A SIMPLE MODEL FOR EVALUATING THE STRUCTURAL RESPONSE OF MASONRY STRUCTURES STRENGTHENED WITH FRP

E. Grande[1] , M. Imbimbo[2], E. Sacco[3]

[1] *PhD, Dept. of Mechanics, Structures and Environment , University of Cassino, Cassino. Italy*
[2] *Associate Professor, Dept. of Mechanics, Structures and Environment , University of Cassino, Cassino. Italy*
[3] *Professor, Dept. of Mechanics, Structures and Environment , University of Cassino, Cassino. Italy*
mimbimbo@unicas.it

ABSTRACT :

The paper presents some numerical applications regarding the nonlinear analysis of masonry elements strengthened with fiber reinforced plastic (FRP) materials. In particular, for the modeling approach of the examined structures, a special finite element based on the Timoshenko's theory considering a particular interpolation of the dependent variables has been proposed by the authors. The model includes special features such as interfaces and rigid offsets in order to account some peculiarities characterizing the behavior of masonry structures.
The paper is organized into two main parts. In the first part the proposed model is described whilst in the second part some numerical applications are discussed.

KEYWORDS: Masonry, strengthening, FRP, FE analysis

1. INTRODUCTION

One of the main drawback in the use of the pushover analysis method for evaluating the safety level of masonry structures refers to the modeling approach. In fact, even if the finite element method with the use of nonlinear constitutive laws and special yield domains provides a good estimation of the nonlinear response of masonry structures (Luciano and Sacco, 1997; Lourenço, 1998; Marfia and Sacco, 2005; Alfano and Sacco, 2006), it also presents some drawbacks (Bazant, 1998) in terms of computational effort and computational problems (localization stabilities, spurious mesh sensitivity, etc.). These aspects become much more evident for masonry structures strengthened with FRP (fibre reinforced plastic) materials (Milani et al., 2006; Grande et al., 2008) because, in this case, the interaction between the strengthening and the masonry support represents a further aspect to take into account in the modeling process.
In the last years several authors have proposed simple models for evaluating the nonlinear response of unstrengthened masonry structures (Como and Grimaldi, 1983; 1987; Braga et al., 1997 Gambarotta and Lagomarsino, 1996; Roca, 2005). In particular, the method based on the equivalent frame model approach (Magenes et al., 2000) represents an attractive tool for structural engineers because it proposes a procedure similar to the approach adopted for framed structures.
Aim of this paper is the development of a simple model based on the equivalent frame approach able to reproduce and, hence, predict the response of masonry structures strengthened with FRP materials. In particular, starting from the work of Marfia et al. (2007), a new finite element has been defined in order to model masonry structures strengthened by FRP.

2. FINITE ELEMENT

The proposed finite element is characterized by three main components (see Figure 1):
- a three-node beam element located at the center of the FE element (CDE);
- two rigid elements located at the ends (AB, FG);
- two interface elements with zero thickness which connect the beam element and the rigid elements (BC, EF).

The beam element is based on the Timoshenko's theory for accounting both the bending and the shear deformations. Moreover, in order to overcome the locking problem which affects the classical finite elements (Marfia et al., 2007), the element has been assumed composed of three joints characterized by a different number and type of the kinematics variables (see Figure 1):
- the joints at the ends are characterized by three variables: the axial displacement w, the transversal displacement v and the rotation φ;
- the central joint presents the same variables of the other joints plus the slope θ_D of this node.

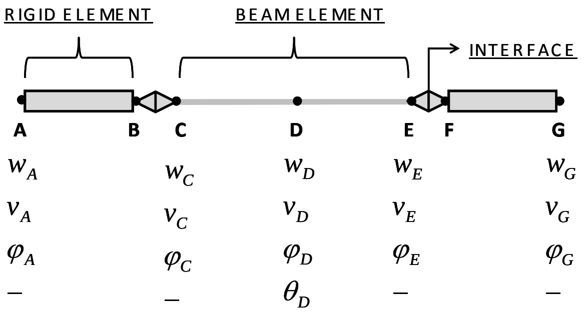


Figure 1 Proposed finite element

On the basis of the chosen variables, different shape functions have been selected in order to approximate the displacement field. In particular, while quadratic functions have been used for approximating w, v, φ, cubic function has been used for approximating θ:

$$\begin{aligned} w &= N_C^w w_C + N_E^w w_E + N_D^w w_D \\ v &= N_C^v v_C + N_E^v v_E + N_D^v v_D + N_D^\theta \theta_D \\ \varphi &= N_C^\varphi \varphi_C + N_E^\varphi \varphi_E + N_D^\varphi \varphi_D \end{aligned} \tag{2.1}$$

where:

$$\begin{aligned} N_C^v &= N_C^w = N_C^\varphi (\xi - 1) \cdot 0.5 \cdot \xi \\ N_E^v &= N_E^w = N_E^\varphi = (\xi + 1) \cdot 0.5 \cdot \xi \\ N_D^v &= N_D^w = N_D^\varphi = 1 - \xi^2 \\ N_D^\theta &= (\xi^2 - 1) \cdot \xi \end{aligned} \tag{2.2}$$

The cross section characterizing the beam element is composed of two parts (see Figure 2): the masonry and the FRP. A perfect adhesion between the two parts has been assumed considering two different constitutive laws for the two materials:

- masonry: no-tensile strength and elastic-plastic behavior in compression;
- FRP: no-compressive strength and linear-elastic behavior in tension.

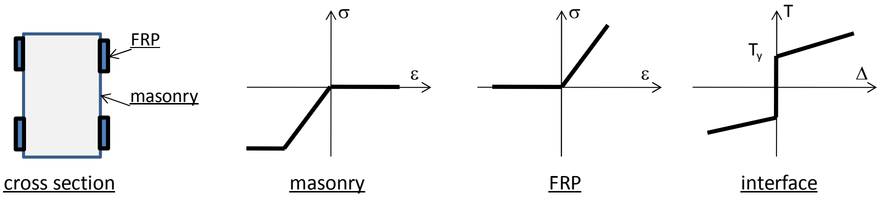


Figure 2 Constitutive laws

The node at the ends of the beam element are connected to the node at the ends of the element through the interfaces and the rigid offsets.
The interfaces elements are characterized by a rigid behavior for the DOFs w and φ whilst presents a rigid-plastic behavior for the transversal displacement v. In particular, the activation of the plastic behavior depends on the shear strength T_y of the cross-section:

$$\begin{cases} T \le T_y \rightarrow \Delta = o \\ T > T_y \rightarrow \Delta = \dfrac{T - T_y}{H_v} \end{cases} \tag{2.3}$$

where Δ is the relative displacement between the edge nodes of the interface ($\Delta = v_B - v_C$; $\Delta = v_E - v_F$) and H_v is the slope of the plastic branch.
The finite element (beam element + interfaces + rigid offsets) has been implemented in the program MatLab (2001) considering only five nodes (ACDEG; see Figure 1) because the displacements of the nodes B and F which connect the interfaces to the rigid offsets depend on the displacements of the external nodes A and G of the rigid offsets:

$$\begin{cases} w_B = w_A \\ v_B = v_A - a\varphi_A \\ \varphi_B = \varphi_A \\ w_F = w_G \\ v_F = v_G - b\varphi_G \\ \varphi_F = \varphi_G \end{cases} \tag{2.4}$$

where a and b are the lengths of the rigid offsets. The program developed in MatLab allows the assemblages of more than one element also considering inclined configurations. Conditions about the maximum strain of both masonry and FRP are also introduced in order to account of the failure modes due to crash of masonry or rupture or debonding of FRP.

3. STUDY CASES

The proposed model has been used with reference to three study cases in order to check the capability of the model to reproduce the response of masonry elements strengthened with FRP and for evaluating the effectiveness of FRP elements to improve the nonlinear response of masonry elements. It is important to underline that, in this work, the activation of the interfaces has been avoid for all the examined cases.

3.1. Study case 1: cantilever beam

The first study case is a cantilever masonry element subjected to both a constant axial force N and an incremental bending moment M. The axial force value, the geometrical characteristics and the mechanical properties of the material are summarized in Table 1.

In Figure 3 the moment-rotation curves of the cantilever beam considering both the cases of the un-strengthened (curve in black color) and FRP-strengthened section (curve in grey color) are shown. In the same graph, the solutions obtained by solving the equilibrium and compatibility equations for an assigned position of the neutral axis y_n and a specific value of the maximum strain ε_{max}, have been also reported (triangular symbols). From the plot it is possible to observe that the cracking of the cross section (y_n=300 mm) occurs at the same load level whilst the attainment of the yield strain ($\varepsilon_{max}=\varepsilon_y$) is influenced by the presence of the FRP. These effects depend respectively on the fact that the FRP works only in tension and provides an additional contribution to the resistant mechanism of the masonry, when it begins to be active.

The accuracy of the implemented model has been checked comparing the results obtained by matlab with the corresponding ones deduced from the solution of the equilibrium and compatibility equations.

3.2. Study case 2: masonry panels

The second study case refers to two masonry panels deduced from the literature (see Figure 4). In particular, the first panel, unstrengthened, (see Figure 4.a) was examined by Fantoni (1981) and the second one, strengthened by both horizontal and vertical FRP strips, was examined by Marcari (2005) (Figure 4.b). Both the panels were tested considering a constant vertical load and an incremental horizontal force applied to the upper part of the panel.

In Figure 5 the numerical force-displacement curves obtained using the developed model are compared with the experimental data.

Table 1 Characteristics of the cantilever beam

axial force	N [N]	1.5E6
beam length	L [mm]	1000
cross section wide	t [mm]	300
cross-section deep	H [mm]	600
masonry Young's modulus	E [MPa]	12500
masonry shear modulus	G [MPa]	5000
masonry plastic strain	ε_y [%]	0.2
FRP Young's modulus	E_f [GPa]	230
FRP cross section area	A_f [mm^2]	150

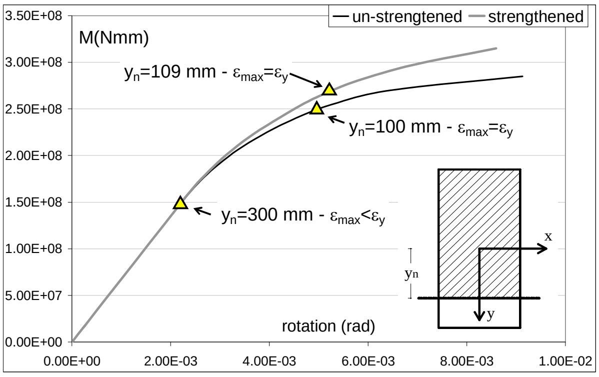


Figure 3 Test beam: moment-rotation curve

From the plots it is possible to observe a good agreement between the numerical and the experimental curves underlining the capability of the developed model to predict the experimental response of the panels. In the case of the FRP-strengthened panel the ultimate displacement deduced by the model is smaller than the experimental one. This is due to the fact that the program stops the analysis when one of the cross sections of the element is completely in tension. It is clear that the experimental behavior is affected by additional contributions, not considered in the numerical model, which allow to sustain further increasing of the external load and, consequently, to exhibit greater ultimate displacement values.

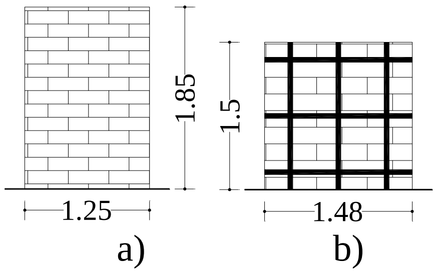


Figure 4 Geometry of the examined masonry panels: a) un-strengthened panel, Fantoni, 1981; b) FRP-strengthened panel, Marcari, 2005

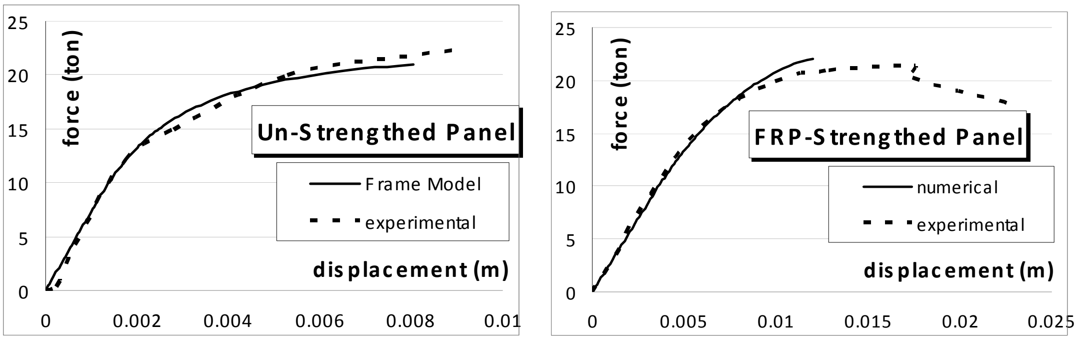
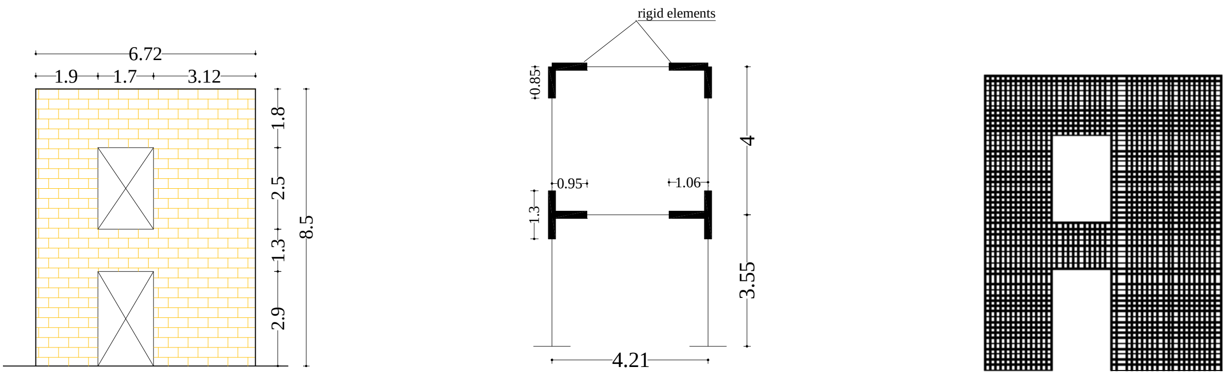


Figure 5 Horizontal load vs. top displacement curves

3.3. Study case 3: masonry façade

The third study case refers to a two-story masonry facade (see Figure 6.a). This element has been firstly analyzed considering the un-strengthened solution and selecting two different modeling approaches:

- the first model (denoted 'Frame Model') is based on the use of the developed beam element both for modeling the piers and the spandrels of the façade (see Figure 6.b);
- the second approach (denoted 'FE model A') has been developed using the commercial FE code DIANA 9.1 (2000). For this model four-node isoparametric plan shell elements have been considered for the discretization of the panel and an elastic plastic material law with the Rankine-Mises yield criterion has been chosen. The mechanical parameters characterizing the material behavior for both models are summarized in Table 2.



a) b) c)

Figure 6 Masonry façade: a) geometry; b) equivalent frame; c) FE model A

Table 2

Masonry Young's modulus	E (MPa)	380
Masonry Poisson's ratio	ν	0.2
Masonry compressive strength	fc (MPa)	1.0
Masonry weight for unit volume	γ (kg/m^3)	1600
Masonry yield strain	ε_y	0.002
FRP Young's modulus	E_f (GPa)	230
FRP max tensile strain	$\varepsilon_{f,max}$	0.2%

Nonlinear static analyses have been performed considering a constant vertical load and incremental horizontal forces with a triangular shape distribution. The obtained results in terms of force-displacement curves are reported in Figure 7 for both the selected models. From the plot it is possible to observe a good agreement between the two modeling approaches. In particular, the frame model presents a smaller initial stiffness and a smaller ultimate displacement which corresponds to the cross section of the left pier is completely in tension.

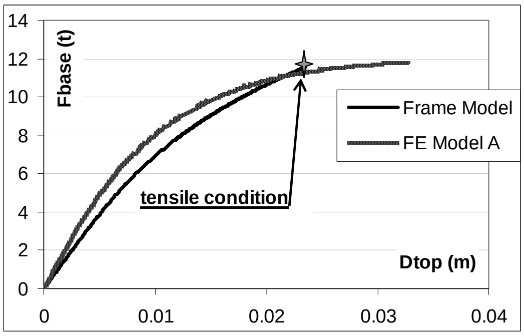


Figure 7 Push-over curves

In order to study the effect of the strengthening on the response of the masonry façade, a typical strengthening configuration characterized by vertical FRP strips located along the piers has been considered (see Figure 8). The façade has been modeled using the frame model and considering two different amount of the FRP: 1 ply (each FRP strip presents a cross-section equal to 60 mm^2) and 2 plyies (each FRP strip presents a cross section equal to 120 mm^2).
In Figure 9 the force-displacement curves of the un-strengthened and FRP-strengthened façade are reported. From the plot it is clear the effect of the FRP-reinforcement both in terms of strength and ultimate displacement.

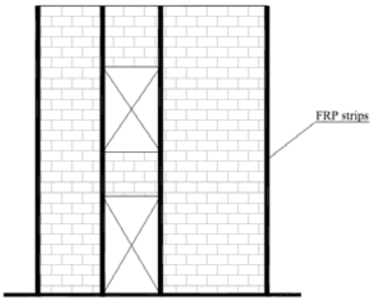


Figure 8 Configuration of the FRP-strengthening system

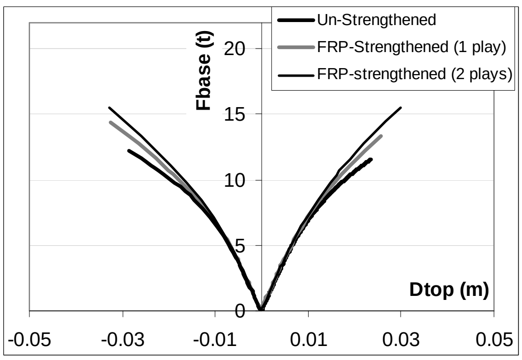


Figure 9 Pushover curves: effect of the FRP-strengthening

4. CONCLUSIONS

The present paper is part of a research activity devoted to develop numerical models for the analysis of masonry structures strengthened by FRP materials. In this paper, the use of a simplified modeling approach based on a special finite element developed by the authors has been presented and applied to different study cases.

The obtained results have been examined in order to check the reliability of the proposed model to reproduce the experimental behavior of masonry elements strengthened by FRP, and for studying the effect of the FRP on the structural response of masonry facades.

The proposed model is a valid tool for the preliminary design of the FRP-reinforcing system devoted to improve the seismic response of masonry structures.

REFERENCES

Luciano R, Sacco E. (1997). *Homogenization technique and damage model for old masonry material*. **Int. J. Solids and Structures 34:4, 3191-3208.**

Lourenço PB. (1998). *Continuum model for masonry: parameter estimation and validation*. **ASCE Journal of Structural Engineering; 124: 6, 642-652.**

Marfia S, Sacco E. (2005). *Numerical procedure for elasto-plastic no-tension model*. **Int. J. for Computational Methods in Eng. Science and Mechanics. 6: 187-199.**

Alfano G, Sacco, E. (2006). *Combining interface damage and friction in a cohesive-zone model*. **International Journal for Numerical Methods in Engineering. 68:5, 542-582.**

Bazant ZP (1998), Fracture and size effect in concrete and other quasi brittle materials. **CRC PRESS, Boca Raton.**

Milani G, Lourenço P., Tralli A. (2006). *Homogenized limit analysis of masonry walls. Part I: failure surfaces*. **Comp. Struct. 84: 166-180.**

E. Grande, G. Milani, E. Sacco (2008). *Modelling and analysis of FRP-strengthened masonry panels.* **Engineering Structures. 30:7, 1842-1860.**

Como M., Grimaldi A.(1983). *An unilateral model for the limit analysis of masonry walls*. **Inter. Congr. On Unilateral Problems in Struc. Analysis, Ravello.**

Braga F., Liberatore D., Spera G. (1997). *A computer program for the seismic analysis of complex masonry buildings*. **In Proceedings of STRUMAS IV, Firenze.**

L. Gambarotta and S. Lagomarsino (1996). *A finite element damage model for the evaluation and rehabilitation of brick masonry shear walls.* **in Worldwide Advances in Structural Concrete and Masonry (A.E. Schultz and S.L. McCabe Eds.), Proc. of the CCMS-ASCE Symposium, Chicago, Illinois.**

Roca, P. (2005). *Simplified models for the assessment of masonry shear walls*. **Construction and Building Materials. 20: 229-238.**

Magenes, G. (2000). *A Method for Pushover Analysis in Seismic assessment of Masonry Buildings.* **Proceedings of the 12th World Conference on Earthquake Engineering, Auckland, New Zealand, CD-ROM.**

Marfia S., Sacco E., Reddy J.N. (2007). *A new docking-free Timoshenko beam finite element*. **Internal Report, DiMSAT, University of Cassino.**

Mathworks, Inc., 2002 Natick, Massachusetts.

Matlab reference manual, Version 6.5.

DIANA 9.1. Displacement analysis finite element software, Version 9.1, TNO-Building Division, Delf, The Netherlands, 2000.

Acknowledgements

The research reported in this paper is part of a project which has received financial support by the Italian Research Program RELUIS.