DYNAMIC CONSTITUTIVE MODEL FOR MATERIAL OF ROCKFILL DAMS

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ABSTRACT :

The recent development in dynamic constitutive models for material of rockfill dams is reviewed and summarized. Their advantages and disadvantages are compared in describing stress-strain relations under various conditions for granular materials. The P-Z constitutive model, which is a kind of the multi-mechanism plastic model based on generalized plasticity, is mainly introduced. It doesn't have to definitely define yield surface and plastic potential surface. Meanwhile, the model can describe the dynamic and static features of sands and clay. Based on the available tests, the determination of the parameters for P-Z model is also given. Then, the shaking table test is carried.

KEYWORDS: dynamic constitutive relation, generalized plastic mechanics, P-Z model, shaking table test

1. INTRODUCTION

The dynamic constitutive relation is the basic relation of describing the dynamic character of soil. Presently, the built dynamic constitutive models can be divided to viscid-elastic model and elastic-plastic model.

In dynamic response analysis, viscid-elastic model is the mainstream theory, which contains two common model, equivalent linear model and Mansing nonlinear model[KONG Liang, WANG Yan-chang, ZHENG Ying-ren,2001]. Although it is formally simple and can calculate the pore water pressure and average development progress of permanent deformation under cyclic loading, it can't consider strain softening, influence of stress path and anisotropy of soil.

Since 1970's,there are extensive studies on elastic-plastic model about saturation soil under dynamic loading. ①Building model under monotonic loading and using relatively complicated hardening law, such as the model based on modifier Cambridge model by Carter[Carter J P, Booker J R, Wrothu C P,1982], the Desai model with single yield surface built in 1984[Desai C S, Gallagher R H,1984].②The dynamic model based on other types of plasticity theory, such as multi-surface model built by Provest, Mroz, Norris and ienkiewicz[Mroz Z, Norris V A , Zienkiewicz O C,1981],[Provest J H,1985], secondary loading surface model built by Hashiguchi[Hashiguchi K,1993], the plasticity model of sand based on multi-mechanism conception under cyclic loading by Kabilamany, Pastor and Zienkiewicz[Iai S, Matsunaga Y, Kaneoka T,1992],[Paster M, Zienkiewicz O C, Chan A H C,1990].

In the paper, we will emphasize on the P-Z model which is based on generalized plasticity theory built by Pastor and Zienkiewicz[M.Pastor and O.C.Zienkiewicz,1986],[O.C, Zienkiewicz and Mroz,1984],[O.C, Pastor and Zienkiewicz[M.Pastor and O.C.Zienkiewicz,1986],[O.C, Zienkiewicz and Z.Mroz,1984],[O.C, Zienkiewicz, K.H.Leung and M.Pastor,1985], meanwhile the shaking table test is done. The comparative analysis between computation and observation results is done.

2. P-Z CONSTITUTIVE MODEL

2.1 Generalized Plasticity Theory

Incremental non-linear elastic-plastic relations between increments of strain and co-rotational stress can be specified as:

$$d\hat{\sigma}_i = D : d\varepsilon \tag{2.1}$$

Where $d\sigma, d\varepsilon$ and D are Cartesian tensors.

Inverse relations can be written in a similar form as

$$d\varepsilon = C : d\sigma \tag{2.2}$$

In the above expressions, tensors D and C will generally depend on the current state variables and on the direction of loading.

Usually, a direction vector n in the stress space discriminating between loading and unloading is introduced. This of course defines a set of surfaces which are equivalent to those used in classical plasticity, but these surfaces need never be explicitly defined.

$$\begin{cases} d\sigma^T n > 0 & \text{loading} \\ d\sigma^T n = 0 & \text{Neutral loading} \\ d\sigma^T n < 0 & \text{Unloading} \end{cases} \tag{2.3}$$

Incremental stress-strain relations for a single mechanism thus have the form

$$\begin{cases} d\sigma = D_L d\varepsilon & \text{Loading} \\ d\sigma = D_U d\varepsilon & \text{Unloading} \end{cases} \tag{2.4}$$

Continuity between loading and unloading processes requires that constitutive tensors D_L^{-1} and D_U^{-1} are of the form

$$D_L^{-1} = D_e^{-1} + \frac{n_{gL} n^T}{H_L}, D_U^{-1} = D_e^{-1} + \frac{n_{gU} n^T}{H_U} \tag{2.5}$$

Where $n_{gL/U}$ are arbitrary unit tensors and $H_{L/U}$ are plastic moduli corresponding to loading and unloading.

The total increment of strain can be assumed to have two components, elastic and plastic,

$$d\varepsilon = d\varepsilon_e + d\varepsilon_p \tag{2.6}$$

Where
$$d\varepsilon_e = D_e^{-1} d\sigma \tag{2.7}$$

$$\begin{cases} d\varepsilon_p = \dfrac{n_{gL}(n^T d\sigma)}{H_L} & \text{Loading} \\ d\varepsilon_p = \dfrac{n_{gU}(n^T d\sigma)}{H_U} & \text{Unloading} \end{cases} \tag{2.8}$$

Then the equation (2.1) is converted as：

$$\begin{cases} d\sigma = D_{Lep} d\varepsilon & \text{Loading} \\ d\sigma = D_{Uep} d\varepsilon & \text{Unloading} \end{cases} \tag{2.9}$$

Through the conversion， D_{ep} is given：

$$D_{Lep} = D_e - \frac{D_e n_{gL} n^T D_e}{H_L + n_{gL} D_e n}, D_{Uep} = D_e - \frac{D_e n_{gL} n^T D_e}{H_U + n_{gU} D_e n} \tag{2.10}$$

Values of $n_{gL/U}$ can be defined directly in any manner convenient to model the problem, if these values differ from n then the flow is non-associative and will lead to non-symmetric tangent matrices; if these values is equivalent to n then the flow is associative and will lead to symmetric tangent matrices. The integrated elastic-plastic characteristic is composed of n、$n_{gL/U}$、$H_{L/U}$ and D_e in stress space。Thus，there are not yield and plastic potential surfaces in elastic-plastic relations.

2.2 P-Z Constitutive Model

We assume that deformation of the material can be considered as the result of deformations produced by M separate mechanisms, all of these subjected to the same state of stress. The strain increment can thus be written as

$$d\varepsilon = \sum_{m=1}^{M} C^{(m)} : d\sigma \tag{2.11}$$

From which
$$C = \sum_{m=1}^{M} C^{(m)} \tag{2.12}$$

total increment of strain is given by the summation of all mechanisms, i.e.

$$d\varepsilon = \sum_{m=1}^{M} C^{e(m)} : d\sigma + \sum_{m=1}^{M} \frac{1}{H_{L/U}^{(m)}} [n_{gL/U}^{(m)} \otimes n^{(m)}] : d\sigma \tag{2.13}$$

Or
$$d\varepsilon = C^e : d\sigma + \sum_{m=1}^{M} \frac{1}{H_{L/U}^{(m)}} [n_{gL/U}^{(m)} \otimes n^{(m)}] : d\sigma$$
$$d\varepsilon = C_{L/U}^{e/p} : d\sigma \tag{2.14}$$

$C^{e/p}$ is the inversion of $D^{e/p}$。

2.2.1 Plastic potential direction vectors under loading and unloading

Following constant q/p' test results obtained by Balasubramanian and Chaudhry, soil dilatancy d can be assumed to be a linear function of q/p', given as

$$d_g = \frac{d\varepsilon_v^p}{d\varepsilon_s^p} = (1+\alpha_g)(M_g - \eta) \tag{2.15}$$

Where α_g is a constant, M_g is the critical state line slope $\ln(p', q)$ space, and $\eta = q/p'$ is the stress ratio. When loading, direction n_{gL} can be determined in the (p, q) space

$$n_{gL}^p = d_g = (1+\alpha_g)(M_g - \eta), n_{gL}^q = 1 \tag{2.16}$$

It is of course possible to define n_{gL} by specifying the shape of the potential surfaces G .such surfaces can be described by the following expression

$$G = q - M_g p\left(1 + \frac{1}{\alpha_g}\right)\left[1 - \left(\frac{p}{p_g}\right)^{\alpha_g}\right] \tag{2.17}$$

In above p_g is a 'size' parameter which has no influence on n_{gL}.

In the triaxial space, Plastic potential flow direction

$$n_{gL} = \left(n_{gL}^p, n_{gL}^q, n_{gL}^\theta\right)^T \tag{2.18}$$

where
$$n_{gL}^p = (1+\alpha_g)(M_g - \eta), n_{gL}^q = 1, n_{gL}^\theta = \frac{\partial G}{\partial \theta} = -\frac{1}{2} q M_g \cos 3\theta \tag{2.19}$$

When unloading, direction n_{gU}

$$n_{gU} = \left(n_{gU}^p, n_{gU}^q, n_{gU}^\theta\right) \tag{2.20}$$

where
$$n_{gU}^p = -\left|n_{gL}^p\right|, n_{gU}^q = n_{gL}^q, n_{gU}^\theta = n_{gL}^\theta \tag{2.21}$$

2.2.2 Loading direction vector

Loading direction vector n has the same style as $n_{gL/U}$

$$n = \left(n^p, n^q, n^\theta\right)^T \tag{2.22}$$

where $$n^p = \left(1+\alpha_f\right)\left(M_f - \eta\right), n^q = 1, n^\theta = -\frac{1}{2} q M_f \cos 3\theta \tag{2.23}$$

Loading direction vector n isn't based on yield surface, however, the yield surface can be derived from the integration of equation(2.5).

$$F = q - M_f p \left(1 + \frac{1}{\alpha_f}\right)\left[1 - \left(\frac{p}{p_f}\right)^{\alpha_f}\right] \tag{2.24}$$

In above p_f is a 'size' parameter which has no influence on n.

2.2.3 The plastic moduli under loading and unloading

The plastic modulus H_L under loading is written as

$$H_L = H_0 p \left(1 - \frac{\eta}{\eta_f}\right)^4 \left(H_v + H_s\right)\left(\frac{\eta}{\eta_{\max}}\right)^{-\gamma_{DM}} \tag{2.25}$$

where $$\eta_f = \left(1 + \frac{1}{\alpha_f}\right) M_f, H_v = \left(1 - \frac{\eta}{M_g}\right), H_s = \beta_0 \beta_1 \exp(-\beta_0 \xi) \tag{2.26}$$

ξ is the accumulated deviatoric plastic strain

$$\xi = \int \left| d\varepsilon_s \right| \tag{2.27}$$

The plastic modulus H_U under unloading is written as

$$H_U = H_{u0} \left(\frac{\eta_u}{M_g}\right)^{-\gamma_u} \tag{2.28}$$

Where H_{u0}、γ_u are material parameters, η_u is called the unloading stress ratio.

2.2.4 Elastic modulus

About isotropic materials, elastic matrix is defined by bulk modulus K and shear modulus G. K and G can be assumed to be a linear function of average stress p

$$K = K_{evo}\,p, G = K_{eso}\,p \qquad (2.29)$$

Where K_{evo}, K_{eso} are parameters measuring elastic modulus.

Therefore, P-Z model doesn't definitely define yield surface and plastic potential surface. It contains twelve parameters, $M_g, M_f, a_g, a_f, \beta_0, \beta_1, H_0, \gamma_{DM}, H_{u0}, \gamma_u, K_{evo}, G_{eso}$ .These parameters can be defined by triaxial test. The values of these parameters which are unavailable in test in some case can be derived experientially.

3. SHAKING TABLE TEST AND THENUMERICAL SIMULATION

In the chapter, the shaking table test is carried out. The author wanted to use finite element program GEHOMadrid to analyse the dynamic character of soil based on P-Z model and Duncan-Chang model before. Because of time limit, Duncan-Chang(E-B) model is applied to analyse the dynamic character of soil lonely. Then the comparative analysis between calculation and test results is done. The test was carried out by using the shaking table in HoHai University, the main parameters of which is given as, The size: 2.0m×2.8m, Ultimate bearing capacity: 6t, Acceleration: horizontal 1.2g,vertical 0.8g

3.1 Arrangement Scheme Of Sensors

The figures of model box and arrangement scheme of sensors are given in figure 1 and figure 2. The mode box is 3.0m long,0.5m wide and 1.5m high. The soil is trapezoid, the size of which is 0.5×3.0×0.7m. There are seven acceleration sensors.

Fig 1 Model box on shaking table

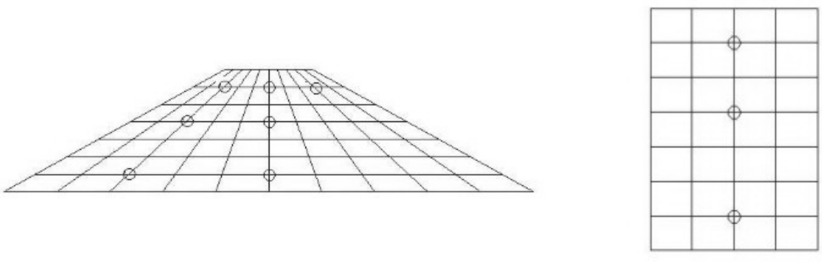


Fig 2 Arrangement scheme of acceleration sensors

3.2 The Loading Mode

Table 3.1 the loading mode of test

Loading mode	1	2	3
Seismic wave(g)	White noise(x0.1,z0.1)	Standard wave(x0.2,z0.14)	Sichuan wave(x0.2,z0.14)

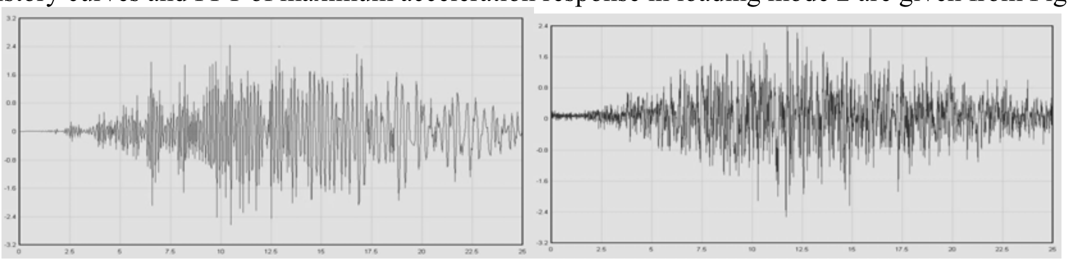
3.3 The Amplification Coefficient Of Acceleration

Table 3.2 Acceleration amplification coefficients between the calculation and test

Loading mode	Calculation results		Test results		Relative error(%)	
	X	Z	X	Z	X	Z
2	1.22	1.05	1.20	1.02	1.7	2.9
3	1.02	0.92	0.99	0.89	3.0	3.4

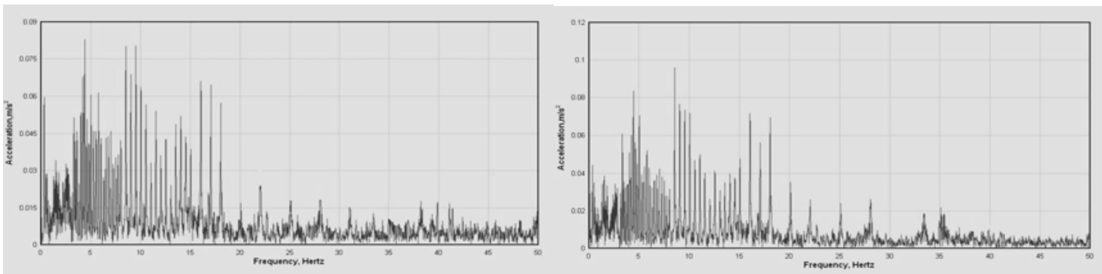
3.4 The Time-history Curve Of Acceleration Response

The time-history curves and FFT of maximum acceleration response in loading mode 2 are given from Fig 3 to 6.



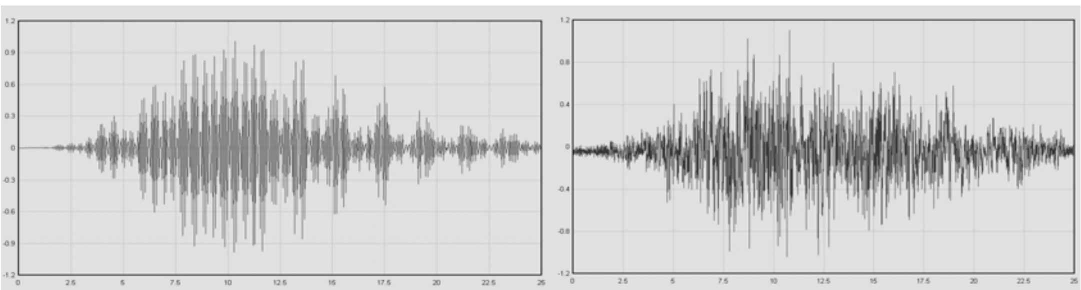
(a) calculation results (b) test results

Fig 3 maximum x-acceleration history in loading mode 2



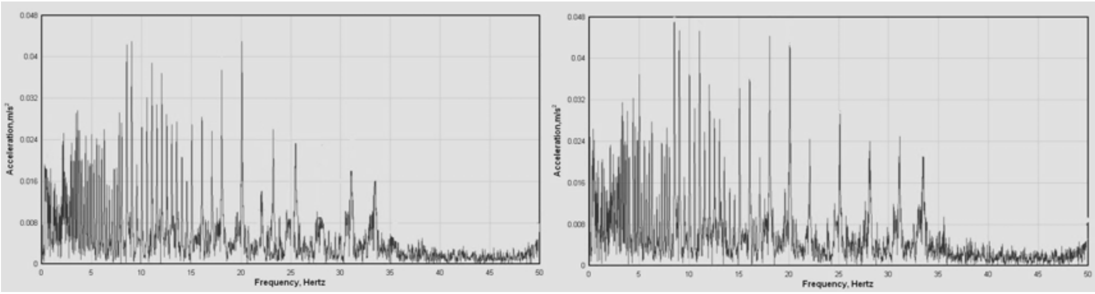
(a) calculation results (b) test results

Fig.4 maximum x-acceleration FFT in loading mode 2



(a) calculation results (b) test results

Fig.5 maximum z-acceleration FFT in loading mode 2



(a) calculation results (b) test results

Fig.6 maximum z-acceleration FFT in loading mode 2

From the figures, it is shown that the calculation results and test results which contain amplitude, waveform and frequency of the wave are approximately identical.

4. CONCLUSION

The paper mainly introduces the P-Z constitutive model, which is a kind of the multi-mechanism plastic model based on generalized plasticity. Meanwhile, The shaking table test of soil and the dynamic analysis of soil using the Duncan-Chang model are carried out. Because of time limit, the analysis based on P-Z model is not done. However, introducing the P-Z model in analyzing soil is a helpful attempt and from the introduction before, we know the P-Z model is simple. The author will lay strong emphasis on the analysis of soil based on P-Z model. The comparative analysis between calculation and test is also to be carried out in next stage work.

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