DESIGN METHOD OF TRIGGER EVENT DEFINITION OF CAT BOND FOCUSSING ON REDUCTION IN BASIS RISK

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ABSTRACT:
It is pointed out that both risk control and risk financing are needed to reduce the seismic risk. Since an earthquake insurance, which is the most popular measure, is limited from the viewpoint of its market size, focused are the other measures that are called alternative risk transfers (hereinafter called ARTs). In many cases ARTs employ full parametric trigger to evaluate compensation, so that basis risk occurs as the difference between the real loss and the compensation, causing the higher risk cost. In this paper, the schematic design method to determine the parametric trigger that was characterized by several grids and forfeiture functions are proposed. A model portfolio consisting of 10 buildings was applied in analyzing the performance of the method. Through the application, the following findings were obtained; the existing method has a room for improvement, the proposed method can reduce the basis risk corresponding to the surplus compensation and the risk cost can be reduced without increasing the risk hedger's risk.

KEYWORDS:
Seismic risk, Catastrophe bond, Trigger event definition, Alternative risk transfer, Basis risk, Risk management

1. INTRODUCTION

Catastrophe bond - CAT bond -, which is a typical ARTs, helps risk management entities to establish seismic risk finance scheme combining earthquake insurance and self retention. On the other hand they take additional risks insurance does not have such as liquidity risk, basis risk and credit risk etc. In this paper basis risk is defined as a financial difference between payment amount of CAT bond and actual loss which risk management entities want to compensate. Though basis risk is controllable, if it remains ignored it turns out they have undesirable detriment that compensation is not sufficient to cover actual loss or cost of risk transfer increase.
The purpose of this paper is to propose schematic design method to determine reasonable trigger event definition under control of basis risk from the standpoint of risk management entities.

2. FRAMWORK

2.1. Definition of Basis Risk

As mentioned above "basis risk" is defined as financial difference between loss to be compensated and actual payment amount due to trigger event from the viewpoint of risk management entities. Therefore CAT bond has basis risk, while earthquake insurance has no basis risk based on indemnity contracts. In this paper, basis risk is defined as the following equation.

$$br_1 = c_I - c_P, \quad if \;\; c_I > c_P \tag{2.1}$$

$$br_2 = c_P - c_I, \quad if \;\; c_I < c_P \tag{2.2}$$

where C_I is insurance coverage to be paid according to indemnity contract and C_P is payment amount due to trigger event of CAT bond.

br_1 causes compensation to be insufficient, while br_2 does risk cost to increase more than risk management entities expected.

2.2. Trigger Event Definition

Though simplified regions such as circular zones or rectangular grids are commonly used as trigger event definition of CAT bond, it is difficult to reflect spatial distribution of seismic hazard and building portfolio on principal reduction factor properly.

In this paper two trigger event definitions are examined. One is defined as a traditional simplified rectangular region and the other is as segmentalized multi-grids with each inherent principal reduction factor.

Procedure to determine trigger event definition is shown in Fig.1. Two step screening of event is characterized as the feature of this approach. In first step screening grids that affect target loss are extracted and in second step relationship between magnitude and target loss is obtained.

The illustrated idea on determination of trigger event definition is shown in Fig. 2.

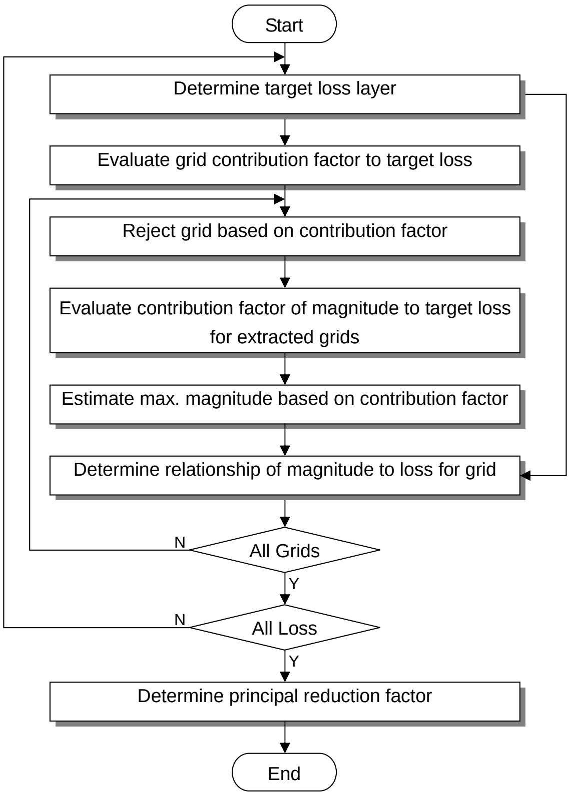


Fig.1 Determination of trigger event definition

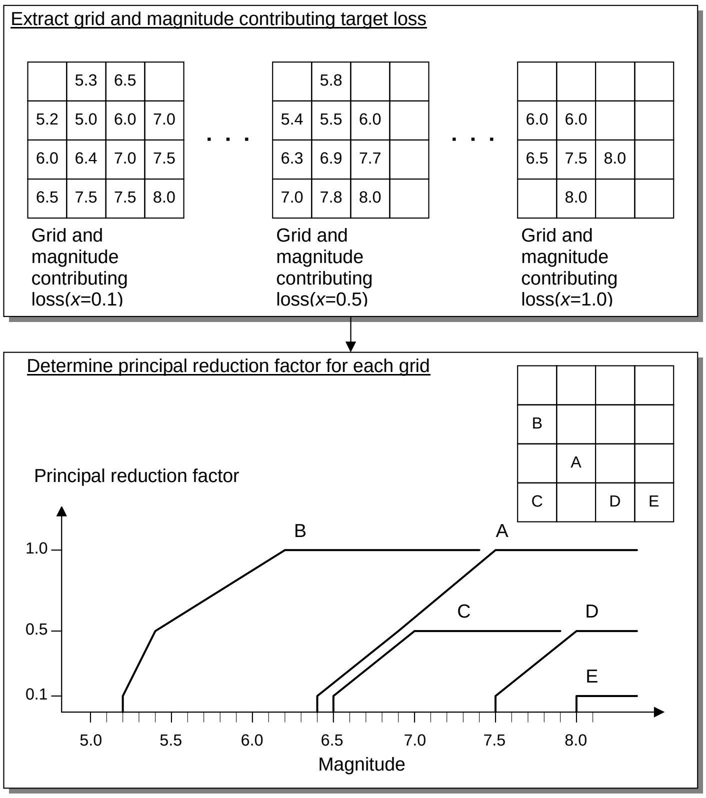


Fig.2 Determination of principal reduction factor

The grid of which contribution factor is lower than a given threshold is rejected.

Contribution factor of the grid j to loss x_c is defined as Eqn. (2.3).

$$\alpha_j(x_c) = \sum_{i \in S_j} v_{i|j} \Big/ \sum_{j=1}^{n} \sum_{i \in S_j} v_{i|j} \tag{2.3}$$

where $v_{i|j}$ is frequency of event i generated in grid j and n is the number of grid.

$v_{i|j}$ is calculated according to Eqn. (2.4).

$$v_{i|j} = v_i \times p_{i|j} \tag{2.4}$$

where $p_{i|j}$ is ratio of area including seismic source to total of the grid.

Class S_j is defined as Eqn. (2.5).

$$S_j = \{i \mid x_{i|j} \geq x_c\} \tag{2.5}$$

where $x_{i|j}$ is loss caused by event i generated in grid j.

Magnitude m_j as trigger event is determined as the largest value that doesn't exceed a given threshold of contribution factor represented by the following Eqn.

$$\beta_{m_j}(x_c) = \int_0^{m_j} f_{m|j,xc}(m)dm \tag{2.6}$$

where $f_{m|j,xc}(m)$ is probability density function in magnitude of event generated in grid j and give loss $x_{i|j}$ being x_c or larger.

3. APPLICATION

3.1. Sample portfolio and source model

Portfolio consisting of 10 buildings in Kanto district is employed in the application. Fig.3 shows the arrangement of the buildings.
Table 1 summarizes the fragility parameters and cost for each building according to Fukushima and Yashiro.
Seismic source models are determined based on Annaka & Yashiro. Seismic source models in which large earthquakes occur are the regions where earthquakes with magnitude of 7.0 or greater occur in land and those with magnitude of 7.5 or greater in sea bottom. The relationship between magnitude and frequency is modeled as characteristic earthquake. On the other hands, the regions where small earthquakes, occur are set along the plate and in the cluster. The relationship of magnitude 6.0 or larger to frequency for these earthquakes is modeled by Gutenberg and Richter equation whose parameters are obtained from the observation records from January 1885 to July 1977, while that of magnitude 5.0 or larger is from January 1926 and July 1997 in case that there are no earthquakes larger than magnitude 6.0 or larger.
Fig. 4 shows the seismic source models employed in the analysis. Table 2 shows the specification of each source model.

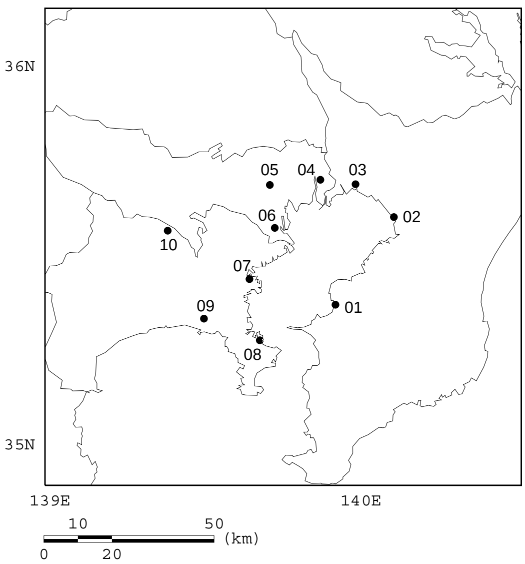


Fig.3 Arrangement of Buildings

Table 1 Fragility parameters and cost

Damage State	Fragility Parameter		Cost	
	Median (cm/s/s)	Logarithmic S.D.	Replace-ment	Damage
Slight	200	0.4	100	5
Moderate	600			10
Severe	1000			30
Collapse	1400			100

Table 2 Specification of each source model

Source Zone for Large EQs

Source ID	*Magnitude*	*Return Period (Year)*	*Souce ID*	*Magnitude*	*Return Period (Year)*
01	7.0-7.6	1182	02	6.9-7.3	5212
03	7.0-7.4	79283	04	6.8-7.2	5931
05	7.1-7.5	2842	06	7.0-7.4	2639
07	6.8-7.2	5676	08	7.1-7.5	8710
09	6.6-7.0	1365	10	6.9-7.3	7239
11	7.5-7.9	1625	12	7.1-7.5	877
13	6.8-7.2	1917	14	7.1-7.5	2851
S1	7.8-8.2	200	S2	7.8-8.2	1000
S3	6.8-7.2	73	N1	7.6-8.0	130

Source Zone for Small EQs

Source ID	*Magnitude*	*A*	*Source ID*	*Magnitude*	*A*
A1	5.0-7.0	2.344	A2	5.0-7.0	4.235
A3	5.0-7.0	1.645	A4	5.0-7.0	3.344

In the regions where small earthquakes, b-value is determined as 0.9 equally.

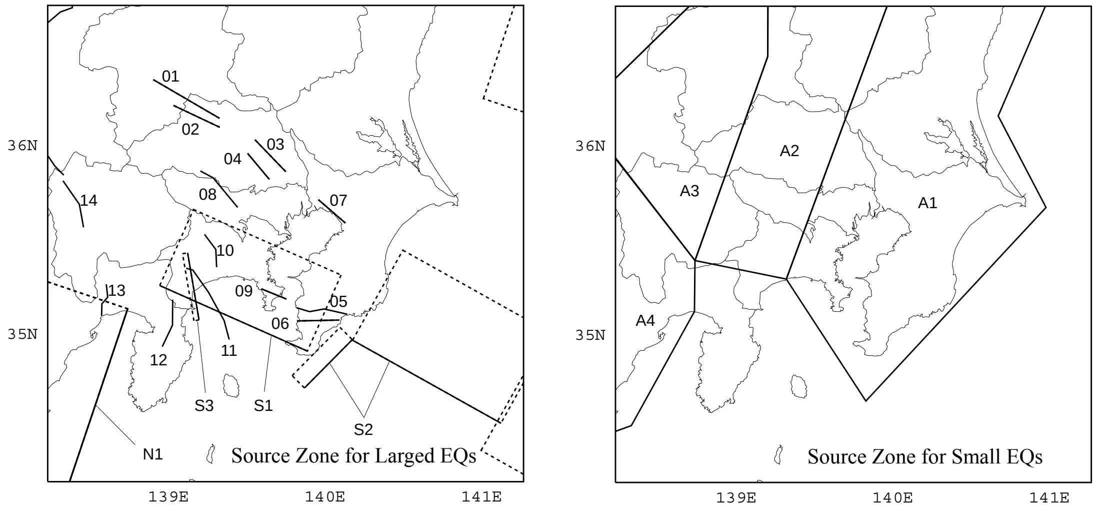


Fig.4 Seismic souce models

Also based on Annaka & Yashiro, following attenuation relation is used in the analysis,

$$\log a = 0.61M + 0.00501h - 2.203\log(d) + 1.377$$
$$d = \sqrt{\Delta^2 + 0.45h^2} + 0.22\exp(0.699M) \quad (3.1)$$

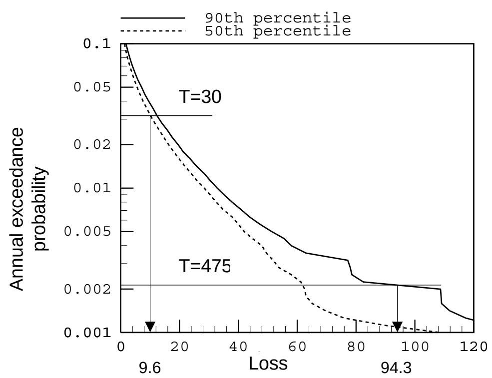
where, a is a peak ground acceleration, M is a magnitude, h is a focal depth and Δ is a epicenter distance, respectively. The standard deviation expressing the uncertainty of attenuation relation is 0.5 in natural logarithm.

3.2. Target Loss and Grid Setting

Insurance coverage is determined based on both deductible and loss limit derived from risk curve without effects of risk financing. In this paper the former is set corresponding to annual frequency 1/30 (50 % exceedance probability in 20 years) on 50th percentile risk curve, while the latter is to 1/475 (10 % exceedance probability in 50 years) on 90th percentile risk curve.

Based on risk curves shown in Fig.5, lower limit of covered layer (deductible), $l_A = 10$, and upper limit of covered layer, (loss limit) $l_E = 100$, are determined and loss x_c is set in the 10 to 100 range by 10.

The grid is divided into eight in the east-west direction and seven in the north-south direction, which is determined as a rectangular that encircle buildings portfolio equidistant from about 50 km around the perimeter shown in Fig. 6.



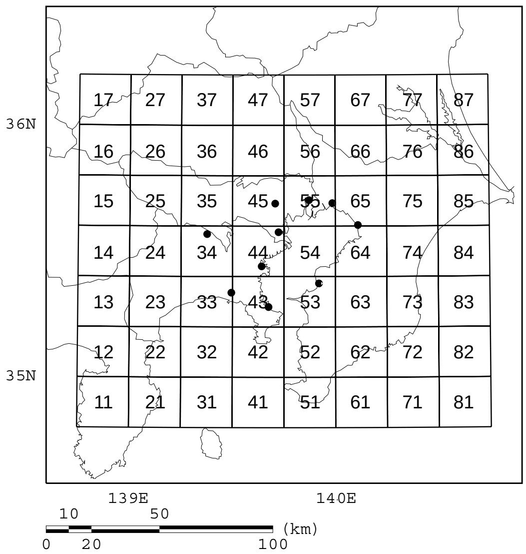


Fig.5 Risk curves (above)
Fig.6 Multi-Grids setting (right)

3.3. Extracting Grids and Evaluation of Magnitude-Loss Relationship

Grid contribution factor is evaluated according to Eqn. (2.3). The Fig.7 shows differences of grid distributions corresponding to threshold 0.01, 0.05 and 0.10, which is a basis for grid extraction. The larger threshold, in other words, the more rejected events are, the more limited to adjacent seismic source S1, Kanto Earthquake, the grid is located.

Relationship between target loss and magnitude is calculated for each grid (Grid ID: 22, 23, 24, 33, 34, 42, 43, 44, 52 and 53) extracted based on threshold of grid contribution factor 0.05 whose thresholds of magnitude correspond to 0.01, 0.05 and 0.10 represented in Fig.8. Principal reduction factor is determined based on the relationships in Fig. 8.

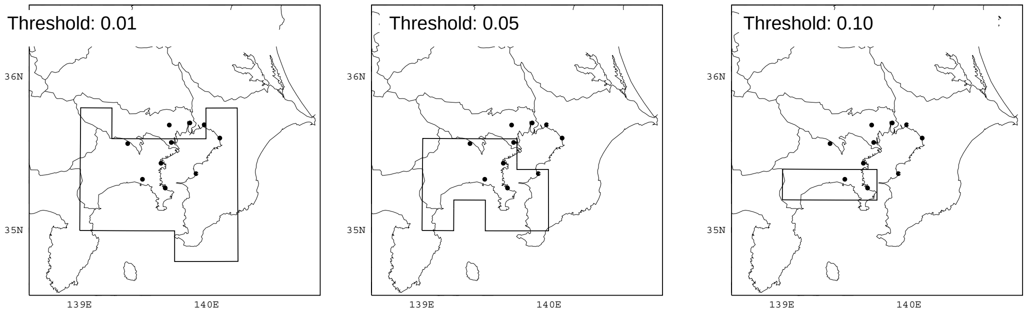


Fig.7 Differences of grid distributions

4. VERIFICATION

4.1. Condition

In this section effectiveness of proposed method is investigated by performing a comparative verification of two different trigger event definitions, that is proposed and traditional one. This decision making is based on an expectation that risk management entities would prefer to reduce both basis risk br_1 and br_2.

The grid corresponding to threshold 0.05 in Fig.7 is adopted as grid by proposed method since that includes predominating seismic source of Kanto earthquake, while excessive grids are not included.

In a similar way principal reduction factor is determined as the one corresponding to magnitude threshold 0.05 in Fig.8 considering deductible $l_A = 10$.

The grid and principal reduction factor based on proposed definition are shown in Fig. 9 and 11 respectively.

On the other hand the ones based on traditional definition are shown in Fig. 10 and 12 respectively.

4.2. Evaluation

Portfolio seismic risk analysis is carried out based on Fukushima et al. Therefore basis risk is represented as probabilistic risk curve, which describes relationship between basis risk and annual exceedance probability.

Fig.13 shows risk curves for two basis risks corresponding to 90th percentile. In Fig.13 full line is the risk curve based on proposed definition, and broken line and dotted line are on traditional grid 1 and 2 respectively.

Table 3 shows PML and AEL. PML is 475-return period loss in 90th percentile, while AEL is annual expected loss in 50th percentile.

Figs.13 and Table 3 expose the tendency that br_2 is more reduced than br_1. That is because proposed trigger event definition is determined as br_2 are lessened with br_1 being low level.

This suggests that risk cost is expected to be limited since br_2 is positively correlated with risk cost.

Fig.14 shows risk curves for risk hedger (risk management entities) and risk taker. Definition of line type is same as Fig.13. Table 4 shows PML and AEL for risk hedger and risk taker.

It is found dependence of br_1 on trigger event definition is not pronounced compared to br_2, which is most effectively reduced by proposed definition.

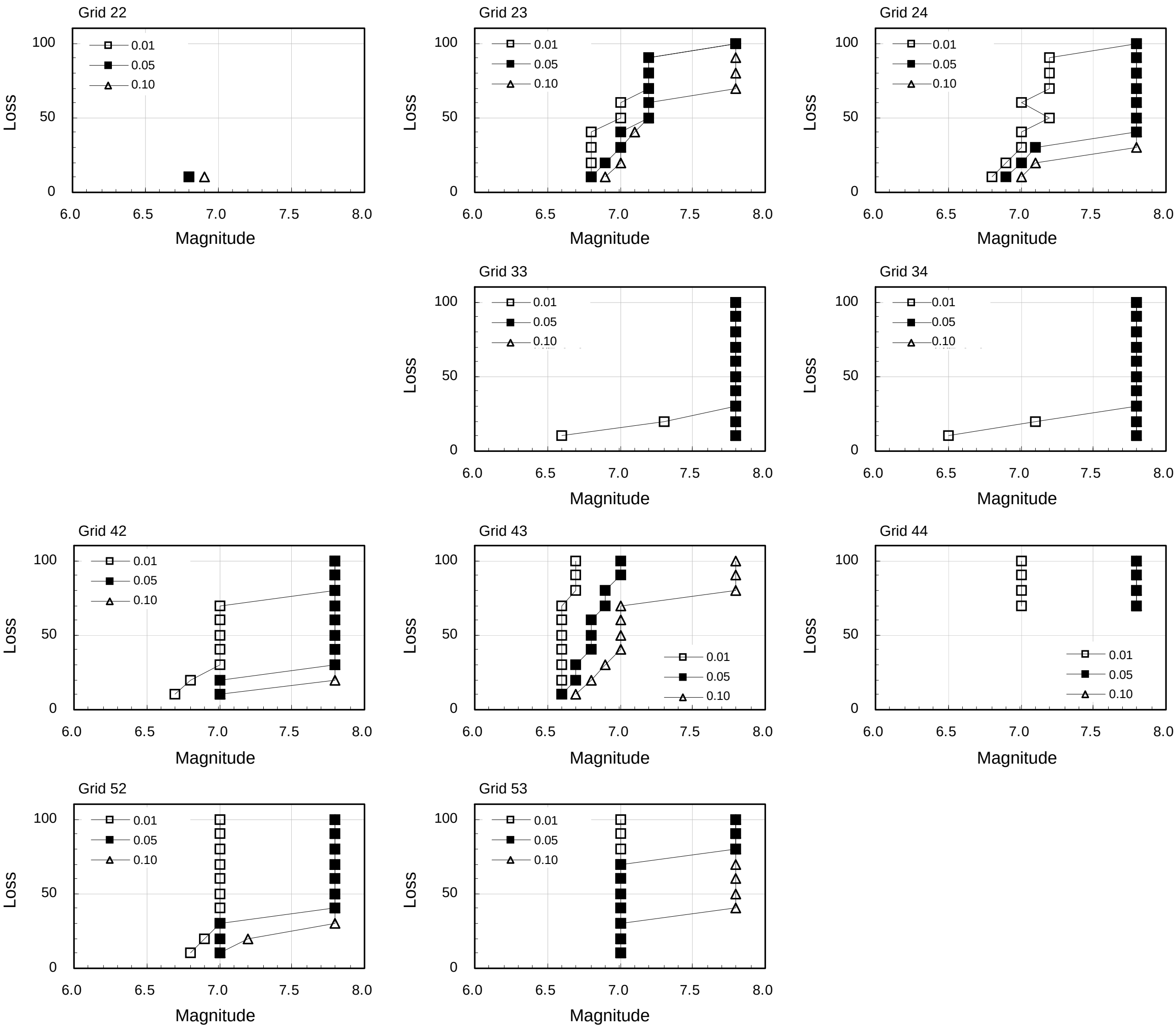


Fig.8 Relationship between magnitude and loss

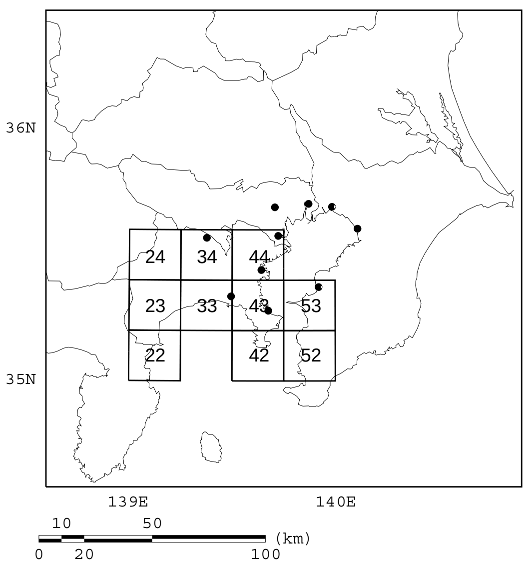


Fig.9 Grid based on proposed method

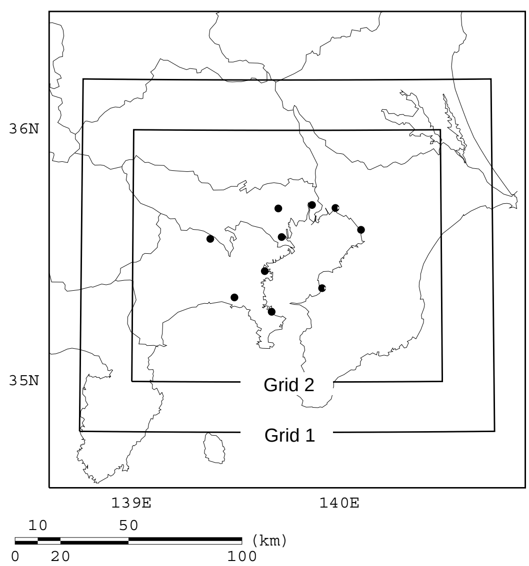


Fig.10 Grid based on traditional method

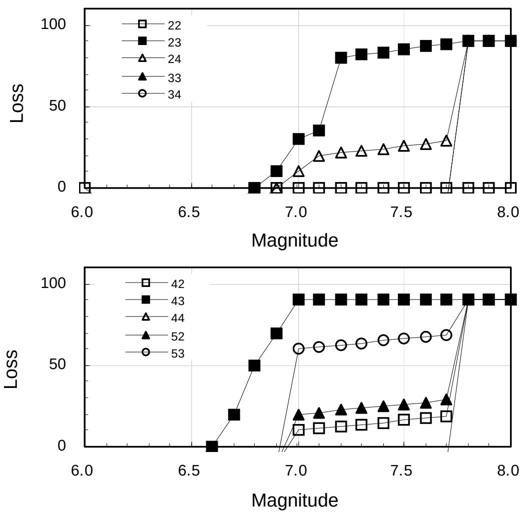


Fig.11 Principal reduction factor based on proposed method

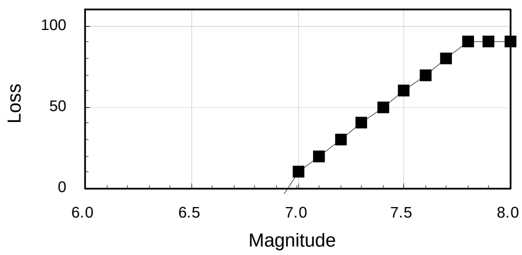


Fig.12 Principal reduction factor based on traditional method

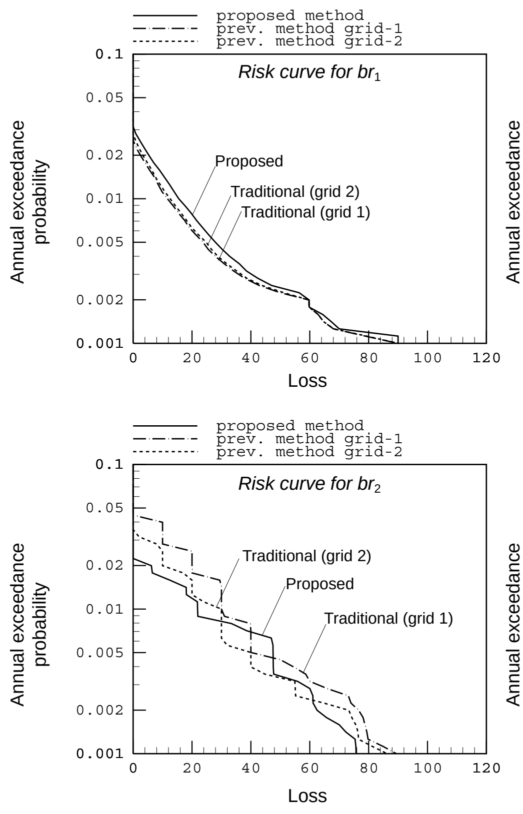


Fig.13 Risk curves of br_1 and br_2

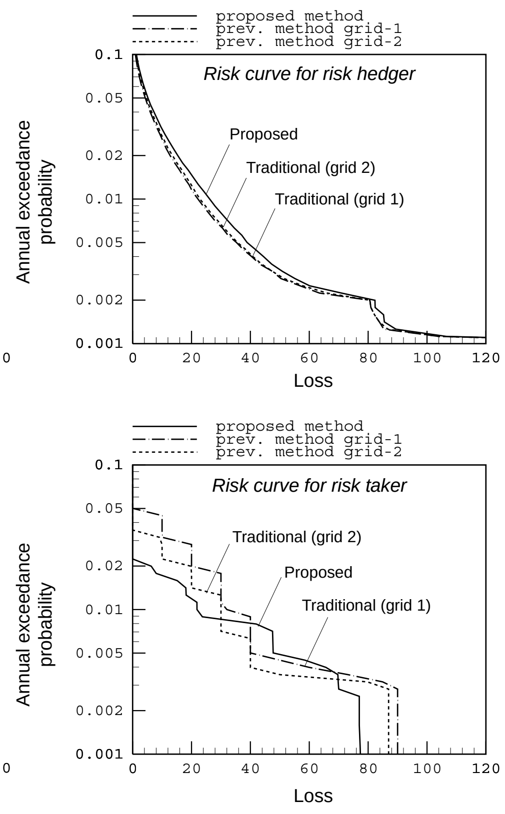


Fig.14 Risk curves of risk hedger and taker

Table 3 PML and AEL of Basis risk

Trigger event definition		Basis Risk : br_1		Basis Risk : br_2	
		AEL_{50}	PML_{90}	AEL_{50}	PML_{90}
Proposed		0.387	59.7	0.550	61.2
Traditional	Grid 01	0.307	57.2	1.006	75.7
	Grid 02	0.325	59.1	0.667	71.3

AEL_{50} : Annual expected loss in 50th percentile
PML_{90} : 475 return period loss in 90th percentile

Table 4 PML and AEL of Risk Hedger/Taker

Trigger event definition		Risk Hedger		Risk Taker	
		AEL_{50}	PML_{90}	AEL_{50}	PML_{90}
Not applicable		1.278	94.3	-	-
Proposed		0.559	82.9	0.717	77.0
Traditional	Grid 01	0.024	76.3	1.252	90.0
	Grid 02	0.378	77.1	0.898	86.9

AEL_{50} : Annual expected loss in 50th percentile
PML_{90} : 475 return period loss in 90th percentile

5. CONCLUSIONS

The framework to determine trigger event definition from the viewpoint of optimization of basis risk is proposed. This proposed method is applied to the model portfolio in Kanto district and effectiveness of the proposed method is examined. Consequently, it is found basis risk can be effectively reduced by proposed method. In other words, risk management entities can establish reasonable risk financial structure by proposed method.

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