A STUDY ON The UNCERTAINTIES IN EARTHQUAKE TIME-DEPENDENT PROBABILITIES IN JAPAN

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ABSTRACT :
In 2005, the Headquarters for Earthquake Research Promotion (HERP), in Japan, released a comprehensive seismic hazard model.. The model provides detailed information on the long term magnitude rate distribution of all subduction zones and regional crustal faults. For a number of seismic sources, the model estimates time-dependent rupture probabilities with strong implications on hazard and loss analysis. In general, the results of time-dependent rupture probability analysis depend strongly on the mean and aperiodicity values of the assumed recurrence density distributions. Often, there are large uncertainties in estimating these parameters due to the fact that data on the occurrences of large historic earthquakes on subduction zones and crustal faults are limited. It is important to account for these uncertainties and evaluate their impact on the results of the time-dependent analysis. It is the objective of this study to formulate and evaluate the effects of parametric and model uncertainty on the results of the renewal time-dependent rupture models in Japan. The results provide insight on the importance, and the scale, of such uncertainty on regional loss distributions.

KEYWORDS:
Time-dependent model, conditional probability, recurrence interval, uncertainty, likelihood

1. INTRODUCTION

The recent national hazard map of Japan, released by the Headquarters for Earthquake Promotion (HERP) in 2005 is a time-dependent model. HERP considers time-dependent rupture probability models for most segments of the subduction zones, along with a number of crustal faults. The Brownian Passage Time (BPT) renewal model (Matthews et al., 2002) is used to model the stochastic inter-arrival times. To calculate the time-dependent occurrence probability within an assumed time window, the BPT model requires information on: the mean recurrence, aperiodicity, date of the last rupture, and the duration of the time window of interest. HERP estimates the mean and aperiodicity parameters based on the rupture history and the displacement of the last ruptures on faults, when such data is available, for all time-dependent subduction zones and faults. There are uncertainties in these estimates due to the scarcity of data on the occurrences of large historic earthquakes on subduction zones and crustal faults. All time-dependent analyses, including the BPT, are highly nonlinear processes. This implies that time-dependent analysis, with or without consideration of the effects of the parametric uncertainties, will not produce the same mean conditional probability values. The critical effects of recurrence interval uncertainty on the results of the conditional probability analysis have been recognized and addressed by different authors. Parsons (2005) used the Monte Carlo simulation to estimate uncertainties in the repeat times and aperiodicity values on different segments of the San Andreas fault in southern California, and on the Nankai Trough in Japan. Team Tokyo (Stein et al., 2006) implemented Parson's methodology to estimate the time-dependent probabilities for the M≥7.9 earthquake on Sagami Trough. Their estimates of the mean recurrence and aperiodicity reflect larger uncertainty in the inter-event arrival times than those that are used in the HERP report. Sykes and Menke [2006] used the maximum likelihood method to estimate the uncertainty in the inter-arrival times of earthquake on a few major active faults, and on subduction zones around the world. In general, they estimated lower level of aperiodicity for recurrence models of several of the faults compared to other studies. These studies included the USGS Working Group 2003 (WGCEP, 2003), which studied the time dependency of faults in the San Francisco Bay area. Their findings suggest that the uncertainties in fault recurrence models, for cases where the estimates of the inter-arrival times are based on historic dates, are lower than those for which the recurrence models are dominated by the paleoseismic-related data. However, in both cases, uncertainties in the estimates of the mean and aperiodicity values of fault recurrence models have nonlinear effects on the results of the time-dependent conditional probability analysis. It is important to use a systematic method of capturing such uncertainties in order tocarry the effects through the conditional probability analysis.

2. PARAMETER SENSITIVITY OF TIME-DEPENDENT PROBABILITY CALCULATIONS

For this discussion, we use the Nankai subduction zone rupture history, and the related HERP time-dependent model, to demonstrate the potential effects of parametric uncertainties on the results of the time-dependent analysis. Table 1 shows the chronology of the Tonankai and the multi-segment ruptures that included the Tokai segment of the Nankai Trough. Also shown are the expected recurrence interval and the aperiodicity values used in the HERP report for the time-dependent analysis. The expected recurrence interval in the HERP report for the Tonankai segment is based on the historic data and the rupture detail information on the 1944 earthquake. The expected recurrence for the Tokai segment is the mean of the historic rupture intervals. In both cases, HERP uses the aperiodicity value of 0.2 for the analysis. Due to the scarcity of data, there are uncertainties in the HERPestimates of both the recurrence and aperiodicity values. Additionally, for the Tokai segment, there is another source of uncertainty due to the fact that the multi-segment rupture history of the Nankai, Tonankai, and Tokai segments is used to formulate the recurrence model for the Tokai segment. To understand the scale of uncertainty due to the scarcity of data, we conducted a stochastic simulation of the set of rupture intervals for the historic data on the Tonankai and Tokai segments as reported by HERP. For each case, we assumed a range for the possible mean recurrence and aperiodicity values. We digitized these ranges and, for each set of recurrence interval and aperiodicity values, defined a recurrence density distribution. The distribution is used to draw 100,000 sets of N random recurrence intervals, where N is the number of observed recurrence intervals for each segment. Any set that simulates all observed recurrence intervals within ± 2 years is accepted as a viable recurrence model for the segment. From the total of 65 million sets of draws for each segment, there are 781 matches for the Tonankai segment and 1568 matches for the Tokai segments. More successful simulated scenarios were observed for the Tokai segment since it has three historic ruptures to match whereas the Tonankai segment has four. For the simulation, we used lognormal distribution rather than BPT, because it, provides an easier way to formulate the stochastic simulation and the likelihood function, as will be discussed shortly. Furthermore, in a number of studies the results of the time-dependent analysis based on the lognormal and BPT distributions are very close (e.g., Matthews et al., 2002). However, for conditional probability calculation we used BPT. Figures 1 and 2 show the histograms of the recurrence intervals and aperiodicity (coefficient of variance, CV, for lognormal distribution) for all sets that simulate the observed recurrence intervals for the Tonankai and Tokai segments. Figure 3 shows the histograms of the conditional probability values based on input data from Figures 1 and 2.

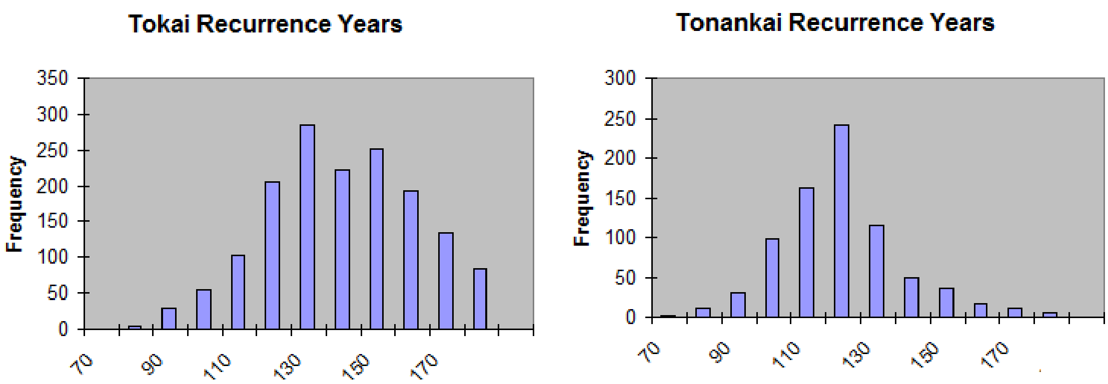

Figure 1. Histograms of the recurrence intervals (in years) for the Tokai and Tonankai segments. Data shown in Figure 2 are used to simulate the observed recurrence intervals for the segments.

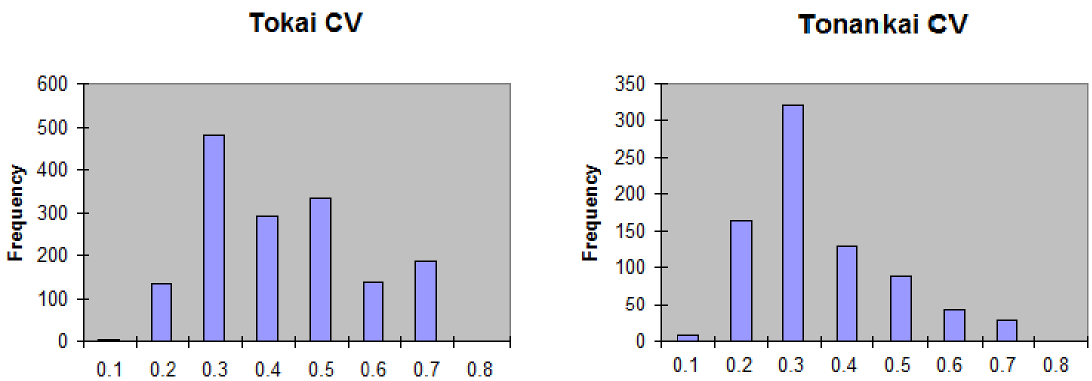

Figure 2. Histograms of the CV of the recurrence distributions for the Tokai and Tonankai segments. that when used with the data in Figure 1, simulated the observed recurrence intervals for the segments.

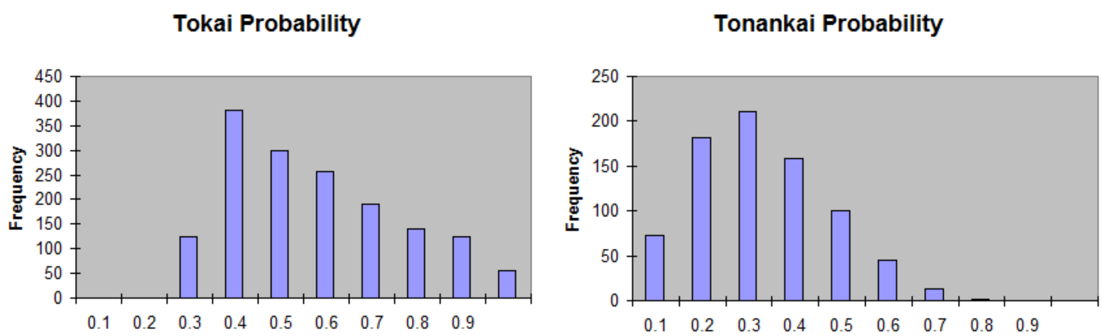

Figure 3. Histograms of the conditional probability values based on the input data shown in Figures 1 and 2.

The results of this simulation indicate that, when the numbers of observed data points are limited, different stochastic processes with different means and aperiodicity values may be the causative sources. Therefore, the mean recurrence and aperiodicity values calculated from the observed data maydiffer from the true values. This introduces epistemic uncertainty in parameters that define the recurrence models and could have strong implications on the results of the time-dependent conditional probability analysis (Figure 3). For example, the mean 30-year conditional probability (P30) value from this simulation for the Tokai segment is about 0.53.

This is very different than the HERPP30 value of 0.86, which is based on the mean recurrence of 118.6 and aperiodicity of 0.2. For the Tonankai segment, the HERP mean recurrence differs fromfrom the mean of the recurrence intervals based on historic data. Other considerations regarding the rupture detail of the last earthquake was also used to formulate the expected mean recurrence. Therefore, we cannot directly compare the HERP P30 value for the Tonankai segment with the average value from this study. However, regardless of the choice of mean recurrence interval, it is clear that there are epistemic uncertainties in the mean recurrence and aperiodicity values that need to be taken into consideration for time-dependent analysis. The time-dependent analysis for the Tokai segment has an extra layer of complexity due to the fact that the historic data do not show any single rupture for this segment that is independent of the Tonanakai and Nankai segments. Obviously, using the recurrence intervals of multi-segment ruptures, as is the case for Tokai, to construct a recurrence model for a single segment rupture adds some model uncertainty. In general, the available information on the rupture history of faults is limite,d and in many cases uncertain. Therefore, it is not possible to uniquely define a recurrence model for each fault for the time-dependent analysis. In this study we used the likelihood statistics to formulate distributions for the ranges of viable mean recurrence and aperiodicity values for faults. These distributions are based on the faults' historic rupture information and/or paleoseismic data, along with related uncertainties.

Table 1a. Great earthquakes along Nankai, Tonankai, and Tokai segments of Nankai Trough. Note that "O" denotes that the whole segment was ruptured in the event, and "Δ" means that the segment was partially ruptured.

Date	Nankai	Tonankai	Tokai
1498.09.20		O	Δ
1605.02.03	O	O	Δ
1707.10.28	O	O	Δ~O
1854.12.23		O	O
1854.12.24	O		
1944.12.07		O	
1946.12.21	O		

Table 1b. Parameters used in HERP BPT time-dependent model.

Parameters	Nankai	Tonankai	Tokai
Mean Recurrence (years)	90.1	86.4	118.8
Aperiodicity	0.2	0.2	0.2

3. MODEL FORMULATION

Consider a fault with n rupture interval values $(t_1, t_2, ..., t_n)$ from historic data. Let us assume that the causative process can be modeled by a lognormal probability distribution, with the median recurrence interval of λ and coefficient of variance of σ. The likelihood function for n recurrence intervals can be written as

$$l(\lambda,\sigma \mid t) = \prod_{i=1}^{n} f(t_i \mid \lambda,\sigma) \tag{1}$$

where $f(t_i \mid \lambda,\sigma)$ is the density function. For a lognormal distribution, the likelihood function can be written as

$$l(\lambda,\sigma \mid t) \propto \prod_{i=1}^{n}\left[\frac{1}{\sigma * t_i}\exp\left(-\frac{1}{2}\frac{(\ln(t_i)-\lambda)^2}{\sigma^2}\right)\right]. \tag{2}$$

It is common practice to use the maximum likelihood method to estimate the most likely values for λ and σ. However, considering the scarcity of data on fault rupture history and the strong nonlinear nature of the time-dependent analysis, using this method to obtain the most likely estimates for θ and σ , rather than using their distributions, can introduce biases in the time-dependent probability estimates. Figure 4 shows plots of three sets of likelihood functions for different assumed values of σ (CV for lognormal distribution) for the Tonankai segment. We propose to use these distributions, with some constraints on the acceptable ranges for the recurrence intervals and CV, to estimate the mean time-dependent occurrence probability $\overline{P}_{30}$ as follows

$$\overline{P}_{30} = K \int_{\sigma=\sigma_1}^{\sigma_2} \int_{\lambda=\lambda_1}^{\lambda_2} l(\lambda \mid \sigma,t) * p_{30}(\lambda,\sigma,t_{past}) \tag{3}$$

where $p_{30}(\lambda,\sigma,t_{past})$ is the 30-year conditional probability. The choices for λ_1, λ_2, σ_1, and σ_2 can be dictated by data or expert opinion, and K is a normalizing factor. The term $l(\lambda \mid \sigma,t)$ represents the likelihood of λ for an assumed value of σ given the observed data. This formulation provides a practical way of capturing and incorporating epistemic uncertainties in the mean and aperiodicity of all recurrence models that are consistent with the observed data. Also, the likelihood formulation allows us to incorporate other data and model related uncertainty into the time-dependent analysis. For example, the paleoseismic data for Kanto type earthquake includes a significant amount of uncertainty due to the inherent uncertainty in carbon dating of the paleo-tsunami data. A good example for model uncertainty is the the Tokai segment, where all available information reflects multi-segment rupture scenarios whereas the time-dependent analysis is conducted for a single segment rupture. This creates some model uncertainty that needs to be accounted for and can be easily included in the analysis. In the case of data uncertainty, the σ of the individual record in the likelihood function can be adjusted to reflect the uncertainty. For model uncertainty, the mean σ values, which are used for constructing the recurrence distributions, can be adjusted to reflect the model uncertainty.

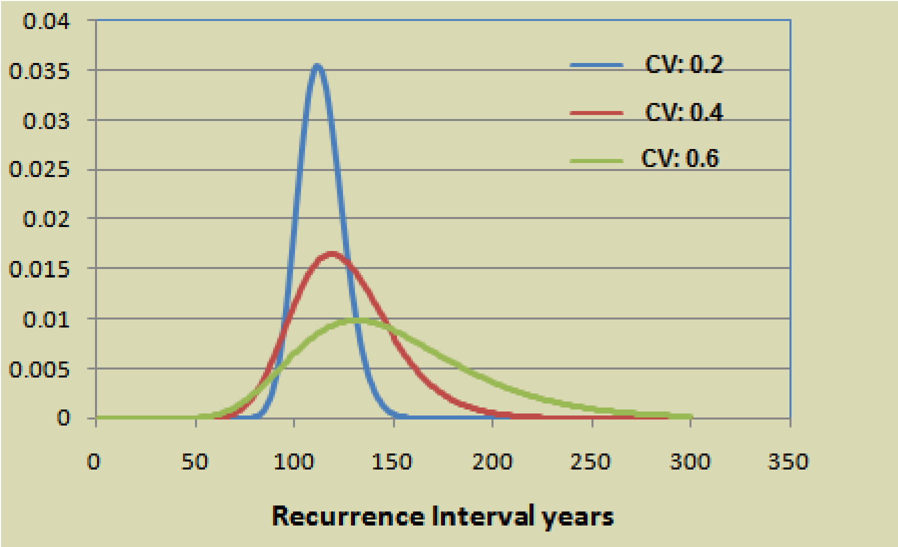

Figure 4. Plots of the likelihood functions for the Tonankai segment of the Nankai Trough for different values of coefficient of variance.

4. SUMMARY AND CONCLUSIONS

The results of time-dependent rupture probability analysis strongly depend on the mean and aperiodicity of the assumed recurrence models. There are uncertainties in these parameters due to the scarcity of the historic earthquake data and the uncertainties in the dates of the paleoearthquakes used to estimate the recurrence intervals. Because time-dependent analysis is a nonlinear process, the estimates of the expected conditional rupture probabilities, with or without accounting for uncertainties, are not the same. In order to estimate the statistically unbiased time-dependent probability , uncertainties need to be accounted for in the analysis. We used the likelihood statistical method to formulate a practical way to define distributions on the mean and aperiodicity values of the feasible recurrence models that are compatible with the observed data. The formulation accounts for various epistemic uncertainties. We use this formulation to evaluate the effects of uncertainties on Japan's HERP time-dependent rupture probability values. We conducted two sets of time-dependent analysis to evaluate the effects of uncertainties on the subduction zones and crustal faults in Japan. In one set we used the HERP's mean and aperiodicity values for the recurrence intervals with an assumed fixed uncertainty on the aperiodicity values. In the other set we used the dates of the historic earthquakes on faults and subduction zones, as reported by HERP, to formulate distributions for the mean and aperiodicity values using the likelihood method. In the second case, we limit the sampling range of the aperiodicity values to ± one sigma value. For Tokai segment, we incorporated model uncertainty by adjusting the sampled aperiodicity values from the distribution upward. For Kanto segment, we used the reported uncertainties on the paleo-tsunami data to formulate uncertainties on the inter-event times. This information is used in the formulation of the likelihood function for the Kanto segment. Table 2 shows the results of the analysis for the subduction zone segments and a few selected faults. The table shows the HERP 30-year time-dependent probabilities, Possionian probabilities, the 30-year time-dependent probabilities using the HERP mean and recurrence intervals with some uncertainties around the values (Case 1), and the 30-year time-dependent probabilities using the dates of the historic earthquakes on faults and subduction zones as reported by HERP (Case 2). The last column in Table 2 lists the corresponding fault ID in the HERP report for crustal faults for reference. The results of the both case studies indicate that accounting for uncertainties can have important effects on the expected time-dependent probabilities. The main source of the discrepancy between the HERP

P30 values and the corresponding values for the cases with uncertainty, is the nonlinear nature of the time-dependent analysis. The severity of the uncertainty effect on P30, however, is not the same for all faults and subduction segments. The effect depends upon the detail of the elapsed time since the last rupture relative to the mean recurrence interval, the value of aperiodicity, and the quantity and quality of the observed data. In general, the results of this study indicate that the expected conditional occurrence probabilities for the majority

Table 2. Comparisons of time-dependent conditional probabilities on subduction segments and some selected crustal faults in Japan.

Names of Fault or subduction segment	P30 HERP	P30 Poisson	P30 Case 1	P30 Case 2	Fault ID in HERP Report
Nankai segment	0.56487	0.2832	0.43739	0.18208	-
Tonankai segment	0.67385	0.29335	0.51146	0.21959	-
Tokai segment	0.87049	0.22316	0.66042	0.65466	-
Miyagi-ken-Oki	0.997	0.55453	0.96765	0.96311	-
Southern Sanriku-Oki close to trench	0.80299	0.24955	0.73599	0.67962	-
Northern Sanriku-Oki	0.04106	0.26602	0.09272	0.06649	-
Tokachi-Oki	0.00568	0.34	0.05368	0.06547	-
Nemuro-Oki	0.40132	0.34	0.43894	0.44106	-
Shikotanto-Oki	0.48093	0.34	0.48445	0.48046	-
Etorofuto-Oki	0.58483	0.34	0.5406	0.52823	-
Northwestern Hokkaido-Oki	0.00048	0.00766	0.00242	0.00243	-
Kanto segment 1923 Taisho type	0.00111	0.12747	0.02908	0.00026	-
Tobetsu fault	0.00082	0.00266	0.00142	0.00135	501
Main part- Ishikari-teichi-toen fault	0.01687	0.00623	0.01827	0.01613	601
Kuromatsunai-teichi fault zone	0.03666	0.00695	0.04927	0.0431	701
Yamagata-bonchi fault zone	0.03909	0.00995	0.04658	0.04014	1801
Shonai-heiya-toen fault zone	0.00021	0.00853	0.00137	0.00148	1901
Kushigata-sanmyaku fault zone	0.01888	0.00853	0.01678	0.01545	2501
Tsukioka fault zone	0.00022	0.00399	0.00103	0.00104	2601
Tachikawa fault zone	0.01346	0.0024	0.019	0.0166	3401
Kannawa/Kozu-Matsuda fault zone	0.04274	0.02817	0.0298	0.01032	3601
Miura-hanto fault	0.00005	0.00879	0.00106	0.00114	3701
Miura-hanto fault group	0.08378	0.017	0.09847	0.1857	3702
Itoigawa-Shizuoka-kozosen fault zone	0.14337	0.02955	0.17269	0.15209	4101
Fujikawa-kako fault zone	0.05205	0.01749	0.05595	0.04944	4301
Sanage-Takahama fault zone	0.00124	0.00283	0.00157	0.00169	5303
Kurehayama fault zone	0.00078	0.00352	0.0016	0.00155	5601
Tonami-heiya/Kurehayama fault zone	0.01149	0.00598	0.01204	0.01072	5602
Morimoto-Togashi fault zone	0.00306	0.01489	0.0065	0.00629	5701
Biwako-seigan fault zone	0.01912	0.00933	0.01997	0.01777	6501

of the subduction zone segments are reduced when compared to the corresponding HERP values. The obvious effect of this on regional losses is that the exceedance rates of losses in the areas dominated by large subduction-related characteristic earthquakes will decrease. The amount of decrease on the average annual loss depends upon the overall relative contribution of these types of earthquakes compared to those from the background and deep seismicity.

The results presented here demonstrate the importance of the epistemic uncertainty on the mean and aperiodicity values of faults recurrence intervals in time-dependent probability analysis. All time-dependent earthquake models suffer from lack of detailed information on paleoseismic and historic earthquake data. It is not possible to formulate a unique recurrence model without uncertainty. The formulation presented here is a practical way of addressing and capturing such uncertainties by integrating the various types of data and expert opinion into a likelihood model in order to obtain viable distributions on the mean and aperiodicity values for time-dependent analysis.

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