PROBABILITIC ASSESSMENT OF EARTHQUAKE DISASTER OCCURANCE IN A BIG CITY FOR PRICING CAT BONDS

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ABSTRACT :

As well known, pricing for earthquake insurance or CAT bonds, depends on the seismic risk in a city, in brief, the occurring probability of earthquake disaster. From a definition of the disaster, this probability can be derived from the probabilities of economic losses and casualties in a given future time period. A scenario earthquake approach is introduced for the risk analysis in this paper. The seismic hazard is characterized by some earthquakes with estimated pairs of magnitude and distance, the losses and casualties are calculated by means of GIS based information and decision-making support system, in which the vulnerability matrix of the building environment and the total property in the buildings are stored and organized well. As an example, a case study for a big city is presented. The result of occurring probability of earthquake disaster is compared with those from traditional method.

KEYWORDS: occurring probability, scenario earthquake, GIS

1. INTRODUCTION

Engineering seismic risk assessment, consisting of seismic hazard analysis and vulnerability evaluation, is a quite comprehensive process as the first and fundamental step in earthquake disaster prevention and reduction. In China, seismic hazards and vulnerabilities are assessed for many cities in recent dozen years. Seismic hazard analysis is a fundamental part of seismic risk assessment. The widely adopted probabilistic approach of seismic hazard analysis involves with regional seismicity, potential earthquake sources and ground motion attenuation. The vulnerability evaluation is to infer a conditional probability of the damage state of engineering structure given various levels of earthquake intensity. It describes a relationship between damage and ground motion for a specific type of structure. In general, these two are completed respectively by seismological team and engineering team, and are combined together for loss estimation. In some cases, there will be gaps in between. The expression of seismic hazard must be improved to combine them together, especially for the application in financial instruments such as earthquake insurance and CAT bonds.

The result of seismic hazard analysis is the occurring or exceeding probability of an earthquake action in a city or region for a specified exposure time. Even if for high intensity, it is still the probability of the intensity in the area. High intensity areas are always limited and cannot cover the whole metropolis because of the fast attenuation in epicenter area. And it never happens that several earthquakes occur in a given period to cause the same intensity and cover the whole metropolis one by one (Tao and Tao, 2006).

For illustration, a case study is presented in this paper. High intensities and their probabilities of occurrence are converted into three scenario earthquakes to represent the probabilistic meaning in the deterministic way. In order to produce the exceeding probability curves of loss rate, it is necessary to analyze the damage from the scenario earthquakes. Many spatial operations must be involved in the calculation, and the amount of data available for each event is in tens of thousands of items. A GIS based decision-making support system for earthquake disaster reduction can be adopted for its spatial operating capacity. By this way, the overestimation of losses in metropolis is avoided. Finally, the exceeding probability curves of loss rate, the ratio of earthquake losses to Industry Value Added (IVA) in the last year, from both of the traditional and modified approaches are drawn and compared with each other.

2. TABLES ASSESS THE OCCURRING PROBABILITY OF EARTHQUAKE DISASTER

2.1. Assessment at Low Intensity Range

In general, seismic hazard is described by a hazard curve, which shows the exceeding probabilities of a set of ground motion amplitudes. The probability of S_{th} type buildings being in k_{th} damage state is referred to as the engineering seismic risk, can be described as

$$P_S(D_k) = \sum_{I=6}^{9} P_S(D_k|I) \cdot P(I) \tag{1}$$

where, $P_S(D_k|I)$ is the conditional probability of S_{th} type buildings being in k_{th} damage state given intensity; $P(I)$ is the occurring possibility of intensity I, and can be derived from $P(I \geqslant i)$, so-called seismic hazard. The latter depends on the regional seismic environment and attenuation relationship of ground motion, and is generally referred to as seismic hazard curve. In nature, earthquake intensity is a sequential classified variable, so $P(I \geqslant i)$ is not really a continuous curve. Surely, intensity I in Eqn. 1 can be substituted with other ground motion parameter Y, by this way, the symbol "Σ" must be substituted to "∫", then $P(Y>y)$ is a continuous curve.

According to the Total Probability Theorem, the probability that intensity I greater than given value i at a site, from all the earthquakes in i_{th} potential source area with magnitude in j_{th} magnitude interval, can be calculated from Eqn. 2,

$$P_{ij}(I \geq i) = \sum_{n=0}^{\infty} P_{ij}(n) \cdot P_{ij}\left(I \geq i | n\right) \tag{2}$$

Presume earthquakes in future period are independent each other, $1\text{-}P(I<i \mid E_{ij})^n$ is adopted instead of $P_{ij}(I \geqslant i \mid n)$ in Eqn. 2, then

$$P_{ij}(I \geq i) = \sum_{n=0}^{\infty} P_{ij}(n) \cdot \left\{1.0 - [1.0 - P(I \geq i | E_{ij})]^n\right\} \tag{3}$$

where, E_{ij} is for the condition of one earthquake occurs in i_{th} potential source area with magnitude in j_{th} magnitude interval; n is the number of earthquakes; $P_{ij}(n)$ is the probability of n earthquakes occurs in i_{th} potential source area with magnitude in j_{th} magnitude interval.

Under the hypothesis of independence, the probability of n earthquakes occur in given period t can be calculated by

$$P(n) = C_t^n P^n (1-P)^{t-n} \tag{4}$$

Since earthquake is infrequent, i.e. $t \gg n$, $P(n)$ can be approximated by

$$P(n) \approx \frac{t^n}{n!} P^n (1-P)^t = \frac{(Pt)^n}{n!}(1-P)^t \tag{5}$$

Let μ be the average annual occurring probability. $P\rightarrow\mu$ and $(1-\mu)^t \rightarrow e^{-\mu t}$ if $t\rightarrow\infty$, according to the Large Number Law. $P_{ij}(n)$ can be represented in Poisson model as

$$P_{ij}(n) = \frac{(\mu_{ij}\cdot t)^n}{n!}\cdot e^{-\mu_{ij}\cdot t} \qquad (6)$$

where, μ_{ij} is the annual probability of one earthquake occurs in i_{th} potential source area with magnitude in j_{th} magnitude interval.

Substituting Eqn. 6 into Eqn. 3, the result is

$$P_{ij}(I\geq i) = 1.0 - e^{-\mu_{ij}\cdot t}\cdot\sum_{n=0}^{\infty}[1.0 - P(I\geq i|E_{ij})]^n\cdot(\mu_{ij}\cdot t)^n/n! = 1.0 - e^{-\mu_{ij}\cdot t\cdot P(I\geq i|E_{ij})} \qquad (7)$$

Then, the probability that intensity I greater than given value i, from all the earthquakes in all potential source areas with magnitudes in all magnitude intervals, can be expressed by

$$P(I\geq i) = 1.0 - e^{-\sum_i\sum_j \mu_{ij}\cdot t\cdot P(I\geq i|E_{ij})} \qquad (8)$$

where, $P(I\geqslant i \mid E_{ij})$ depends on the attenuation relationship and is related with the type of potential source areas.

In general, $P(I)$ is calculated by

$$P(I) = P(I\geq i) - P(I\geq i+1) \qquad (9)$$

In some cases, the $P(I)$ may be less than $P(I+1)$, if I is low. The reason is that the Eqn. 9 is exactly correct only under the condition of one earthquake occurs. Dealing with the above procedure, one can find that $P(I\geqslant i)$ is contributed by earthquakes in many potential source areas with various magnitudes and various occurring times. It means that $P(I\geqslant i)$ consists of not only $P(I=i)$ and $P(I\geq i+1)$, but also $P(I=i$ and $I>i)$. The later cannot be ignored in earthquake active regions for intensities less than Ⅶ and long evaluated period, like 50 or 100 years. The authors suggested a solution. Firstly the exceeding probability in short period t can be calculated from the hazard in long period T by

$$P_t(I\geq i) = 1-[1-P_T(I\geq i)]^{\frac{t}{T}} \qquad (10)$$

Obviously, $P(I=i$ *and* $I>i)$ can be ignored when the period is short. Say, one month is short enough for a general region; for region with high seismisity, it must be shorten to days. The occurring probability in the short period can be obtained by

$$P_t(I=i) = P_t(I\geq i) - P_t(I\geq i+1) \qquad (11)$$

Then, the occurring probability in the long period is

$$P_T(I=i) = 1-[1-P_t(I=i)]^{\frac{T}{t}} \qquad (12)$$

2.2. *GIS based method*

There will be an overestimation if the exceeding probabilities of high intensity events are applied directly in loss estimation, even the probability is quite low, since the loss ratios in severe damage or collapse states is much higher than those in low intensity areas, and it is impossible that many earthquakes occur at the same time covering the whole city with high intensities. In general, high intensity areas are always limited and the whole city will not be completely in area with intensity Ⅷ or greater.

Scenario earthquake approach can be adopted as a solution for this overestimation. One or more scenario earthquakes can characterize the hazard with the same exceeding probability. It must be consistent with the regional seismic environment and determined by the regional attenuation relationship of ground motion. By means of the procedure developed by the authors, a scenario earthquake can cause the same intensity in the city with that on the hazard curve, given exceeding probability. The magnitude of the scenario earthquake should be less than the upper bound magnitude of the dominant potential source area, and the distance should be comparative with the potential source area. Many spatial operations are involved in the loss estimation from scenario earthquakes. GIS based system, built for damage evaluation in many cities and regions worldwide, is a powerful tool to perform spatial analysis and map the expected losses due to a scenario earthquake (Zaicenco, 2005).

A GIS based decision-making support system for earthquake disaster reduction (Xie and Tao et al., 2000), constructed for a city in northeast of China, can be adopted for seismic hazard assessment, seismic damage forecast, post-earthquake quick evaluation of seismic losses and decision-making for emergency response as well as post-quake recovering. There are two parts in the system, one is for daily management, and the other is for damage analysis. The former is adopted to manage basic information, involving regional seismic environment, site conditions, engineering environment, lifeline systems environment and relieving basic information. The later is adopted to analyze damages, involving scenario earthquake parameters input, damage and loss assessment, damage inquiry, emergency response plan and report generation. The system consists of 68 coverages and 28 analytical modules. The spatial distributions of casualties, the caused shaking, damage and loss of engineering structures from a scenario earthquake can be assessed very fast by this system.

3. AN EXAMPLE

The city in northeast China is adopted as an example. The result from Eqn. 9, $P_{50}(I=i)$ listed in Table 1, shows $P(I=\text{III})$ is less than $P(I=\text{IV})$, one can see there must be something wrong. In nature, $P(I)$ must be a monotone decreasing function, i.e. $P(I=i)$ must be greater than $P(I=i+1)$. So $P_{50}(I\geqslant i)$ is conversed to the exceeding probability in a short period, like one month, by Eqn. 3, and the occurring probability in 50 years is derived from Eqn. 4 and Eqn. 5, as $P'_{50}(I=i)$ in Table 1.

Table 1. Exceeding probability and occurring probability of intensity in 50 years

Probability	Intensity				
	III	IV	V	VI	VII
$P_{50}(I\geqslant i)$	0.9076	0.6165	0.2545	0.0485	0.0045
$P_{50}(I=i)$	0.2911	0.3620	0.2060	0.0439	0.0044
$P'_{50}(I=i)$	0.7589	0.4855	0.2165	0.0441	0.0044

Comparing values of $P_{50}(I=i)$ with those of $P'_{50}(I=i)$ in Table 1, one can see the difference on low intensity range clearly, those on high intensity range are not so clear, since the seismicity of the site is low. Another site is selected to show the underestimation for higher intensities in a region with high seismicity. The exceeding probabilities of high intensities and the occurring probabilities from the two methods are listed in Table 2.

Table 2. Exceeding probability and occurring probability of high intensity in 50 years

Probability	Intensity				
	III	IV	V	VI	VII
$P_{50}(I \geqslant i)$	0.6069	0.3605	0.2178	0.1088	0.0296
$P_{50}(I=i)$	0.2464	0.1427	0.1090	0.0792	0.0246
$P'_{50}(I=i)$	0.3853	0.1824	0.1223	0.0816	0.0247

Seismic hazard analysis in the first case shows that the occurring probabilities are 4.85%, 0.45% and 0.02% in 50 years corresponding to intensity VI, VII, VIII, so three scenario earthquakes with magnitudes 5.0, 5.5 and 6.0 are designed respectively. The damage distributions from these earthquakes are shown in Figure 1 - Figure 3. In each figure, (a) is for building damage, (b) for road network, (c) for electric power system, and (d) for water supply system. From these figures, one can find the areas with high intensities, VII and VIII, are very limited.

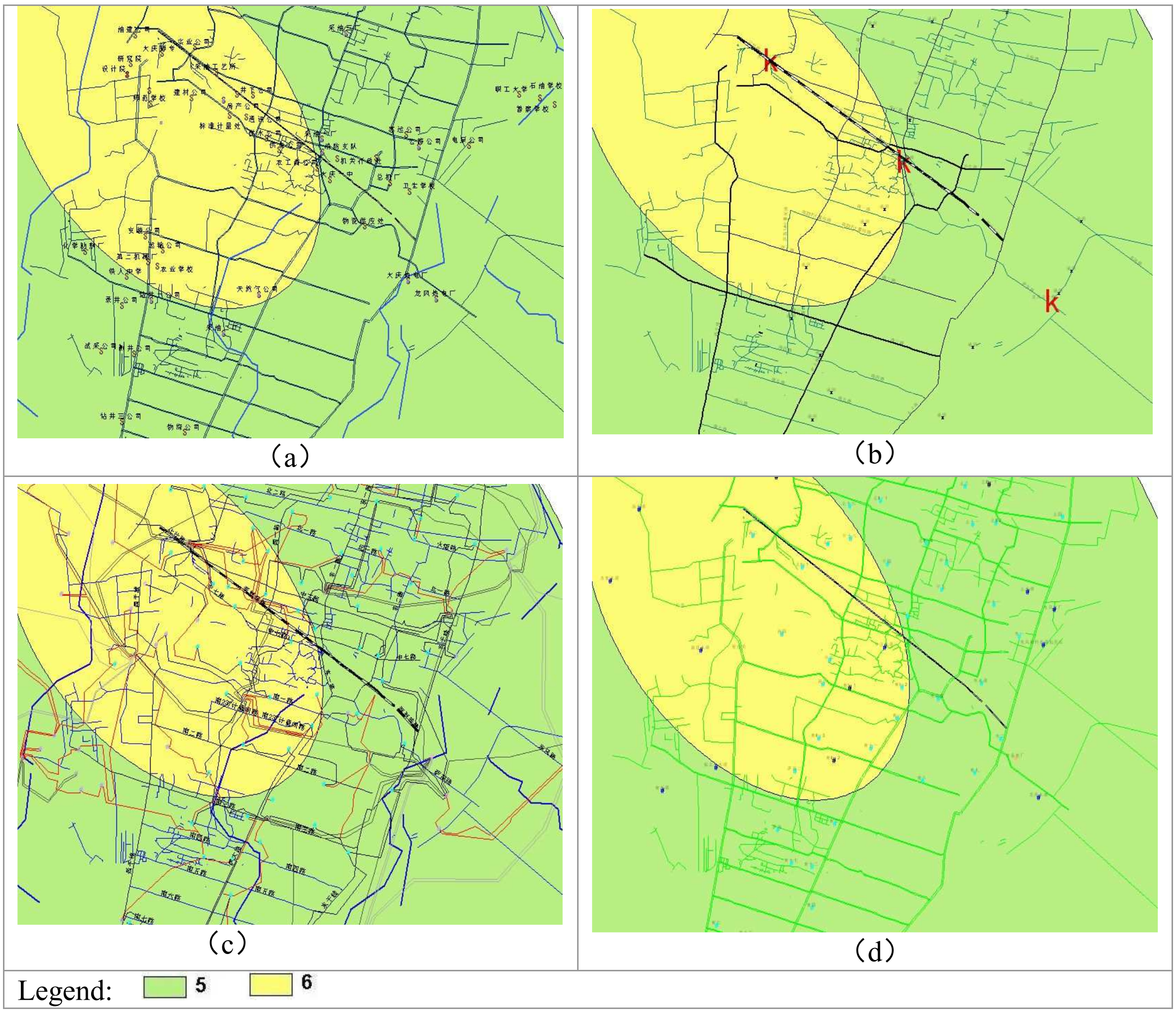


Figure 1 Damage caused by an earthquake with magnitude 5.0 in the case study

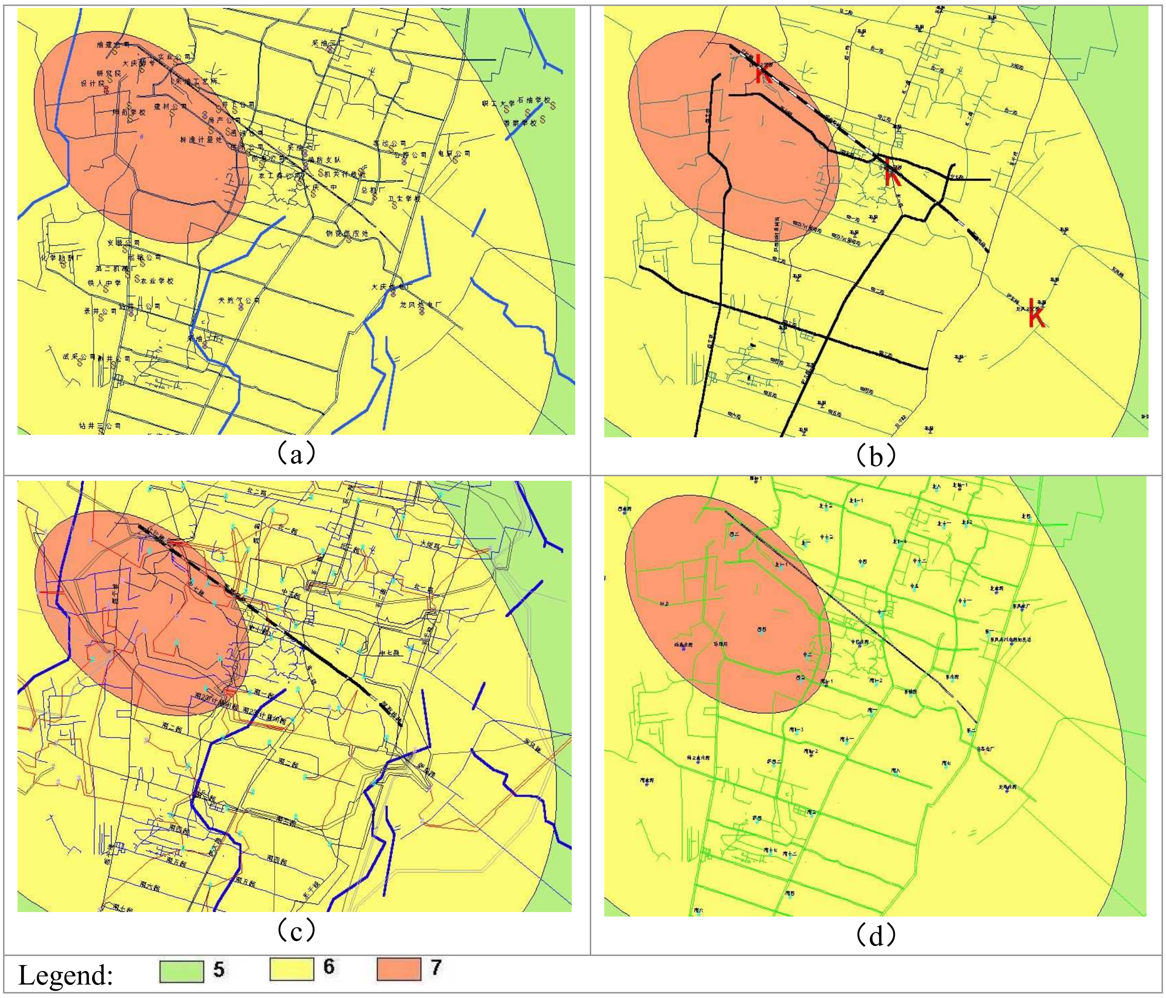
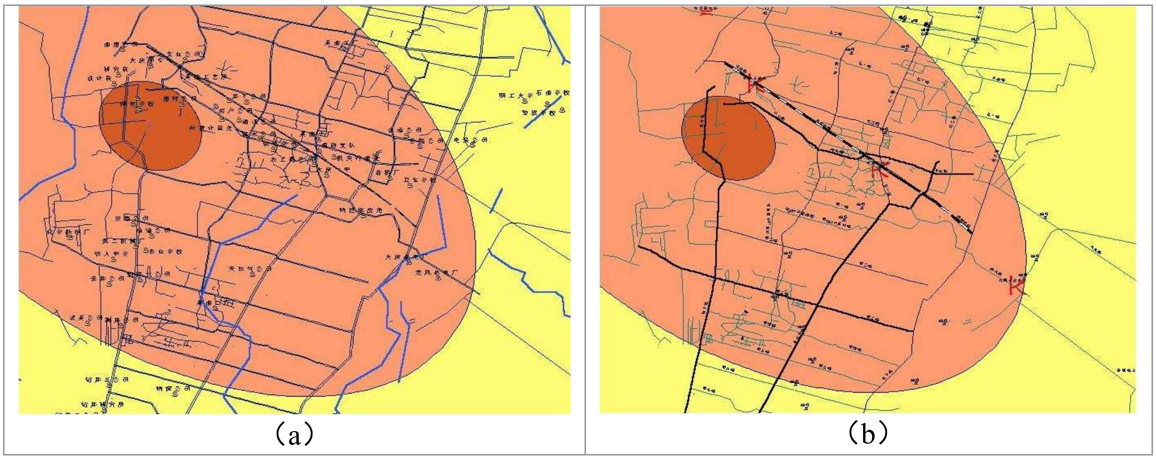


Figure 2 Damage caused by an earthquake with magnitude 5.5 in the case study



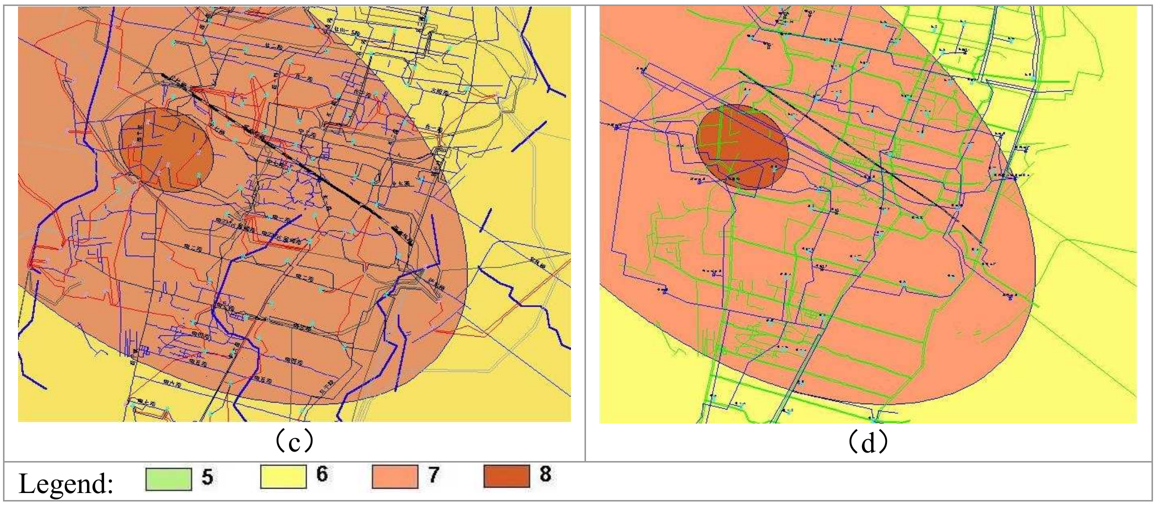


Figure 3 Damage caused by an earthquake with magnitude 6.0 in the case study

The system can estimate gross loss, death and injuries, and their spatial distribution quickly by spatial operating capacity of GIS. The results in this case are listed in Table 3.

Table 3. Gross loss, death, injuries and homeless

Scenario EQ	Loss (RMB Million)	Death	Injuries	Homeless
5.0	28.66	0	0	310
5.5	218.90	2	8	14939
6.0	998.19	100	400	96084

In order to price a CAT bond (Tao and Tao, 2007), the occurring probability of a catastrophe is adopted as an input. According to the definition of a catastrophe (Tang, 1999), the exceeding probability curve of loss ratio, the ratio of loss to IVA in last year, is the dominant parameters. The data in Table 3 and loss in this case estimated by the traditional method (Institute of Engineering Mechanics, 1994) are discounted to values in 2004, considering the influence of interest rate and inflation. Then, the curves of loss to IVA ratio from the two methods are shown in Figure 4. One can find there is an obvious overestimation by the traditional method.

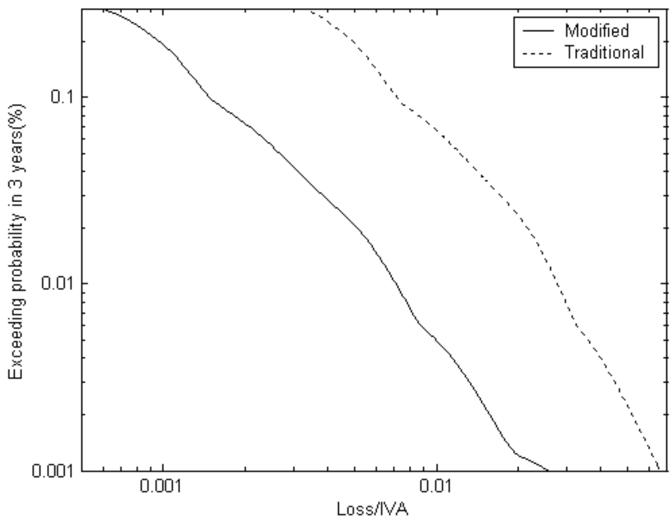


Figure 4 The exceeding probability curves from the two methods

4. CONCLUSIONS

Occurring probability of an earthquake is basic and important information for loss estimation. The traditional method of intensity occurring is modified, especially for low intensity range. For high intensity range, loss is estimated by scenario earthquake method. A city in northeast China is adopted as an example, the hazard curves and exceeding probability curves of loss to IVA ratio from the traditional and modified methods are compared with each other, which shows the loss is underestimated in low intensity range and overestimated in high intensity range traditionally.

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