Jerk and Jerk Sensor

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ABSTRACT :

This paper presents the principles and specifications of a new sensor for measuring the derivatives of acceleration. Derivative of acceleration is the first order differentiation of acceleration. It is also referred to as "Jerk". Jerk is directly related to a physical damage process of materials and structures. The derivative of acceleration (jerk) has been found to have many useful applications. Examples are: high dynamic motion aerial vehicle trace measurement, earthquake-resistant mechanisms of structures, mechanisms of high speed auto-control of machines, human responses in high-speed moving vehicle and high-speed elevators.

KEYWORDS:

Derivative of acceleration, Jerk, Jerk sensor, Earthquake engineering.

1. Introduction

Derivative of acceleration (jerk) is the first order differentiation of acceleration. It is also referred to as Jerk. Assume that we can measure the derivative of acceleration (jerk) and denote as $x(t)$, then, from Newton's second law of motion,

$$\frac{dF}{dt} = Mx(t) \tag{1}$$

where M is the mass and

$$x(t) = \frac{da(t)}{dt} \tag{2}$$

in which $a(t)$ is the acceleration time history.

In the time history description of jerk, the non-zero segments represent the dynamic process. Understanding of this dynamic process requires proper measurement of the derivation of acceleration (jerk). The derivative of acceleration (jerk) is directly related to the physical damage process of a physical system under dynamic loads initiated by molecular bond separation in the material.

In earthquake engineering, the damaging effect of an earthquake to structures is largely described by the peak values of the ground motion (acceleration, velocity and displacement). Recent studies have shown that in near fault ground motions, PGA and PGV are sometimes directly related. Their relationship may be determined by the derivative of acceleration (jerk), which gives the measurement of a component of acceleration transformed into speed, and vice versa. Current study of jerk for earthquake engineering applications is mainly in structural damage detection [1] by using the moving coordinate system to decompose ground acceleration in order to analyze potential near-source characteristics [2]. Jerk in near-source ground motion is also a current topic of research interest.

According to [1], jerk in 3D motion consists of three components that can be expressed as:

$$da/dt = d^3S/dt^3T + d(\kappa V^2)/dtN + \kappa|V|^3(-\kappa T + (1/|V|^2)(d|V|/dt)N + \gamma B) \quad (3)$$

where the first term is the rate of linear acceleration change , the second term is the rate of normal acceleration change , and the last term is the fundamental Frenet vector , in which γ is the only parameter that cannot be determined without da/dt . In general, da/dt may cause temporal torsion. However, Eq. (3) shows that

da/dt also contains other effects to the motion. A study on Kobe earthquake ground motion records shows that $\kappa|V|^3\gamma$ was not uniformly distributed and the magnitudes of κ and γ may reach in 10^2rad /s/cm. It contains sharp spikes with short duration
(< 1 00 ms). However, this is only the jerk component that rotates acceleration out of the tangential plan. Further investigation is to be carried out, in particular for rate of linear acceleration change.

Derivative of acceleration (jerk) is also an important comfort indicator for high-speed elevators. For elevator acceleration, deceleration and vibration control, derivative of acceleration (jerk) is one of the parameters to be controlled according to code requirement. Typically, it is kept less than 2(m/s^3).

In monitoring the working condition of the wheels of a train, particularly for problems of losing contact with rail surface and slipping, the derivative of acceleration (jerk) is one of the three typical characteristics (Δv, dv/dt, and d^2v/dt^2). The automated anti -slipping control is based on the threshold and real time values of the three signals.

The dynamic performance of global position device requires to response to the derivative of acceleration (jerk) up to 60(m/s^3).

Fussy and neuro-network based vehicle suspension control systems require the measurement of vertical acceleration and jerk to be combined into the evaluation function for minimization and optimization.

In this paper, we present the development of a jerk sensor that will enable us to measure directly the derivatives of accelerations in a dynamic system. The sensor has been registered as a Chinese patent.

2. Sensor Development

Theoretically speaking, a direct differentiation of measured acceleration can give the derivative of acceleration (jerk). However, it is not practically feasible because differentiation is very sensitive to noise. Typically, a normally acceptable low noise level in the acceleration signal can easily be amplified resulting in unacceptably high level of error in the derivative of acceleration (jerk) signal. The error level of a differential circuit is measured by

$$\gamma_d \approx -\frac{1}{2}(\omega RC) \qquad (4)$$

where RC is the vibration frequency of the measuring object . Error in the high frequency range can easily cause high frequency noise. Therefore, development of a high accuracy sensor for measuring the derivative of acceleration (jerk) requires new approaches.

For a true differential circuitry, the output voltage is given by

$$u_0 = -RC\frac{du_i}{dt} \qquad (5)$$

where RC is the differential resistor and capacitor, u_i is the input voltage . It is seen from Equation (5) that u_0 is sensitive to the abrupt change of u_i, therefore, its noise resistance is poor. In addition, RC brings delay effect to the feedback signal, which can easily cause resonance. When the input voltage experiences a quick change, the output voltage may exceed the maximal output voltage of the integrated amplifier circuitry, and results in malfunction. Since the accelerometer is already very sensitive to high frequency signals （$\ddot{X} = (2\pi f)^2 X$）, by taking differential of acceleration directly, the error level will be unacceptable. Therefore, to acquire high accuracy jerk sensor, a different approach is used as described in the following. Figure 1 illustrates the basic elements and principle of the jerk sensor mechanism. It is an absolute motion based, coupled oscillator sensing device.

In this sensor unit, m is the movable mass, k is the supporting stiffness and b is the damping coefficient (including aero-dynamic damping), G = BL is he electro-mechanical coupling coefficient. B is the coefficient of magnetic inductance and L is the length of the conduct wire. i is the current and x is the relative displacement between the movable mass and the sensor cover , X is the base derivative of acceleration (jerk) (to be measured by the Sensor), e is the output voltage of the jerk sensor.

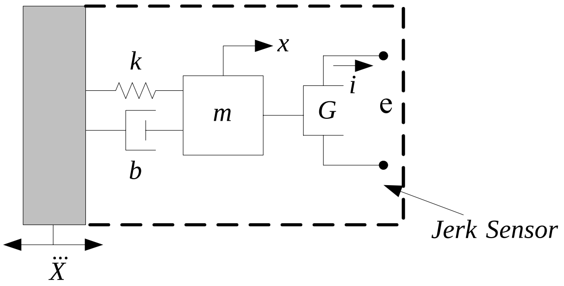

Fig.1 The principle of the jerk sensor

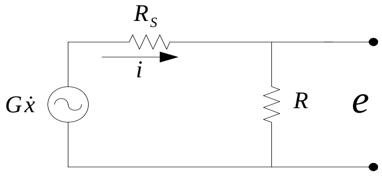

Fig.2 The principle of the circuit

The differential equation of motion is

$$m\ddot{x}+b\dot{x}+kx+Gi=-m\ddot{X} \qquad (6)$$

The principle of the circuit is illustrated in Fig.2

$$\left.\begin{aligned} R_S i+e=G\dot{x} \\ \frac{1}{R}e=i \end{aligned}\right\} \qquad (7)$$

Neglecting the aero-dynamic damping, solutions of Eqs. (6) and (7) in the frequency domain are given by:

$$x(s)=-\frac{s^2X(s)}{s^2+2D_0n_0s+\omega_0{}^2} \qquad (8)$$

$$e(s)=-AG\frac{s^3X(s)}{s^2+2D_0\omega_0s+\omega_0{}^2} \qquad (9)$$

where $D_0=\frac{G^2}{2m\omega_0(R_S+R)}$ is the damping ratio, $A=\frac{R}{R_S+R}$, $s=j\omega$ № $j=\sqrt{-1}$, $\omega_0=\sqrt{\frac{k}{m}}$, ω_0 is natural frequency of jerk sensor，ω is natural frequency of the ground. When $\omega_0>>\omega$ ， $D_0<1$

$$e(s)=G\dot{x}A=A\cdot G\cdot H(s)\cdot s^3X(s) \qquad (10)$$

$e(s)$ is the output voltage of the jerk sensor, $H(s)=\frac{1}{s^2+2D_0\omega_0s+\omega_0{}^2}$ is the frequency characteristic， $s^3X(s)$ is derivative of acceleration (jerk). The sensitivity of the jerk sensor is given by

$$S_{\dddot{X}}=\frac{e(s)}{s^3X(s)}=A\cdot G\cdot\frac{1}{s^2+2D_0\omega_0s+\omega_0{}^2} \qquad (11)$$

When $\omega_0>>\omega$ ， $D_0<1$ the sensitivity of the jerk sensor is

$$S_{\dddot{X}}\doteq\frac{AG}{\omega_0{}^2} \qquad (12)$$

Equation (12) indicates that with increasing natural frequency ω_0, the sensitivity of the jerk sensor decreases.

3. Specifications of jerk sensor

Figure 3 is the photo of JW-3D triaxial Jerk sensor. The specifications of jerk sensor is provided in Table 1. Typical frequency response for JW-1, JW-2 and JW-3 jerk sensor is illustrated in Fig.4, Fig.5 and Fig.6.

Fig. 3 JW-3D Jerk Sensor

Table 1. Specifications of jerk sensor

Model	JW-1	JW-2	JW-3	JW2-3D
Sensitivity [mV /(m/s^3)]	0.0 8	0. 2	1. 2	0. 2
Bandwidth (Hz, -3 dB)	0.3~100	0.3~60	0.05~30	0.3~60
Measurable Range(m/s^3)	10000	5000	4000	5000
Dynamic range (dB)	100	100	100	100
Linearity (%)	±1	±1	±1	±1
Max. transverse sensitive (%)				±2
Temperature range (^{o}C)	-30~70	-30~70	-30~70	-30~70

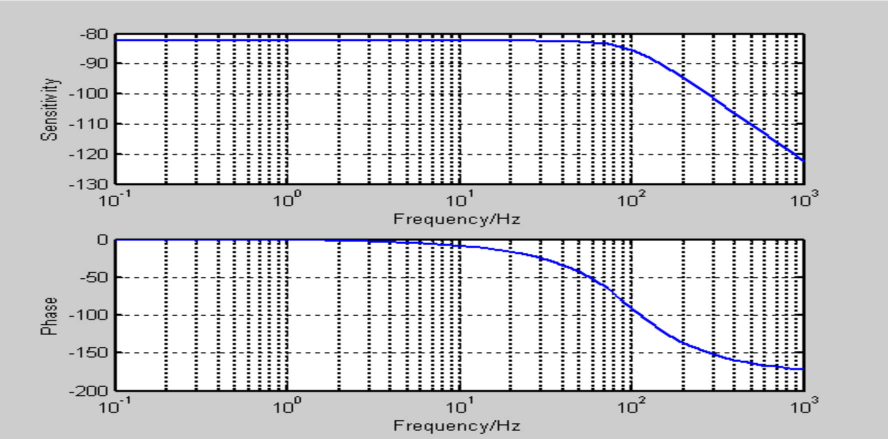

Fig.4 Typical frequency response for JW-1 jerk sensor

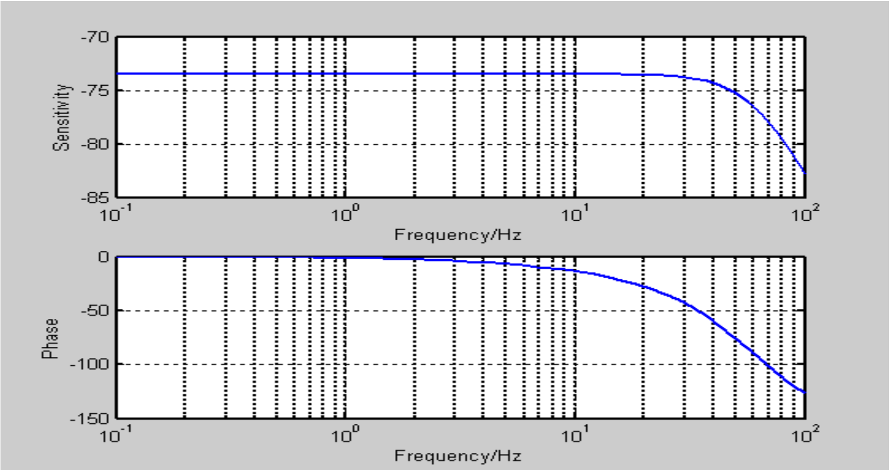

Fig.5 Typical frequency response for JW-2 jerk sensor

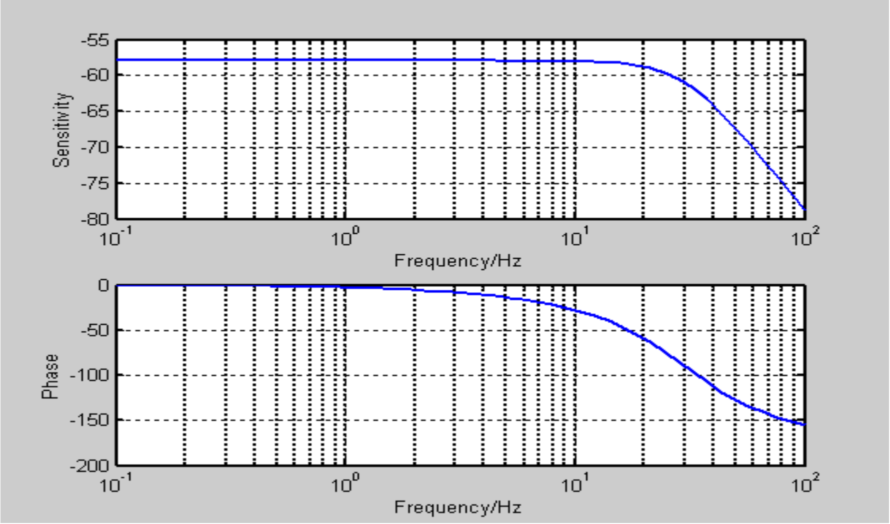

Fig.6 Typical frequency response for JW-3 jerk sensor

4. Experimental result

4.1The calibration result of jerk sensor

The jerk sensor has been tested and calibrated by using a low frequency vibration table. The sensitivities, frequency responses (amplitude–frequency characteristic), linearity and maximum transverse sensitivities are measured. Results are given in Table 2 to Table 8.

Table 2. Calibration result of amplitude-frequency characteristic for model JW-1 jerk sensor

f（Hz）	0.5	1.0	2.0	5.0	10.0	20.0	40.0	60.0	80.0	100.0
Sensitive $S_{\dddot{x}}$ $[10^{-5}V/(m/s^3)]$	8.098	8.131	8.233	8.274	8.332	8.190	8.379	8.428	8.053	8.456

Table 3. Calibration result of amplitude-frequency characteristic for model JW-2 jerk sensor

f（Hz）	0.4	0.5	1.0	2.0	5.0	10.0	20.0	40.0	60.0
Sensitive $S_{\dddot{X}}$ $[10^{-4}V/(m/s^3)]$	1.808	1.802	1.778	1.782	1.782	1.808	1.806	1.881	1.683

Table 4. Calibration result of amplitude-frequency characteristic for model JW-3 jerk sensor

f（Hz）	0.5	0.8	1.0	2.0	3.0	5.0	10.0	20.0	30.0
Sensitive $S_{\dddot{X}}$ $[10^{-3}V/(m/s^3)]$	1.227	1.200	1.209	1.213	1.219	1.222	1.238	1.070	0.970

Table 5. Calibration result of linearity for model JW-1 jerk sensor (at 20Hz)

m/s^3	15.82	42.35	62.75	107.9	171.5	253.7	602.2	1165	1568
Sensitive $S_{\dddot{X}}$ $[10^{-5}V/(m/s^3)]$	8.254	8.247	8.124	8.126	8.139	8.186	8.122	8.210	8.360
$S_{\dddot{X}} / \overline{S}_{\dddot{X}}$	1.009	1.008	0.993	0.993	0.995	1.000	0.993	1.003	1.022

$\overline{S}_{\dddot{X}} = 8.183\times[10^{-5}V/(m/s^3)]$ is average sensitive.

Table 6. Calibration result of linearity for model JW-1 jerk sensor (at 80Hz)

m/s^3	1405	1973	2546	3486	4480	5499	6262	7968
Sensitive $S_{\dddot{X}}$ $[10^{-5}V/(m/s^3)]$	7.932	7.989	8.027	8.135	8.232	8.220	8.257	8.125
$S_{\dddot{X}} / \overline{S}_{\dddot{X}}$	0.981	0.988	0.992	1.006	1.018	1.016	1.021	1.004

$\overline{S}_{\dddot{X}} = 8.089\times[10^{-5}\ V/(m/s^3)]$ is average sensitive.

Table 7. Calibration result of linearity for model JW-2 jerk sensor (at 5 Hz)

m/s^3	13.53	50.02	91.92	124.9	153.3	180.2	210.5	239.3	275.1
Sensitive $S_{\dddot{X}}$ $[10^{-4}V/(m/s^3)]$	1.866	1.869	1.889	1.883	1.890	1.890	1.896	1.899	1.903
$S_{\dddot{X}} / \overline{S}_{\dddot{X}}$	0.990	0.991	1.002	0.999	1.002	1.002	1.006	1.007	1.009

Table 8. Sensitive and max. transverse sensitive for model JW2-3D triaxial jerk sensor

Measure direction	x	y	z
Sensitive $[10^{-4}V/(m/s^3)]$ (5Hz)	1.684	1.724	1.840
Sensitive $[10^{-4}V/(m/s^3)]$ (10Hz)	1.686	1.733	1.852
Max.transverse sensitive (%)	0.793	1.046	0.936

4.2 Earthquake wave contrast experiment for MBA accelerometer and JW-1 jerk sensor after once integral

After once integral, the output voltage of the Jerk Sensor is in direct proportion to the acceleration of measuring point, so is the output voltage of MBA accelerometer. The jerk sensor and the MBA accelerometer (Sensitive:0.2V V/(m/s^2), Bandwidth :0—200 Hz, Dynamic range: >110 dB) on the low frequency vibration table，input earthquake wave for low frequency vibration table, and record acceleration time history of MBA accelerometer and JW-1 jerk sensor passing once integral. Acceleration time history record is illustrated in Fig.7, CH1 is acceleration record for MBA accelerometer and CH2 is acceleration record for JW-1 jerk sensor passing once integral. Amplitude analysis result is listed in table.9. Fig.8 illustrates phase analysis result, top curve is power spectrum, bottom is phase spectrum. Relative Phases difference between the two acceleration time history record is listed Table.10.

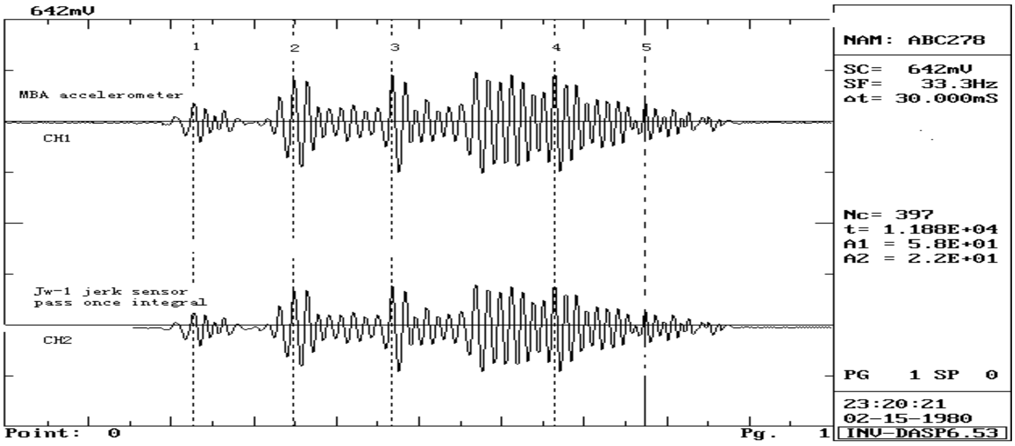

Fig.7

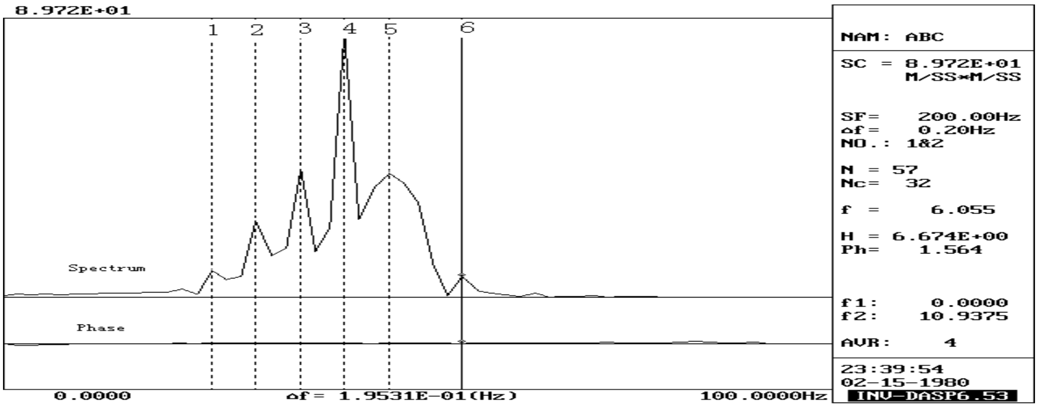

Fig.8

Table 9. Amplitude analysis result

No.	1	2	3	4
CH1(MBA)	58.50	130.5	146.5	154.0
CH2 (JW-1)	23.75	54.25	61.00	63.50
CH2/CH1	0.406	0.416	0.416	0.412
(CH2/CH1)/ (CH2/CH1)*	0.981	1.005	1.005	0.995

(CH2/CH1) is average of CH2/CH1

Table 10. Phase analysis result

No.	1	2	3	4	5	6
f（Hz）	1.578	3.320	3.906	4.492	5.078	6.055
Phase（Degree）	1.750	2.250	2.170	1.800	1.700	1.564

Table 10 and Table 11 indicate that frequency responses (amplitude–frequency characteristic and phase–frequency characteristic) of MBA accelerometer and JW-1 jerk sensor after once integral are approximately identical.

5. Summary

Results of calibration indicate that frequency response and linearity of (JW jerk) sensor are excellent （Table 2,3,4,5,6 and 7）, The maximum transverse sensitive ratio of JW2-3D triaxial jerk sensor is less than 2% (Table 8). Experimental result indicates that the jerk sensor may record the time history of jerk motions (Fig.7).

6. Acknowledgement

The US-PRC Cooperative Research Project is jointly supported by the US National Science Foundation (Award #CMS-0414020) and Technology Foundation of Hei Long Jang Province, China (4eoA519).

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