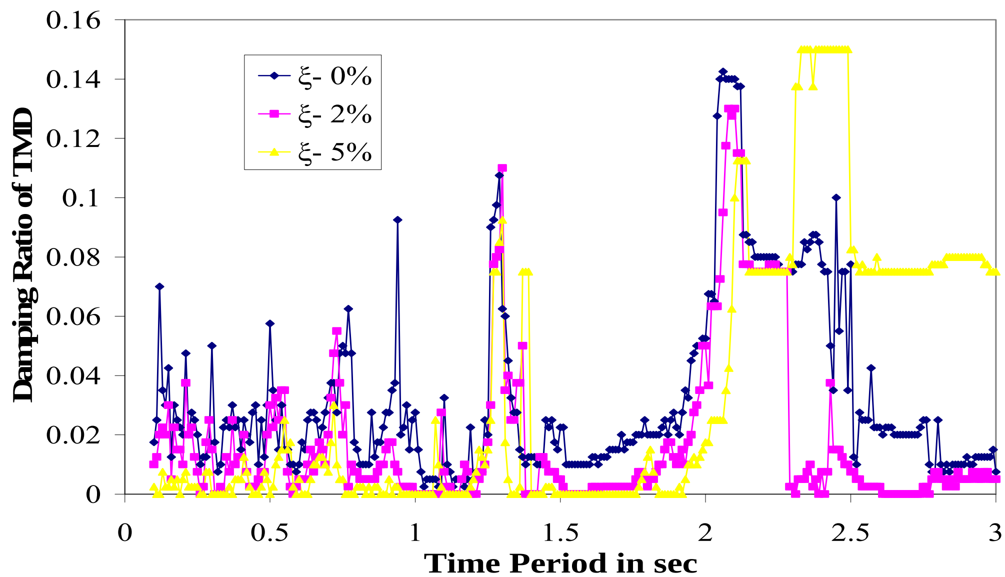

Figure 8 Optimized Values of Damping Ratio of TMD

iii. For structures with time period 0.96 to 1.21 sec the mass ratio is 1.5% for all the three ξ of structure. Hence in this region the ratio may be increased.

iv. The mass ratio of structures in the time period 1.52 to1.75sec, 2.02 to 2.15 sec and 2.77 to 3 sec shows that for ξ =2% and 5% is 1.5 % but for ξ=0% it is less than 1.5% hence for ξ =2% and 5% the mass ratio may be increased.

v. From Figure 5 it can be seen that the frequency ratios are almost near to each other all three values of ξ, except in region 1.6 to 1.86 sec where the frequency ratio for ξ=0% is less than the other two values.

vi. Mass ratio of structures with damped TMD is presented in Figure 6. It shows that structures with time period in the range 0.8 to 1.28sec, 1.45 to 1.77 sec, 2.02 to 2.25sec and 2.75 to 3 sec take a mass ratio of 1.5% immaterial of the damping of the structure. Hence in this region mass ratio may be increased.

vii. For structures with time period 1.77 to 2 sec as ξ increases mass ratio decreases, and for structures in the time period region of 2.27 to 2.72 sec the mass ratio is less for ξ=2%.

viii. Figure 8 shows that as ξ increases the damping ratio of the TMD decreases except in the region 2.29 to 3 sec where for ξ=5% the damping ratio of TMD increases

3 VALIDATION USING LUMPED MODEL

Validation of the optimized parameters is essential, because literature shows that the effectiveness of the TMD in reducing the response of the same structure under different earthquakes or of different structure during the same earthquake is significantly different.

3.1. Model

For validation of the parameters three multi-storey framed structure, namely, 3, 5 and 9 storey were modeled using the routines of ANSYS. The details of the model are shown in Table 2. The columns and beams were designed as I-sections and the slab was designed as concrete section. The material properties of the materials used are given in Table 3.

Table 2. Details of the Models

Model Name	No. of Floors	No. of Bays	Floor Height (m)	Bay Size (m)	Columns	Beam	Slab (m)
M3	3	3x3	3	3	ISLB 350 @ 495N/m	ISLB 325@ 431 N/m	0.10
M5	5	3x3	3	3	ISLB 400 @ 569N/m	ISLB 350@ 495N/m	0.12
M9	9	3x3	3	3	ISLB450 @ 653N/m	ISLB400@ 569N/m	0.15

Table 3. Properties of Materials

Material	Young's Modulus N/m2	Poisson's ratio	Density Kg/m3
Steel	2.01×10^{11}	0.3	7900
Concrete	0.35×10^{10}	0.16	2500

In ANSYS the beams and columns were modeled using 2 node Beam188 element, and the slab was modeled using 4 node Shell63 element.

The ANSYS models are shown in Figure 9.a,b,c. Modal analysis was carried out for the full model and the time period and unit modal mass was found out. Based on T and M of the structure stiffness of structure for the lumped mass system was calculated using equation 4. The values of mass (M) and stiffness (ks) for the lumped model are given in Table.4.

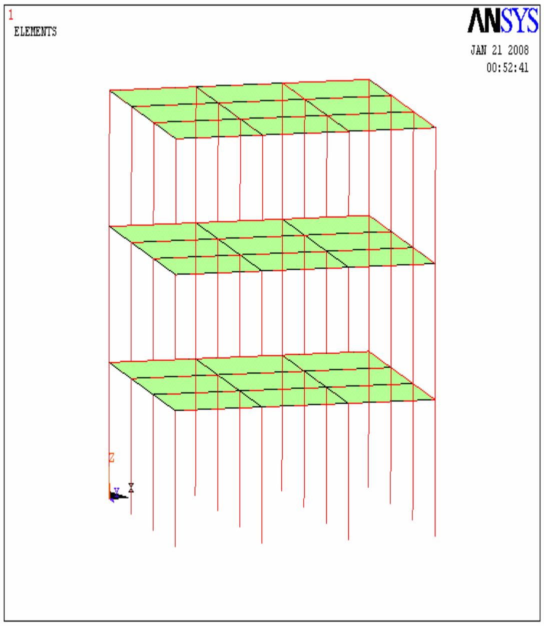

Figure 9.a. Three Storey Model M3

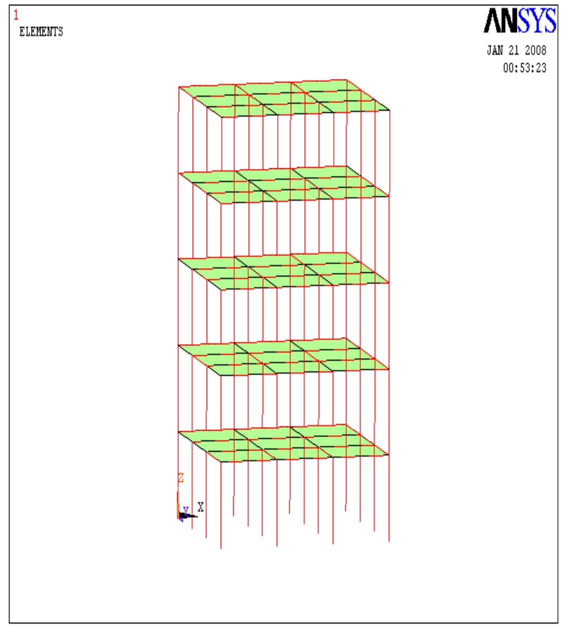

Figure 9 b. Five storey Model M5

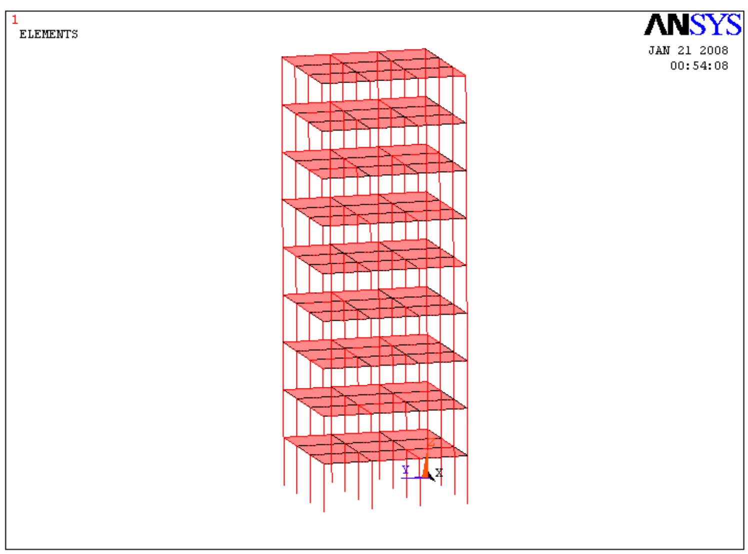

Figure 9.c. Nine Storey Model M9

Table 4: Results of Modal Analysis on Full Models

Model Name	Time Period T in sec	Effective Mass of the structure M (Kg)	Stiffness of the Structure ks (N/m)
M3	0.387	68076	17958968
M5	0.65	130892	12240401
M9	1.247	272559	6925273

The structure was then modeled as lumped single degree of freedom spring- mass system as shown Figure 10a, with mass equal to that of the unit modal mass and stiffness as given in Table 4. ξ is taken as 0%. Combin40 element is used to model the idealized structure in ANSYS.

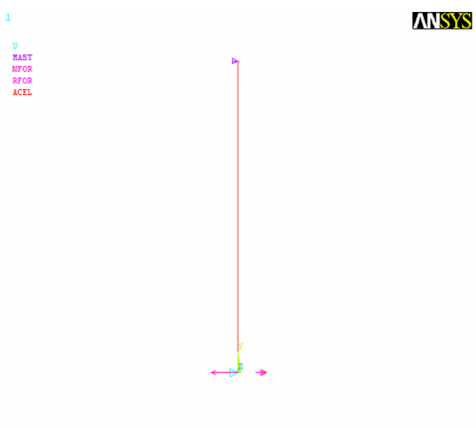

Figure 10.a. Idealized Structure

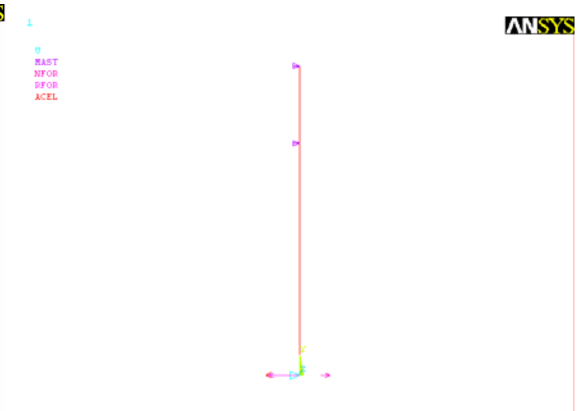

Figure 10.b. Idealized Structure with TMD

To compare the behavior of the full model and the lumped model, time history analysis was done for both the models using kobe (k2) data given in Table1. Full Transient analysis was done to carry out time history analysis and the time-displacement record was plotted. The comparison is shown in Figure 11.a,b,c. From the figures it can be seen that the deflection pattern of the lumped model is matching exactly with that of the full model with maximum variation of deflection of 20%.

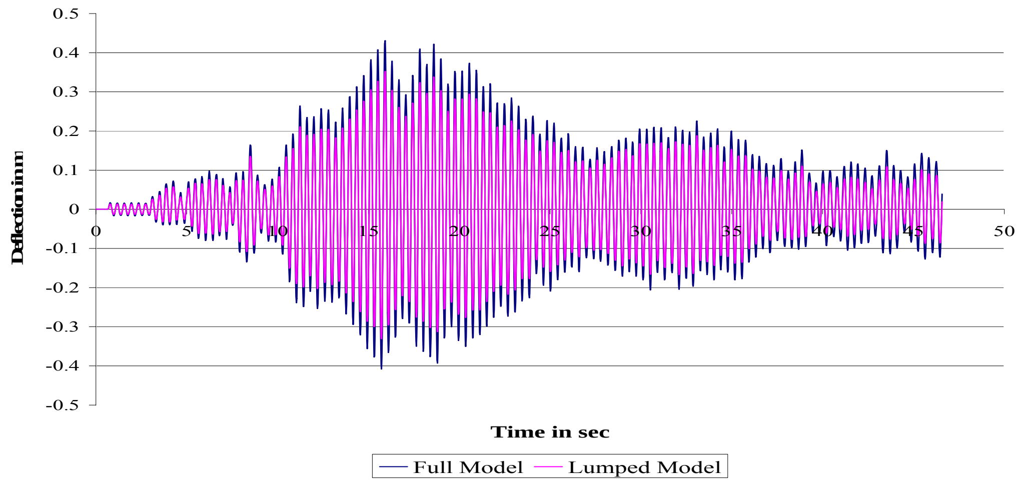

Figure 11.a. Three Storey Model M3

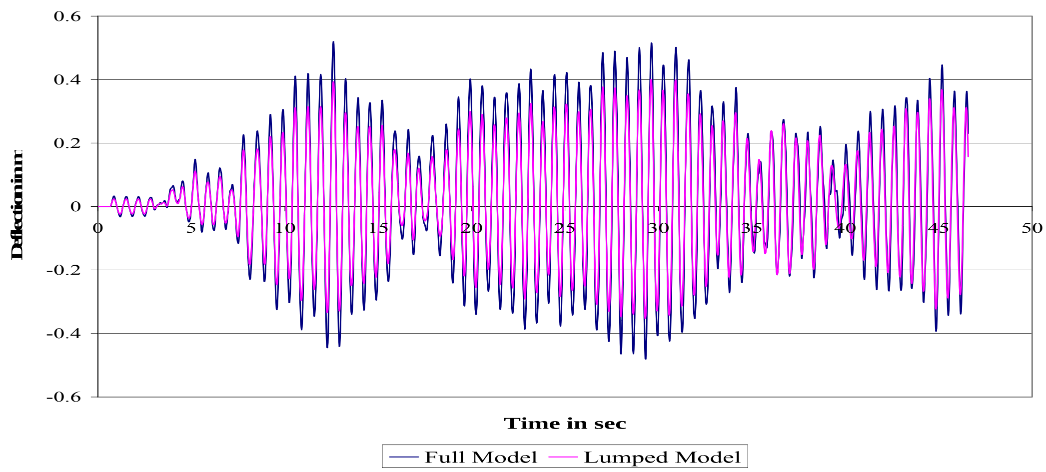

Figure 11.b. Five Storey Model M5

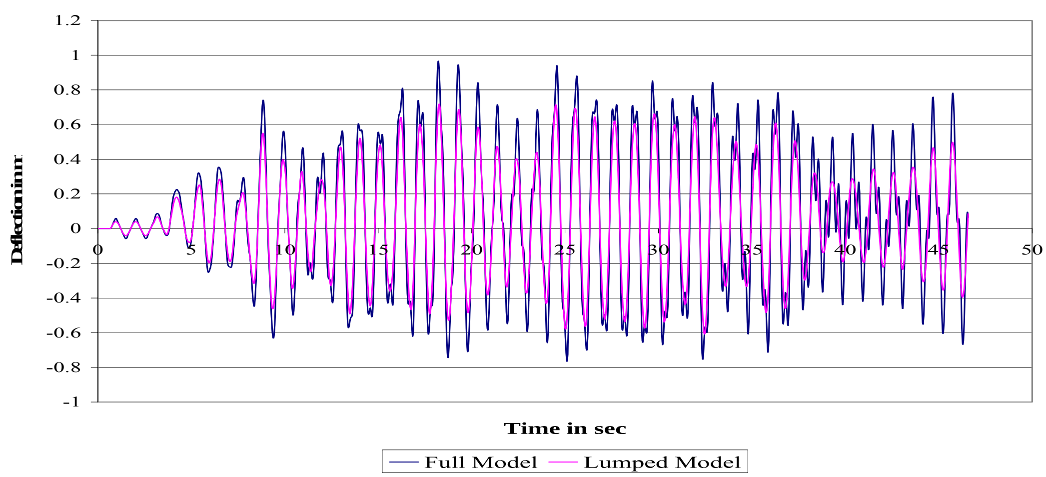

Figure 11.c. Nine Storey Model M9

Figure 11. Comparison of deflection Pattern for full and lumped models for K2 data

The TMD was then attached to the idealized model as shown in Figure 10.b. Combin40 was used to model the TMD also. Analytical investigation to validate the optimized parameters was carried under three cases namely, i) Without TMD ii) with TMD and without considering damping of TMD, ie., γ = 0% and ii) With TMD and considering damping of TMD(γ). For all the cases ξ is taken as 0%. The optimized parameters, namely μ_{opt} α_{opt} γ_{opt} for the three models are taken from Figure 4 and Figure 5. The values for the cases (ii) and case (iii) are given in Table.5.

Table 5. Optimized Parametric Values

		Without γ Case(ii)		With γ Case(iii)		
Model Name	T in sec	μ_{opt}	α_{opt}	μ_{opt}	α_{opt}	γ_{opt}
M3	0.387	1.1	0.9725	1.225	0.965	0.015
M5	0.65	0.95	0.9375	1.325	0.95	0.0275
M9	1.247	1.275	1.0425	1.5	1.0425	0.02

Twenty earthquake records obtained from various stations was used for time history analysis. The details of the records are presented in Table.6. The acceleration of the data varied from 0.07g to 2.086g. Time history analysis was carried out using the lumped model without and with TMD. The peak response of the three models for the three cases, for all the earthquake data was noted. The comparison of the peak response for the three lumped models for the three cases, namely without TMD, with TMD without γ and with TMD and γ are given in Figure 12 to Figure 14.

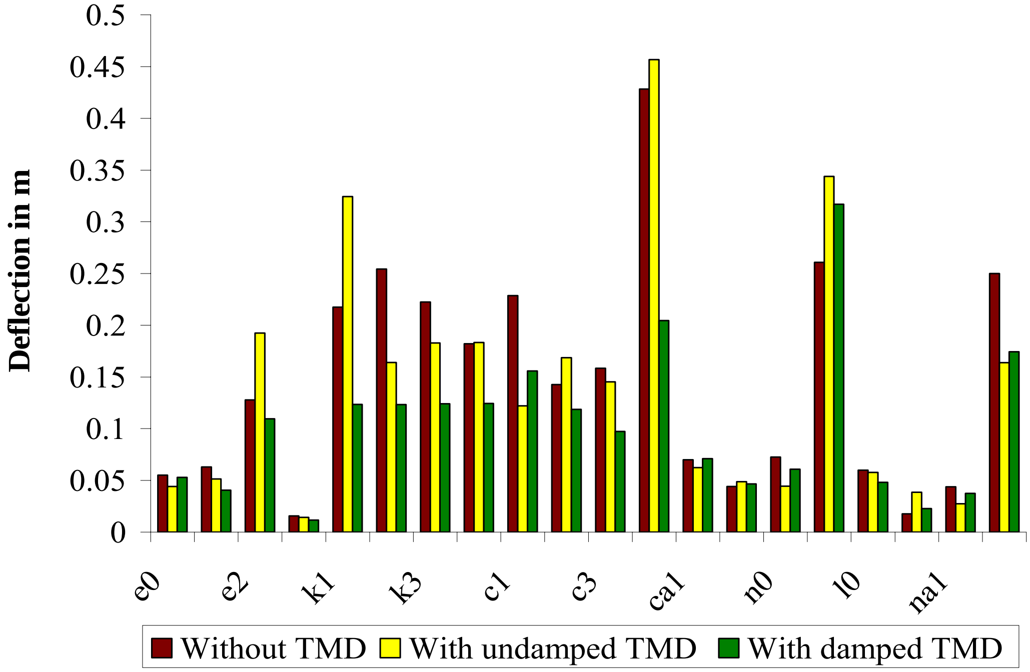

Figure 12. Comparison of Peak Response for M3

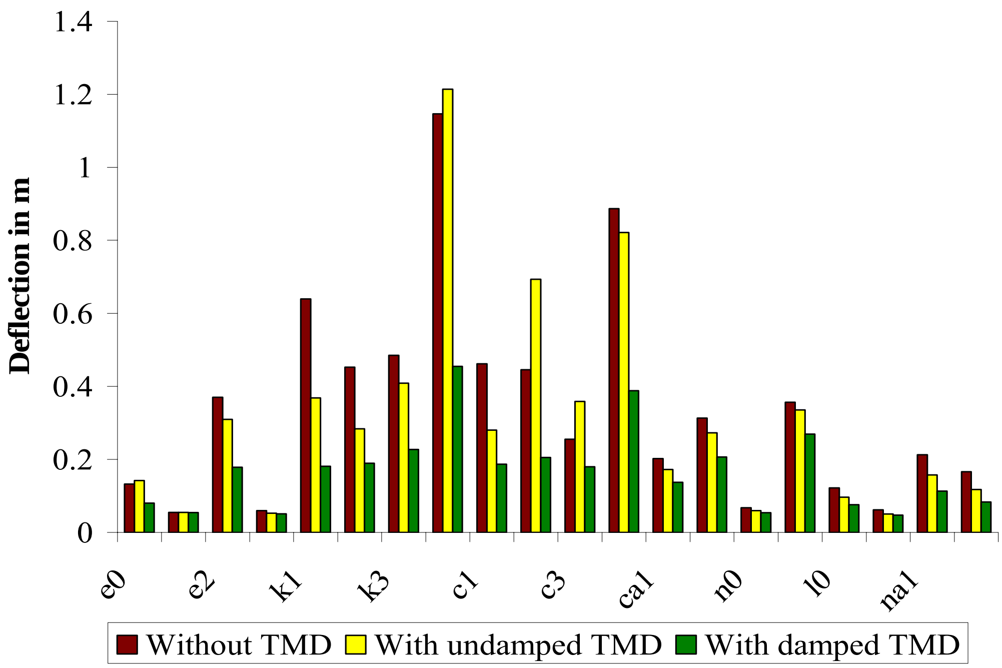

Figure 13. Comparison of Peak Response for M5

Table 6: Earthquake Records for Validation Using Lumped Model

S.No	Earthquake	Abbrev -iation	Magnitude	Date	Station	Closest to fault Rupture (km)	PGA,g	Duration in sec
1	Imperial Valley	e0	7.1	18/05/1940	Elcentro Array	Norm.	0.3	31.18
2	Imperial Valley	e1	6.5	15/10/1979	Elcentro Array	6.5	1.655	39.035
3	Imperial Valley	e2	5.2	15/10/1979	Elcentro Array	13.1	0.366	19.65
4	Imperial Valley	e3	6.5	15/10/1979	St-CoachelleCanal	49.3	0.128	28.53
5	Kobe	k1	6.9	16/01/1995	Takatori	0.3	0.616	40.96
6	Kobe	k2	6.9	16/01/1995	KJMA	0.6	0.821	48
7	Kobe	k3	6.9	16/01/1995	Kakogawa	26.4	0.345	40.96
8	Kobe	k4	6.9	16/01/1995	MZH	69.4	0.07	78
9	Chi-Chi- Taiwan	c1	7.6	20/09/1999	CHY028	7.31	0.821	90
10	Chi-Chi- Taiwan	c2	7.6	20/09/1999	CHY010	25.39	0.125	132
11	Chi-Chi- Taiwan	c3	7.6	20/09/1999	CHY032	39.34	0.088	90
12	Chi-Chi- Taiwan	c4	7.6	20/09/1999	CHY014	41.49	0.263	149
13	Cape Mendocino	ca1	7.1	25/04/1992	Cape Mendocino	8.5	1.497	30
14	Cape Mendocino	ca2	7.1	25/04/1992	Petrolia	9.5	0.662	36
15	Northridge	n0	6.7	17/01/1994		Norm	0.311	40
16	Northridge	n1	6.7	17/01/1994	Tarzana Cedar Hill	17.5	1.779	40
17	Loma Prieta	l0	6.9	18/10/1989		Norm	0.537	39.905
18	Loma Prieta	l1	6.9	18/10/1990	APEEL	47.7	0.156	39.95
19	Nahami, Canada	na1	6.8	23/12/1985	S1UP	6	2.086	20.565
20	San Fernando	s1	6.6	9/2/1971	Pacoima Dam	2.8	1.226	41.64

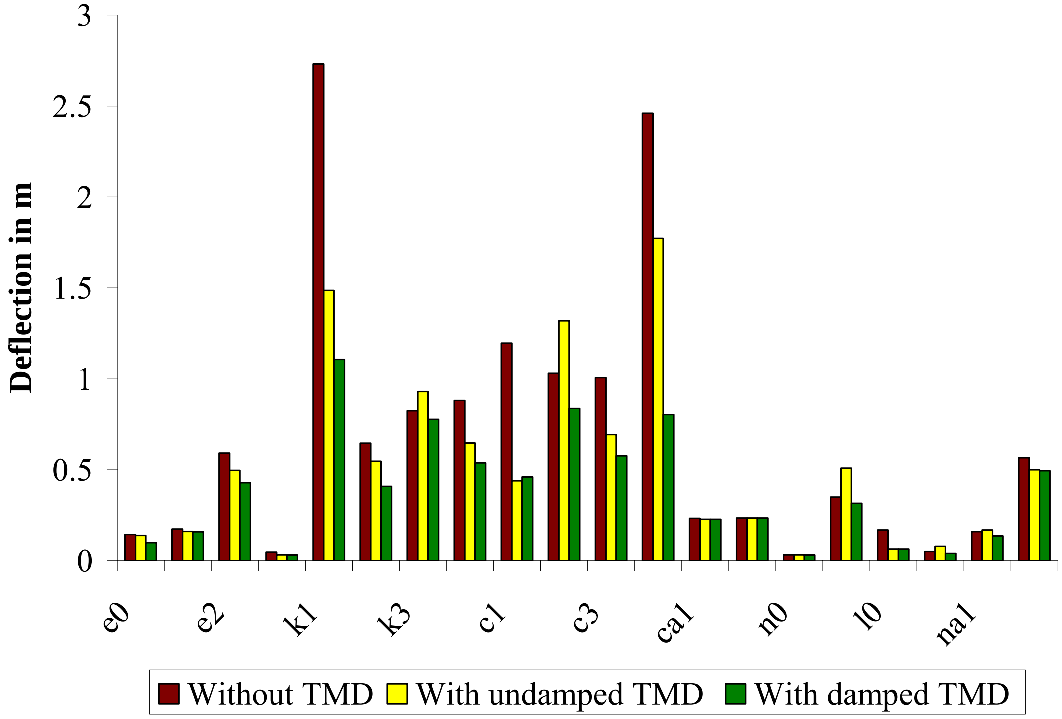

Figure 14. Comparison of Peak Response for M9

From the figures the following inferences can be made,

i. The addition of TMD to the structure reduces the peak response of the structure subjected to seismic excitation.

ii. For case (ii), the maximum reduction in displacement for M3 is 47% for c1 data, for M5 it is 42% for k1 data and for M9 the reduction is 46% for k1 data.

iii. For case(iii), the maximum reduction in displacement for M3 is 51% for k2 data, for M5 it is 71% for k1 data and for M9 the reduction is 67% for c4 data.

iv. For few earthquake data the peak response increases in all the three models for case (ii), ie, with TMD without γ, but for all earthquake data there is reduction in displacement for M5 and M9 for case (iii) and for 16 data there is reduction in response for M3 , ie., with TMD and with γ. Hence it can be seen that addition of damping in the TMD increases the reduction in the peak response.

4. CONCLUSION

A procedure of optimization of the parameters of a passive TMD and validation of the optimized parameters has been presented in the paper. The important conclusions drawn are as follows, i) From the optimum parameters derived it can be seen that for each time period of the structure the parameters are unique, ii) Increasing damping in the TMD increases the mass ratio, iii) TMD may be effective in the range 0.1 to 0.8 sec and 1.77-2.74 sec, beyond this region studied the mass ratio assumes higher values beyond practical consideration, iv) Incorporating Damping in the TMD decreases the peak response of the structures. Hence the optimized parameters may be utilized for designing TMD for better seismic performance.

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