The 14th World Conference on Earthquake Engineering
October 12-17, 2008, Beijing, China

Theoretical bases of the three-dimensional rheologic model of earthquake preparation

Zhiping Song[1], Xiangchu Yin[2] and Shirong Mei[2]

[1] *Professor,National Geophysical Observatory at Sheshan, Earthquake Administration of Shanghai Municipality, Shanghai 200062, China.*
[2] *Professor,Institue of Earthquake Science, China Earthquake Administration, Beijing, 100036,China.*
Email: zpsong1966@sina.com, zpsong@sohu.com

ABSTRACT
Based on the three-dimensional elastic inclusion model proposed by Dobrovolskii, we developed a rheological inclusion model to study earthquake preparation processes. By using the Corresponding Principle in the theory of rheologic mechanics, we derived the analytic expressions of viscoelastic displacement, normal strains, the shear strains，the bulk-strain, the ground tilt , the increment of underground water level, the variation of resistivity at an arbitrary point (x,y,z) in three directions of X-axis, Y-axis and Z-axis produced by a three-dimensional inclusion in the semi-infinite rheologic medium defined by the standard linear rheologic model. After the spatial-temporal variation of bulk-strain being computed on the ground produced by such a spherical rheologic inclusion, some interesting results are obtained, suggesting that the bulk-strain produced by a hard inclusion change with time according to three stages (α,β,γ) with different characteristics, similar to that of geodetic deformation observations, but different with the results of a soft inclusion. These theoretical results can be used to explain the characteristics of spatial-temporal evolution, patterns, quadrant-distribution of earthquake precursors, the changeability, spontaneity and complexity of short-term and imminent-term precursors. It offers a theoretical base to build physical models for earthquake precursors and to predict the earthquakes.
KEYWORDS: three-dimensional elastic inclusion mode, rheologic inclusion model , analytic expressions.

Introduction

Studies on the inclusion model started in the 1950s. Eshelby (1957) gave the analytical solution of the elliptic sphere inclusion model in the elastic medium. Aki and Richards (1980) applied the Eshelby's inclusion theory to analyze the dynamics of seismic wave. Brady (1974) brought forward the soft inclusion model. Dobrovolskii (1991) advanced the elastic analytical solutions of the hard inclusion model. Mei (1994) set up the hard-body earthquake-preparation model. In order to study the earthquake generating process, it is not enough to consider the elastic inclusion theory. We must take into account the time factor.

So we extend Dobrovolskii's the elastic theory of hard inclusion to the theory of rheologic medium inclusion (Song, 1996), and analyze preliminarily the spatio-temporal evolution process of bulk-strain field based on a rheologic inclusion model (Song et al., 2000). In this paper, we will give the displacement field and the strain field of the rheologic inclusion theory (Song et al, 2003; Song et al. 2004).

There are a lot of precursors related to bulk-strain, such as electrical resistivity and underground water level. Therefore, the viscoelastic analytical expressions of these precursor parameters can be derived by the elastic analytic expression of bulk-strain (Song et al, 2006).

1 Basic theory

There exists an inclusion in the semi-infinite medium (Figure 1a). S_0 is the surface of the Earth where the action of the atmosphere pressure is not considered. S_1 is a hypothetical surface in the Earth and its boundary condition may be displacement, stress or other physical parameters. To the first-order approximation, the outside force for shallow focus earthquake may include the body force, whereas the gravity and the tidal force may be neglected. In careful analysis and discussion, although the tidal disturbing effect is small, it may be significant due to its long and circulatory action.

In a rectangular coordinate system, the plane *XOY* is in the surface of the Earth. The *Z*-axis directs to the center of the Earth. The center of the inclusion is placed at the point (0, 0, H). As for the boundary conditions, only $\sigma_{xy}^{\infty} = \sigma_{yx}^{\infty} = -\tau$ (Fig. 1), and all other components are zero. That is, the infinitude is acted by uniform shear stress and the bottom is free surface. If the boundary condition is a function of time, it is expressed as $\tau H(t)$, in which $H(t)$ is a unit jump function. We have carefully analyzed the typicality of rheologic medium models and legibility of theoretical derivation (Yin et al., 1982). Therefore, we choose the standard linear solid model (series connection between a spring and a Kelvin body or parallel connection between a spring and a Maxwell body) as the medium model in and outside the three-dimension inclusion respectively in Fig.1b and Fig.1c (Song et al., 2003).

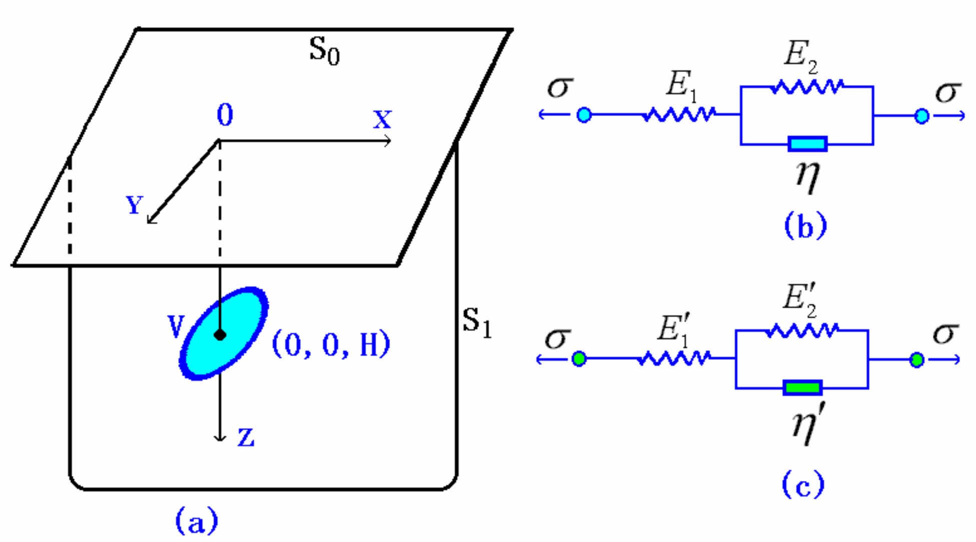

Figure 1 Three-dimension inclusion and its rheologic medium model
(a) Three-dimension inclusion model; (b) Rheologic medium model outside the inclusion;
(c) Rheologic medium model inside the inclusion

Based on Dobrovolskii's hard inclusion elastic theory and the Correspondence Principle of rheologic mechanics (Yin, 1985), we derived the viscoelastic expression of displacement produced by the inclusion in the rheologic medium.

Based on the analytic expression $U_i^e(r)$ of displacement of an elastic inclusion, using the Laplace Transform, we can derive the solution $\overline{U}_i^e(r,s)$ of elastic displacement in the image-plane. Then substituting the elastic parameters μ and K with $\overline{\mu}(s)$ and $\overline{K}(s)$ of the image-plane, we can derive the solution $\overline{U}_i^v(r,s)$ of visco-elastic displacement in the image-plane. At last, using the Inverse Laplace Transform, we can derive the viscoelastic expression $U_i^v(r,t)$.

Our previous work extended Dobrovolskii's the elastic theory of hard inclusion model into rheologic inclusion model (Song, 1996). We analyzed preliminarily the spatial and temporal evolution process of bulk-strain field for this model (Song et al., 2000). In this paper, we give the displacement field, the strain field and precursor field of the rheologic inclusion model, and try to discuss the characteristics of spatial-temporal evolution, patterns, quadrant-distribution of earthquake precursors. Further and detailed derivations can be found in two papers (Song et al, 2003; Song et al. 2004; Song et al. 2006).

2 Displacement field

The analytic solution of visco-elactic displacement $U(r,t)$ in the *X*-axis, $V(r,t)$ in the *Y*-axis and $W(r,t)$ in the *Z*-axis has been obtained in Song et al. (2003). For example, the analytic solution of visco-elactic displacement $U(r,t)$ in the *X*-axis of a point (x,y,z) is as follows :

$$U(r,t)=\frac{Vy\tau}{8\pi}\{G_{u0}H(t)+(G_{u1}+G''_{u1}+tG'_{u1})\exp(-\frac{q_0}{q_1}t)+G_{u2}\exp(-\frac{1}{p'_1}t)$$
$$+G_{u3}\exp(-\frac{3K+2q_0}{3Kp_1+2q_1}t)+G_{u4}\exp(-\frac{6K+q_0}{6Kp_1+q_1}t)\} \quad (1)$$

Defining the time parameters T_1,T_2,T_3,T_4 as

$$T_1=\frac{q_0}{q_1},\quad T_2=\frac{1}{p'_1},\quad T_3=\frac{3K+2q_0}{3Kp_1+2q_1},\quad T_4=\frac{6K+q_0}{6Kp_1+q_1}$$

where the definitions of q_0, q_1, p_1 and q'_0, q'_1, p'_1 can be written as:

$$\begin{cases} p_1=\dfrac{\eta}{E_1+E_2} & q_0=\dfrac{E_1\cdot E_2}{E_1+E_2} & q_1=\dfrac{E_1\eta}{E_1+E_2} \\ p'_1=\dfrac{\eta'}{E'_1+E'_2} & q'_0=\dfrac{E'_1\cdot E'_2}{E'_1+E'_2} & q'_1=\dfrac{E'_1\eta'}{E'_1+E'_2} \end{cases}$$

The parameters G_{ui} in equation (1) are

$$\begin{cases}
G_{u0}=2(\dfrac{q'_0}{q_0}-1)\cdot\left[\dfrac{S_u(0)}{q_0(3K+q_0)}+P_{u2}\cdot\dfrac{3}{3K+2q_0}+P_{u4}\cdot\dfrac{3}{6K+q_0}\right] \\
G_{u1}=\dfrac{2(1-p_1T_1)}{q_0}\begin{Bmatrix}\dfrac{S_u(T_1)}{(3K+2q_0)-(3Kp_1+2q_1)T_1}-\dfrac{3(1-p_1T_1)(q'_0-q'_1T_1)}{1-p'_1T_1}\bullet \\ \left[\dfrac{P_{u2}}{(3K+2q_0)-(3Kp_1+2q_1)T_1}+\dfrac{P_{u4}}{(6K+q_0)-(6Kp_1+q_1)T_1}\right]\end{Bmatrix} \\
G_{u2}=-2(1-p_1T_2)^2\dfrac{q'_0-q'_1T_2}{q_0-q_1T_2}\begin{Bmatrix}\dfrac{S_u(T_2)}{(q_0-q_1T_2)[(3K+2q_0)-(3Kp_1+2q_1)T_2]}+ \\ \dfrac{3P_{u2}}{(3K+2q_0)-(3Kp_1+2q_1)T_1}+\dfrac{3P_{u4}}{(6K+q_0)-(6Kp_1+q_1)T_1}\end{Bmatrix} \\
G_{u3}=-\dfrac{2(1-p_1T_3)}{3K+2q_0}\left[\dfrac{1-p_1T_3}{1-p'_1T_3}\cdot\dfrac{q'_0-q'_1T_3}{q_0-q_1T_3}-1\right]\left[\dfrac{S_u(T_3)}{q_0-q_1T_3}+3P_{u2}\right] \\
G_{u4}=-\dfrac{6(1-p_1T_4)}{6K+q_0}\left[\dfrac{1-p_1T_4}{1-p'_1T_4}\cdot\dfrac{q'_0-q'_1T_4}{q_0-q_1T_4}-1\right]\cdot P_{u4} \\
G'_{u1}=\dfrac{-2(q'_0-q'_1T_1)(1-p_1T_1)^2}{q_0q_1(1-p'_1T_1)[(3K+2q_0)-(3Kp_1+2q_1)T_1]}\cdot S_u(T_1) \\
G''_{u1}=2\{P_{u1}\cdot S(6K+q_0,6Kp_1+q_1)+P_{u3}\cdot S(6K+7q_0,6Kp_1+7q_1)\}
\end{cases} \quad (2)$$

$$S_u(T)=P_{u1}\cdot[6K+q_0-(6Kp_1+q_1)T]+P_{u3}\cdot[6K+7q_0-(6Kp_1+7q_1)T] \quad (3)$$

The universal function $S(m,n)$ is

$$S(m,n)=[AA(3K+2q_0,3Kp_1+2q_1)+BB(3K+2q_0,3Kp_1+2q_1)]\cdot(m-nT_1)$$
$$+CC(3K+2q_0,3Kp_1+2q_1)\cdot n \quad (4)$$

In the equation (5), $AA(m,n)$, $BB(m,n)$, $CC(m,n)$ and $DD(m,n)$ are

$$\begin{cases}
AA(m,n)=-\dfrac{(1-p_1T_1)[q'_1(1-p_1T_1)-2p_1(q'_0-q'_1T_1)]}{q_0q_1(1-p'_1T_1)}\cdot\dfrac{1}{m-nT_1} \\
BB(m,n)=-\dfrac{(1-p_1T_1)^2(q'_0-q'_1T_1)}{q_0(1-p'_1T_1)}\cdot\dfrac{DD(m,n)}{m-nT_1} \\
CC(m,n)=-\dfrac{(1-p_1T_1)^2(q'_0-q'_1T_1)}{q_0q_1(1-p'_1T_1)}\cdot\dfrac{1}{m-nT_1} \\
DD(m,n)=\dfrac{1}{q_0}-\dfrac{p'_1}{q_1(1-p'_1T_1)}-\dfrac{n}{q_1(m-nT_1)}
\end{cases} \quad (5)$$

where
$$\begin{cases} P_{u1} = -\dfrac{6Hz}{r_2^5} + x^2\left(-\dfrac{3}{r_1^5} + \dfrac{30Hz}{r_2^7}\right) \\ P_{u2} = \dfrac{1}{r_2^3} - \dfrac{1}{r_1^3} \\ P_{u3} = -\dfrac{3x^2}{r_2^5} \\ P_{u4} = -\dfrac{4}{r_2(r_2+H+z)^2} + \dfrac{4(3r_2+H+z)}{r_2^3(r_2+H+z)^3}\cdot x^2 \end{cases} \quad (6)$$

where $r=\sqrt{x^2+y^2+z^2}$, $r_1=\sqrt{x^2+y^2+(H-z)^2}$, $r_2=\sqrt{x^2+y^2+(H+z)^2}$. The center of inclusion is at (0, 0, H). The coordinate of the resolved point is (x, y, z), while other parameters were given by SONG *et al* (2003).

When t=0, the displacement $U(r,0)$ in the X-axis is

$$U(r,0)=\frac{Vy\tau}{8\pi}\{G_{u0}+G_{u1}+G''_{u1}+G_{u2}+G_{u3}+G_{u4}\} \quad (7)$$

3 Strain field

The normal strain $\varepsilon_{xx}(r,t)$ in X-axis, $\varepsilon_{yy}(r,t)$ in Y-axis , $\varepsilon_{zz}(r,t)$ in Z-axis and then the bulk-strain $\theta(r,t)$ has been obtained by Song (1996). The viscoelastic analytic expression of the bulk-strain $\theta(r, t)$ at a point (x,y,z) is (Song, et al, 2004).

$$\begin{aligned}\theta(r,t)=\frac{3V\tau xy}{4\pi}\{&M_{\theta 0}H(t)+M_{\theta 1}\exp(-\frac{q_0}{q_1}t)+M_{\theta 2}\exp(-\frac{1}{p'_1}t) \\ &+M_{\theta 3}\exp(-\frac{3K+2q_0}{3Kp_1+2q_1}t)+M_{\theta 4}\exp(-\frac{6K+q_0}{6Kp_1+q_1}t)\}\end{aligned} \quad (8)$$

Where, the parameters $M_{\theta 0}$, $M_{\theta 1}$, $M_{\theta 2}$, $M_{\theta 3}$ and $M_{\theta 4}$ are as follows

$$\begin{cases} M_{\theta 0}=6(\dfrac{q'_0}{q_0}-1)\cdot\dfrac{S_\theta(0)}{3K+2q_0} \\ M_{\theta 1}=-6\cdot\dfrac{(q'_0-q'_1T_1)}{q_0}\cdot\dfrac{(1-p_1T_1)^2}{1-p'_1T_1}\cdot\dfrac{S_\theta(T_1)}{(3K+2q_0)-(3Kp_1+2q_1)T_1} \\ M_{\theta 2}=-6\cdot\dfrac{q'_0-q'_1T_2}{q_0-q_1T_2}\cdot\dfrac{(1-p_1T_2)^2}{(3K+2q_0)-(3Kp_1+2q_1)T_2}\cdot S_\theta(T_2) \\ M_{\theta 3}=-6\cdot\dfrac{(1-p_1T_3)}{(3K+2q_0)}\left[\dfrac{1-p_1T_3}{1-p'_1T_3}\cdot\dfrac{q'_0-q'_1T_3}{q_0-q_1T_3}-1\right]\cdot S_\theta(T_3) \\ M_{\theta 4}=-6\cdot\dfrac{(1-p_1T_4)}{6K+q_0}\cdot\dfrac{(6K+7q_0)-(6Kp_1+7q_1)T_4}{(3K+2q_0)-(3Kp_1+2q_1)T_4}\cdot\left[\dfrac{1-p_1T_4}{1-p'_1T_4}\cdot\dfrac{q'_0-q'_1T_4}{q_0-q_1T_4}-1\right]\cdot P_{\theta 2} \end{cases} \quad (9)$$

where the universal function of bulk-strain $S_\theta(T)$ is given by

$$S_\theta(T)=P_{\theta 1}+P_{\theta 2}\cdot\frac{(6K+7q_0)-(6Kp_1+7q_1)T}{(6K+q_0)-(6Kp_1+q_1)T} \quad (10)$$

where the parameters $P_{\theta 1}$ and $P_{\theta 2}$ which related to the coordinate system are

$$\begin{cases} P_{\theta 1}=\dfrac{1}{r_1^5}-\dfrac{10H(H+Z)}{r_2^7} \\ P_{\theta 2}=\dfrac{1}{r_2^5} \end{cases} \quad (11)$$

When t=0, the bulk-strain $\theta(r,0)$ is

$$\theta(r,0)=\frac{3V\tau xy}{4\pi}\{M_{\theta 0}+M_{\theta 1}+M_{\theta 2}+M_{\theta 3}+M_{\theta 4}\} \quad (12)$$

4 Ground tilt field of three-dimensional rheologic model

4.1 x-direction tilt $\gamma_x(r, t)$

In the theory of three-dimensional elastic inclusion, the x-direction tilt $r_x(r)$ at a point (x, y, z) is equal to the x-direction partial differential of the z-direction displacement $w(r)$. Dobrovolskii (1991) gave the elastic solution of x-direction tilt at a point (x, y, z), $\gamma_x(r)=\partial w/\partial x$, as follows:

$$\gamma_x(r)=\frac{\alpha V\tau}{8\pi\mu(1-\nu)}y\left(F_7+x^2F_{13}\right) \tag{13}$$

where V denotes the inclusion volume, μ and ν the elastic coefficients of medium, K the elastic modulus of medium, α the increment of the ratio of the shear modulus inside the inclusion to that outside the inclusion, respectively. F_7 and F_{13} are expressed as follows:

$$\begin{cases} F_7=\dfrac{3(H-z)}{r_1^5}+\dfrac{3(3-4\nu)(H-z)}{r_2^5}+\dfrac{30Hz(H+z)}{r_2^7}-\dfrac{4(1-2\nu)(1-\nu)(2r_2+H+z)}{r_2^3(r_2+H+z)^2} \\ F_{13}=-\dfrac{15(H-z)}{r_1^7}-\dfrac{15(3-4\nu)(H-z)}{r_2^7}-\dfrac{210Hz(H+z)}{r_2^9}+ \\ \qquad \dfrac{12(1-2\nu)(1-\nu)}{r_2^5(r_2+H+z)}+\dfrac{4(1-2\nu)(1-\nu)(5r_2+3H+3z)}{r_2^4(r_2+H+z)^3} \end{cases} \tag{14}$$

The $\gamma_x(r)$ in equation (13) can be transformed into

$$\gamma_x(r)=\frac{\alpha V\tau}{8\pi\mu(1-\nu)}y\left[P_{\gamma x1}+P_{\gamma x2}(3-4\nu)+P_{\gamma x3}(1-2\nu)(1-\nu)\right] \tag{15}$$

$$\begin{cases} P_{\gamma x1}=\dfrac{3(H-z)}{r_1^5}+\dfrac{30Hz(H+z)}{r_2^7}-x^2\left[\dfrac{15(H-z)}{r_1^7}+\dfrac{210Hz(H+z)}{r_2^9}\right] \\ P_{\gamma x2}=\dfrac{3(H-z)}{r_2^5}-\dfrac{15(H-z)x^2}{r_2^7} \\ P_{\gamma x3}=-\dfrac{4(2r_2+H+z)}{r_2^3(r_2+H+z)^2}+\dfrac{12x^2}{r_2^5(r_2+H+z)}+\dfrac{4(5r_2+3H+3z)x^2}{r_2^4(r_2+H+z)^3} \end{cases} \tag{16}$$

According to the Corresponding Principle, $\gamma_x(r)$ in equation (15) undergoes Laplace transform to obtain the viscoelastic solution on the image-plane:

$$\bar{\gamma}_x^{(v)}(r,s)=\frac{V\tau y}{8\pi}\left[\frac{N_{x0}}{s}+\frac{N_{x1}+N''_{x1}}{s+\dfrac{q_0}{q_1}}+\frac{N'_{x1}}{\left(s+\dfrac{q_0}{q_1}\right)^2}+\frac{N_{x2}}{s+\dfrac{1}{p'_1}}+\frac{N_{x3}}{s+\dfrac{3K+2q_0}{3Kp_1+2q_1}}+\frac{N_{x4}}{s+\dfrac{6K+q_0}{6Kp_1+q_1}}\right] \tag{17}$$

Using the inverse Laplace transform for $\bar{\gamma}_x^{(v)}(r,s)$ in equation (17), the x-direction tilt $\gamma_x(r, t)$ can be obtained:

$$\begin{aligned}\gamma_x(r,t)=\frac{V\tau y}{8\pi}\Bigg[&N_{x0}H(t)+(N_{x1}+N''_{x1}+tN'_{x1})\exp\left(-\frac{q_0}{q_1}t\right)+N_{x2}\exp\left(-\frac{1}{p'_1}t\right)+ \\ &N_{x3}\exp\left(-\frac{3K+2q_0}{3Kp_1+2q_1}t\right)+N_{x4}\exp\left(-\frac{6K+q_0}{6Kp_1+q_1}t\right)\Bigg]\end{aligned} \tag{18}$$

where N_{xi} (i=0, 1, 2, 3, 4) in equations (17) and (18) are expressed as follows:

$$\begin{cases}N_{x0}=2\left(\dfrac{q_0'}{q_0}-1\right)\cdot\left(\dfrac{S_{\gamma x}(0)}{q_0(3K+q_0)}+\dfrac{3P_{\gamma x3}}{6K+q_0}\right)\\ N_{x1}=\dfrac{2(1-p_1T_1)}{q_0}\left[\dfrac{S_{\gamma x}(T_1)}{(3K+2q_0)-(3Kp_1+2q_1)T_1}-\dfrac{1-p_1T_1}{1-p_1'T_1}\cdot\dfrac{3(q_0'-q_1'T_1)P_{\gamma x3}}{(6K+q_0)-(6Kp_1+q_1)T_1}\right]\\ N_{x2}=-2(1-p_1T_2)^2\dfrac{q_0'-q_1'T_2}{q_0-q_1T_2}\left\{\dfrac{S_{\gamma x}(T_2)}{(q_0-q_1T_2)[(3K+2q_0)-(3Kp_1+2q_1)T_2]}+\dfrac{3P_{\gamma x3}}{(6K+q_0)-(6Kp_1+q_1)T_2}\right\}\\ N_{x3}=-\dfrac{2(1-p_1T_3)}{(3K+2q_0)(q_0-q_1T_3)}\left(\dfrac{1-p_1T_3}{1-p_1'T_3}\cdot\dfrac{q_0'-q_1'T_3}{q_0-q_1T_3}-1\right)\cdot S_{\gamma x}(T_3)\\ N_{x4}=-\dfrac{6\,(1-p_1T_4)}{6K+q_0}\left(\dfrac{1-p_1T_4}{1-p_1'T_4}\cdot\dfrac{q_0'-q_1'T_4}{q_0-q_1T_4}-1\right)\cdot P_{\gamma x3}\\ N_{x1}'=-\dfrac{2(q_0'-q_1'T_1)(1-p_1T_1)^2}{q_0q_1(1-p_1'T_1)[(3K+2q_0)-(3Kp_1+2q_1)T_1]}\cdot S_{\gamma x}(T_1)\\ N_{x1}''=2\cdot[P_{\gamma x1}\cdot S(6K+q_0,6Kp_1+q_1)+P_{\gamma x2}\cdot S(6K+7q_0,6Kp_1+7q_1)]\end{cases} \tag{19}$$

Where, $P_{\gamma xi}$ (i=1, 2, 3) are same as those in equation (16), and $S_{\gamma x}(T)$ can be written as:

$$S_{\gamma x}(T)=P_{\gamma x1}\cdot[6K+q_0-(6Kp_1+q_1)T]+P_{\gamma x2}\cdot[6K+7q_0-(6Kp_1+7q_1)T] \tag{20}$$

When t=0, the x-direction tilt $\gamma_x(r, t)$ is

$$\gamma_x(r,0)=\frac{V\tau y}{8\pi}\left(N_{x0}+N_{x1}+N_{x1}''+N_{x2}+N_{x3}+N_{x4}\right) \tag{21}$$

4.2 y-direction tilt $\gamma_y(r, t)$

In the theory of three-dimensional elastic inclusion, the y-direction tilt $\gamma_y(r)$ at a point (x, y, z) is the y-direction partial differential of the z-direction displacement $w(r)$. Dobrovolskii (1991) gave the elastic solution of y-direction tilt at a point (x, y, z), $\gamma_y(r)=\partial w/\partial y$, as follows:

$$\gamma_y(r)=\frac{\alpha V\tau}{8\pi\mu(1-\nu)}x\left(F_7+y^2F_{13}\right) \tag{22}$$

Where, F_7 and F_{13} are same as those in equation (14). The $\gamma_y(r)$ in equation (22) can be transformed as

$$\gamma_y(r)=\frac{\alpha V\tau}{8\pi\mu(1-\nu)}x[P_{\gamma y1}+P_{\gamma y2}(3-4\nu)+P_{\gamma y3}(1-2\nu)(1-\nu)] \tag{23}$$

Where, $P_{\gamma yi}$ (i=1, 2, 3) are

$$\begin{cases}P_{\gamma y1}=\dfrac{3(H-z)}{r_1^5}+\dfrac{30Hz(H+z)}{r_2^7}-y^2\left[\dfrac{15(H-z)}{r_1^7}+\dfrac{210Hz(H+z)}{r_2^9}\right]\\ P_{\gamma y2}=\dfrac{3(H-z)}{r_2^5}-\dfrac{15(H-z)y^2}{r_2^7}\\ P_{\gamma y3}=-\dfrac{4(2r_2+H+z)}{r_2^3(r_2+H+z)^2}+\dfrac{12y^2}{r_2^5(r_2+H+z)}+\dfrac{4(5r_2+3H+3z)y^2}{r_2^4(r_2+H+z)^3}\end{cases} \tag{24}$$

According to the Corresponding Principle, the equation (23) undergoes Laplace transform to obtain the y-direction tilt as follows:

$$\gamma_y(r,t)=\frac{V\tau x}{8\pi}\Bigg[N_{y0}H(t)+(N_{y1}+N_{y1}''+tN_{y1}')\exp\left(-\frac{q_0}{q_1}t\right)+N_{y2}\exp\left(-\frac{1}{p_1'}t\right)+ N_{y3}\exp\left(-\frac{3K+2q_0}{3Kp_1+2q_1}t\right)+N_{y4}\exp\left(-\frac{6K+q_0}{6Kp_1+q_1}t\right)\Bigg] \tag{25}$$

Where, the parameters N_{yi} (i=0, 1, 2, 3, 4) can be written by substituting $P_{\gamma xi}$ in (19) with $P_{\gamma yi}$ in (24). When t=0, y-direction tilt is

$$\gamma_y(r,0)=\frac{V\tau x}{8\pi}(N_{y0}+N_{y1}+N''_{y1}+N_{y2}+N_{y3}+N_{y4}) \tag{26}$$

When $t\to\infty$, y-direction tilt is

$$\gamma_y(r,\infty)=\frac{V\tau x}{8\pi}N_{y0} \tag{27}$$

5 Viscoelastic solution for underground water level of three-dimensional rheologic model

The elevation and subsidence of well water level is the indirect effect of bulk-strain in the aquifer system (Rhoabs and Robinson, 1979; HOU, 1984). The water level variation Δh is related to bulk-strain θ, porosity of medium, density of water, *etc*. It can be written as (HOU, 1984)

$$\Delta h=\frac{\theta}{S_s} \tag{28}$$

Where the unit storage coefficient $S_s=\rho\, g[(1-\varphi)/k_r+\varphi/k_w]$, ρ denotes the density of water, g the gravity acceleration, φ the porosity, k_r the rock modulus of compression, k_w the water modulus of compression, respectively. The unit storage coefficient S_s is different from place to place. Rhoabs and Robinson (1979) showed S_s is in order of 10^{-8} resulted from the calculation of the wells in USA. HOU (1984) showed $S_s=5.0\times10^{-9}$/cm after studying the water levels of ten wells in Jiangsu Province of China.

In order to study the effect of bulk-strain θ on the water level variation Δh, we refer to the bulk-strain expression of rheologic inclusion model given by the equation (8) in this paper. Then, the viscoelastic solution of water-level at a point (x, y, z), the Δh (r, t), can be given by

$$\begin{aligned}\Delta h(r,t)=\frac{3V\tau xy}{4\pi S_s}\Bigg[&M_{\theta0}H(t)+M_{\theta1}\exp\left(-\frac{q_0}{q_1}t\right)+M_{\theta2}\exp\left(-\frac{1}{p_1'}t\right)+\\&M_{\theta3}\exp\left(-\frac{3K+2q_0}{3Kp_1+2q_1}t\right)+M_{\theta4}\exp\left(-\frac{6K+q_0}{6Kp_1+q_1}t\right)\Bigg]\end{aligned} \tag{29}$$

where

$$\begin{cases}M_{\theta0}=6\left(\dfrac{q_0'}{q_0}-1\right)\cdot\dfrac{S_\theta(0)}{3K+2q_0}\\ M_{\theta1}=-6\cdot\dfrac{q_0'-q_1'T_1}{q_0}\cdot\dfrac{(1-p_1T_1)^2}{1-p_1'T_1}\cdot\dfrac{S_\theta(T_1)}{(3K+2q_0)-(3Kp_1+2q_1)T_1}\\ M_{\theta2}=-6\cdot\dfrac{q_0'-q_1'T_2}{q_0-q_1T_2}\cdot\dfrac{(1-p_1T_2)^2}{(3K+2q_0)-(3Kp_1+2q_1)T_2}\cdot S_\theta(T_2)\\ M_{\theta3}=-6\cdot\dfrac{1-p_1T_3}{3K+2q_0}\cdot\left(\dfrac{1-p_1T_3}{1-p_1'T_3}\cdot\dfrac{q_0'-q_1'T_3}{q_0-q_1T_3}-1\right)\cdot S_\theta(T_3)\\ M_{\theta4}=-6\cdot\dfrac{1-p_1T_4}{6K+q_0}\cdot\dfrac{(6K+7q_0)-(6Kp_1+7q_1)T_4}{(3K+2q_0)-(3Kp_1+2q_1)T_4}\cdot\left(\dfrac{1-p_1T_4}{1-p_1'T_4}\cdot\dfrac{q_0'-q_1'T_4}{q_0-q_1T_4}-1\right)\cdot P_{\theta2}\end{cases} \tag{30}$$

When t=0, water level becomes

$$\Delta h(r,0)=\frac{3V\tau xy}{4\pi S_s}\left(M_{\theta0}+M_{\theta1}+M_{\theta2}+M_{\theta3}+M_{\theta4}\right) \tag{31}$$

6 Viscoelastic solution for electrical resistivity of three-dimensional rheologic model

Archie's Law is one of the important theoretical bases to study the micro-mechanism of earth resistivity precursor (QIAN, 1985; MEI *et al*, 1993). It is shown that the earth resistivity is related to deformation, fluid and the humidity of rocks. Here, the resistivity of humid rock follows the Archie law and is given by

$$\rho=\alpha\rho_0\varphi^m S^{-n} \tag{32}$$

where α is a constant and ρ_0 denotes the porous-liquid resistivity, φ the porosity respectively. The porous-liquid saturation level $S=V_{\text{liq}}/V$, m and n are the structure indexes. Usually, m=1.3~1.95, n=2. ZHOU *et al* (1994) showed the porosity of sandstone being usually 0.45%~2.21%. Under the action of stress, the porosity φ and the liquid-in-hole saturation will change, and the variation of resistivity $\Delta\rho/\rho$ is given by

$$\frac{\Delta\rho}{\rho}=\left(\frac{n-m}{\varphi}+m\right)\cdot\frac{\Delta V}{V} \tag{33}$$

Here, $\Delta V/V$ is the bulk-strain θ. In order to study the effect of bulk-strain θ on the resistivity, we refer to the bulk-strain of rheologic inclusion model given by the equation (8) in this paper. Then, the viscoelastic solution of the variation of resistivity at a point (x, y, z), $\Delta\rho/\rho(r, t)$ is given by

$$\begin{aligned}\frac{\Delta\rho}{\rho}(r,t)=\frac{3V\tau xy}{4\pi}\left(\frac{n-m}{\varphi}+m\right)\cdot\Bigg[&M_{\theta 0}H(t)+M_{\theta 1}\exp\left(-\frac{q_0}{q_1}t\right)+M_{\theta 2}\exp\left(-\frac{1}{p_1'}t\right)+\\&M_{\theta 3}\exp\left(-\frac{3K+2q_0}{3Kp_1+2q_1}t\right)+M_{\theta 4}\exp\left(-\frac{6K+q_0}{6Kp_1+q_1}t\right)\Bigg]\end{aligned} \tag{34}$$

When t=0, the viscoelastic solution of resistivity variation is as follows:

$$\frac{\Delta\rho}{\rho}(r,0)=\frac{3V\tau xy}{4\pi}\left(\frac{n-m}{\varphi}+m\right)\cdot\left(M_{\theta 0}+M_{\theta 1}+M_{\theta 2}+M_{\theta 3}+M_{\theta 4}\right) \tag{35}$$

7 Characteristics of bulk-strain on the ground surface

In order to test the correctness of the derived results and computed programs, Song et al. (2000) analyzed comparatively the bulk-strain produced by a hard-inclusion and a soft-inclusion.

7.1 Quadrant of bulk-strain produced by a hard inclusion and a soft one

The characteristics of the bulk-strain spatial field of the hard inclusion and the soft one are similar in that both have quadrant distribution and the distribution inside the region with the size two times the extension of seismic source is contrary to the one outside this region. The dissimilarity is that the bulk-strain high-value regions produced by the hard inclusion correspond rightly to the low-value regions of the soft inclusion.

7.2 Pattern features of the bulk-strain versus time

Pattern features of the bulk-strain versus time produced by a hard-inclusion are as follows: (1) The bulk-strain pattern varies with the distance to the epicenter. The bulk-strain in the source region goes through a process of "increasing → maximum → decreasing → tend towards a stable value" and seems like an upward protruding (Fig. 2). For the points around the region with its distance to seismic foci about two times the extension of seismic source, their bulk-strains go down firstly, then ascend and show the characteristics of "N". The bulk-strains at the points outside the region with the size two times the extension of seismic source show the characteristics of downward protruding. The patterns at points far away from the epicenter are basically similar to the seismic source, yet their variation values are comparatively smaller. (2) The variation value of bulk-strain decreases as the distance increases.

It is meaningful that the pattern of bulk-strain produced in the soft inclusion is different from that in the hard inclusion. The bulk-strain values produced in the soft inclusion are all increase gradually with the different epicenter distances and then tend towards a stable value, but there is no decreasing process after the maximum value.

7.3 Three-stage characteristics of the bulk-strain produced in a hard-inclusion

The bulk-strain in the hard-inclusion possesses three-stage characteristics of "increasing → maximum → decreasing to a stable value". That is, during the stage α, the bulk-strain value increases gradually with the passage of time and the bulk-strains in the near-source region and the far-source region all show outward extension. During the stage β, the bulk-strain value and its range all vary a little with the time and reach to extreme state. Other relevant physical parameters should be extreme too, then earthquake precursors during the stage β should be most significant and occur almost at the same time in the near-source region and the far-source region. During the stage γ, the bulk-strain value weakens as time goes on, and the corresponding spatial characteristics are that the bulk-strain of the far-source region shows contraction to the epicenter and the bulk-strain of the near-source area migrates outwards.

7.4 Characteristics of bulk-strain produced in a soft-inclusion

The study shows that the bulk-strains in the near-source region and the far-source area produced in the soft inclusion all diffuse outwards and their diffusing directions are conjugate. If a soft inclusion is regarded as an earthquake preparation model, the spatial-temporal evolution of precursors should diffuse outwards in the whole seismic preparation process.

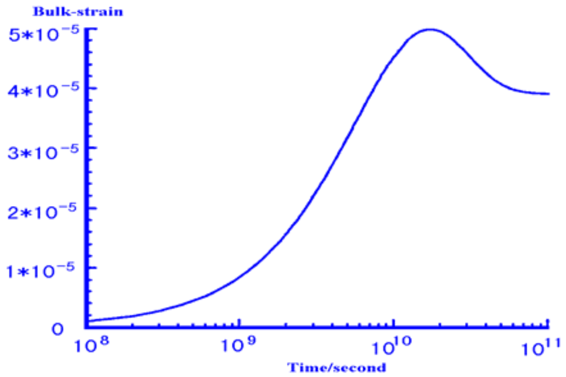

Figure 2 Bulk-strain curve versus time in the near-source region of the hard inclusion

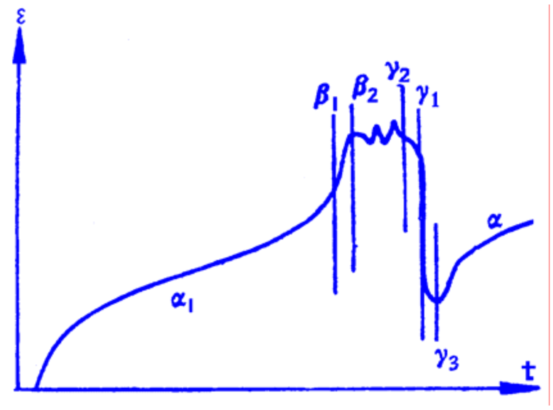

Figure 3 The three stages (α,β and γ) of deformation (Mesherikov, 1968; Fujii, 1974)

8 Application of theoretical result to earthquake precursors

There exist three stages (namely α,β and γ) in the process of bulk-strain versus time in the near-source region caused by the hard inclusion (Figure 2). Having studied abundant observation data of crust deformation, Mesherikov (1968) and Fujii (1974) found that the deformation curves have characteristics of three stages in the earthquake preparation process, namely, α,β and γ (Figure 3). Though our theoretical curves cannot give the relative proportion of the three stages, the similarity of basic patterns of the theoretical and empirical curves supports the scientific reliability of the theoretical curves. This also supports the rationality of the strong-body earthquake preparation model.

The bulk-strain curve pattern of the hard inclusion in the near-source region is "small → big → maximum → decreasing to stability" and shows a upward protruding variation. At points around the region with its distance to foci about two times the extension of seismic source, the pattern of bulk-strain curves is "descending → ascending" and shows the characteristic of " N" type. The bulk-strain types at points near the region of three times the extension of seismic source show downward protruding variation. The bulk-strain at points far away the inclusion has little variation. These are similar to phenomena in the earthquake observation that the bulk-strain abnormal types vary with the increase of the distance (Mei, et al, 1993). In other words, this phenomenon does not occur occasionally, but possesses physical basis.

Li (1981) found that the water-radon anomalies show the development pattern outward half a year before the Tangshan earthquake. Zheng (1979) had studied filtered water-level data and found that the water-level descending anomalies develop outwards in the directions of north-east and north-west half year before the Tangshan earthquake. There also exist the characteristics about half year before the Longling earthquake and the Songpan shock (Mei, et al, 1993). Song's studies (Song,2000; Song et al.,2006) show that the three stages (α,β and γ) are very evident. Especially, there all exist the sub-phase γ_1 of precursors contracting from the periphery to the epicenter region and the sub-phase γ_2 of precursors diffusing from the epicenter area to the periphery during the γ-stage (corresponding to earthquake short-term and imminent-term stage). It can be seen that the existence of α-,β- and γ-stage and the spatial-temporal migration of precursors may be controlled by the earthquake preparation body. It may be a usual phenomenon and should be an important base to predict earthquakes.

Quadrant distribution is one of the common characteristics of the spatial distribution of bulk-strain on the ground with differences in some details associated with the earthquake preparation. The spatial range and the values of the anomaly in bulk-strain all increase with time and decrease after reaching the maximum value in the model of hard inclusion. For the soft model there is only trend variation, however. Experimental results (Xu et al., 1997) show compatible quadrant distribution. In earthquake precursors, there exist some consistent quadrant distribution of anomalies in the deformation and geo-resistivity before the cases of Datong earthquake, Zhangbei earthquake and Baotou earthquake etc (Song, et al., 2001). This feature would be useful for predicting the location of a strong earthquake.

9 Conclusions

So far we have described the derivation of the bulk-strain and ground deformation and the preliminary application based on the theory of rheologic inclusion model in the seismogenic process. The results are as follows:

Applying the theoretical expression of the three-dimension rheologic inclusion model, we computed in detail the spatial-temporal evolution process of bulk-strain on the ground produced by a spherical rheologic inclusion in semi-infinite rheologic medium. The results show that spatial-temporal evolution of bulk-strain produced by a hard inclusion has three stages of different characteristics, which are similar to most of the geodetic deformation curves, but not for the case of soft inclusion. The α-stage characteristic is a long stage in which the precursors in both the near-field and the far-field develop from the focal region to the periphery. The β-stage shows a very rapid propagation of the precursors, which they almost appear everywhere. During the γ-stage, the precursors in the far-field converge from the periphery, and the precursors in the near-field develop outwards.

The theoretical results have been applied to explain tentatively the stage-variation of the spatial-temporal evolution, the pattern-feature and quadrant-distribution characteristics of earthquake precursors. It is a theoretical base to found the physical model of earthquake precursors and a reference to predict physically earthquakes.

There are still some questions to be studied, for example, the viscoelastic expression of geo-electricity, geo-magnetism and wave velocity and so on.

Acknowledgements: Gratefully acknowledge the support from National Geophysical Observatory at Sheshan (grant num: 2007FY220100)，the Chinese Joint Seismological Science Foundation (grant num: 106078) and NSFC (grant num: 40576022).

References

Aki K. and Richards P G. (1980). Quantitative Seismology: Theory and Methods. San Francisco: W. H. Freeman and Co., 50~57.

Brady B T. (1974). Theory of earthquake. *Pure and Applied Geophysics,* **112:4,** 701~725.

Dobrovolskii I P. (1991). Theory on the preparation of tectonic earthquakes, Moscow: Ivz. An SSSR, 1~218 (in Russian).

Eshelby J D. (1957). The determination of the elastic field of an ellipsoidal inclusion and related problem, *Proceedings of the Royal Society*, **A241**, 376~396.

FENG Zhi-sheng, YANG Jian-jun, MEI Wei-ping et al. (2004). Preliminary study on characteristic of seismic precursor of earth resistivity anisotropy degree in East China. *Acta Seismologica Sinica*, **17:2,** 244~250.

Fujii Yoichiro (1974). Relationship between duration time of anomalous vertical deformation of the crust and magnitude before the occurrence of an earthquake. *Earthquake*, **27:3**, 197~213 (in Japanese).

Li Xuanhu (1981). Phenomena of diffusion and contraction of water-radon precursors. *Earthquake*, **3:5,** 340~357 (in Chinese).

HOU Mao-sheng (1984). On the relation between the stress-strain condition at depth and the water level variation in the SU-10 water well. *Acta Seismologica Sinica*, **6:1**, 90~97 (in Chinese).

Mei Shirong, Feng Deyi, Zhang Guomin, et al. (1993). Introduction to Earthquake Prediction in China. Beijing: Seismological Press, 241~263 (in Chinese).

Mei Shirong (1994). A study on the physical model of earthquake precursor field and the mechanism of precursors' time-space distribution ——Origin and evidences of the strong body earthquake-generating model, *Acta Seismologica Sinica*, **7:3**, 273~282.

Mesherikov J A. (1968), Recent Crust movements in seismic region: Geodetic and geomorphic data,

Tectonophysics, **6:1**,29~39.
QIAN Jia-dong (1985). Application on Earthquake Prediction for Earth Resistivity. Beijing: Seismological Press, 1~58 (in Chinese).
Rhoabs G. H. and Robinson E S. 1979. Determination of aquifer parameters from well tides. *J Geophys Res*, **84:11**, 6 071~6 082.
Song Zhiping (1996). On the Spatial Temporal Evolution of the Load/Unload Response Ratio (LURR) and Other Earthquake Precursors and Their Implications, (Ph D. Thesis), Beijing: Institute of Geophysics, China Seismological Bureau, 15~110 (in Chinese).
Song Zhiping, Yin Xiangchu and Mei Shirong (2000). Theoretical analysis of the spatio-temporal evolution of the bulk-strain field based on a rheologic inclusion model, *ACTA Seismologica Sinica*, **13:5**,525-535.
Song Zhiping, Xu Ping, Zhang Hongkui, et al. (2001). Quadrant characteristics of earthquake precursors and its theoretical analysis, *ACTA Seismologica Sinica*, **14:1**,58-65.
Song Zhiping, Yin Xiangchu and Mei Shirong (2003). Three-dimensional rheologic model of earthquake preparation (1) displacement field, *ACTA Seismologica Sinica*, **16:6**,588-597.
Song Zhiping, Yin Xiangchu and Mei Shirong (2004). Three-dimensional rheologic model of earthquake preparation (2)strain field and its applications, *ACTA Seismologica Sinica*, **17:2**,131-143.
Song Zhi-ping，Mei Shi-rong，Yin Xiang-chu (2006). Three-dimensional rheologic model of earthquake preparation (3) precursor field，*ACTA Seismologica Sinica*, **19:1**,20-29.
Song Zhi-ping，Yin Xiang-chu，Mei Shi-rong and et al. (2006). The displacement and strain field of three-dimensional rheologic model of earthquake preparation, *Pure and Applied Geophysics*, **163:9**, 1991-2009.
SUN Qi-zheng, ZHANG Guo-min, LI Xuan-hu et al. (1998). Method of Analysis and Prediction Earthquake in Electromagnetics. Beijing: Seismological Press, 190~191 (in Chinese).
Xu Zhaoyong, Wang Bin, Zhao Jinming, et al. (1997). An experimental study of fractural features of concrete samples containing hard inclusion, *ACTA Seismologica Sinica*, **10:1**,97-105.
XUE Yan, SONG Zhi-ping, SONG Xian-yue et al. (2003). Theoretic analysis for tendency anomalies′ type of deformation precursors in earthquake-generating process. *Earthquake Research in China*, **19:3**,306~314 (in Chinese).
Yin Xiangchu and Zhen Tianyu (1982). Rheologic model of earthquake preparation process, *Scientia Sinica (series B)*, **10**,922~930.
Yin Xiangchu (1985). Solid Mechanics, Beijing: Seismological Press, 397~407. (In Chinese)
Zheng Zhizhen (1979). On generation process of the Tangshan earthquake basing on water-level filtered data, *Acta Geophysica Sinica*, **22:3**, 267~280 (in Chinese).
ZHOU Bo-ran, TIAN Zhong-yuan, ZHAO Bi-hua (1994). Determining the porosity of sandstone with CT technique. *Well Logging Technique*, **18:3**, 178~184 (in Chinese).