# STABILITY AND IDENTIFICATION FOR DISCRETE-TIME RATIONAL APPROXIMATION OF FOUNDATION FREQUENCY RESPONSE

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**ABSTRACT:** Discrete-time rational approximation (DRA) of foundation frequency response is the first step of a systematic procedure for constructing various time-domain recursive evaluations (TDREs) in foundation vibration analysis. The stability and accuracy of DRA determine those of its TDREs as realization. In this paper, the stability and identification of DRA are studied. The DRA can be obtained from a continuous-time rational approximation (CRA) of foundation frequency response. If letting the discrete-time frequency equal to the continuous-time one, the high-frequency loss and the aliasing may occur due to the periodic nature of DRA. To avoid these, the bilinear transform is used in this paper, so that the stability and accuracy of the resulting DRA is identical with those of CRA. The stability conditions of DRA are stated in *z*-plane. The resulting DRA is realized as the direct-form and parallel-form TDREs. The effectiveness of bilinear transform method is verified by analyzing several typical foundation vibration problems using the resulting TDREs and comparing with the results of lumped-parameter models (LPMs) resulting from the same CRA.
**KEYWORDS:** foundation vibration, recursive evaluation, discrete-time rational approximation, bilinear transform

## 1. INTRODUCTION

In an accompanying paper [1] the stability and identification of *continuous-time rational approximation* (CRA) of foundation frequency response realized as various *lumped-parameter models* (LPMs) are studied. Alternatively, the foundation frequency response can be also represented by a rational function in *z* variable of *z*-transform in discrete-time case. Such rational function is here called as *discrete-time rational approximation* (DRA), that can be realized as various types of *time-domain recursive evaluations* (TDREs). The stability and identification of DRA are studied in this paper. The DRA can be obtained via two paths: directly from foundation frequency response [2-4] or indirectly from CRA [5, 6]. This paper concentrates on the second. A practical difficulty in obtaining DRA from CRA with identity of discrete-time and continuous-time frequencies is that the high-frequency lost and aliasing may occur due to the period nature of DRA. (Actually, the high-frequency lost will also occur in the first path mentioned above.) To avoid these, the bilinear transform method is used in this paper.

Unlike foundation-soil system and its LPM, the interinvertible systems of a TDRE are two different linear time-invariant discrete-time (LTID) systems: TDRE of impedance force from foundation displacement with *dynamic-stiffness-form DRA* (SDRA) as frequency response and TDRE of response displacement from foundation force with *dynamic-flexibility-form DRA* (FDRA) as frequency response. TDREs of impedance force are widely studied and used to compute the interaction force of foundation in time-domain soil-structure-interaction analysis.

## 2. DISCRETE-TIME RATIONAL APPROXIMATION

### *2.1. Continuous-Time Rational Approximation*

A stable and accurate CRA of foundation frequency response can be obtained from the accompanying paper [1]. The *dynamic-stiffness-form CRA* (SCRA) can be written as (Eqn. 2.1 in [1])

$$S_C(\overline{\omega}) = S_C(\overline{s}) = S_0 \frac{1 + p_1 \overline{s} + ... + p_{N+1} \overline{s}^{N+1}}{1 + q_1 \overline{s} + ... + q_N \overline{s}^N} \quad (2.1)$$

where $S_0$ is the static stiffness, $\overline{s}$ is the dimensionless complex frequency, $\overline{s} = i\overline{\omega}$ here, $i = \sqrt{-1}$, $\overline{\omega} = \omega d / c_S$ is the dimensionless continuous-time frequency with the conventional Fourier radian frequency $\omega$, the characteristic length $d$ (of foundation) and the (shear) wave velocity $c_S$ (of soil), and $p_j, q_j$ are the real parameters. It is clear that SCRA is singular at high-frequency limit, i.e. $S_C(\overline{s} \to \infty) \to \infty$. Therefore, if applying the bilinear transform to Eqn. 2.1 directly, a marginally stable pole $z = -1$ of the resulting SDRA will be obtained. To avoid this, Eqn. 2.1 is decomposed into a sum of a linear term and a new rational function without singularity that is called as SCRA without singularity and denoted by $\hat{S}_C(\overline{s})$, as follows

$$S_C(\overline{s}) = S_0 \hat{c}_\infty \overline{s} + \hat{S}_C(\overline{s}) \quad (2.2)$$

$$\hat{S}_C(\overline{s}) = S_0 \frac{1 + \hat{p}_1 \overline{s} + ... + \hat{p}_N \overline{s}^N}{1 + q_1 \overline{s} + ... + q_N \overline{s}^N} \quad (2.3)$$

where $\hat{c}_\infty = p_{N+1} / q_N$, and $\hat{p}_j = p_j - \hat{c}_\infty q_{j-1}$ for $j = 1,...,N$ with the definition of $q_0 = 1$. Thus, the bilinear transform can be applied to Eqn. 2.3, and the linear term is modeled by a dashpot. Correspondingly, the *dynamic-flexibility-form CRA* (FCRA) can be written as (Eqn. 2.2 in [1])

$$F_C(\overline{\omega}) = F_C(\overline{s}) = F_0 \frac{1 + q_1 \overline{s} + ... + q_N \overline{s}^N}{1 + p_1 \overline{s} + ... + p_{N+1} \overline{s}^{N+1}} \quad (2.4)$$

where $F_0$ is static flexibility. The bilinear transform can be applied to Eqn. 2.4 without any special treatment required.

### *2.2. Bilinear Transform*

A bilinear transform relation between the dimensionless complex frequency $\overline{s}$ and the $z$ variable of $z$-transform is

$$\overline{s} = \frac{2}{\Delta \overline{t}} \frac{1 - z^{-1}}{1 + z^{-1}} \quad (2.5)$$

where $\Delta \overline{t} = \Delta t c_S / d$ is the dimensionless time-step size, $z = \exp(i\omega_D \Delta t) = \exp(i\overline{\omega}_D \Delta \overline{t})$ with the discrete-time frequency $\omega_D$ and corresponding dimensionless discrete-time frequency $\overline{\omega}_D = \omega_D d / c_S$. The bilinear transform converts the imaginary axis in $\overline{s}$-plane ($\overline{s} = i\overline{\omega}$) into a unit circle in $z$-plane ($|z| = 1$). The left- and right-half plane in $\overline{s}$-plane map into the inside and outside of the unit circle in $z$-plane, respectively. Substituting $\overline{s} = i\overline{\omega}$ and $z = \exp(i\overline{\omega}_D \Delta \overline{t})$ into Eqn. 2.5, the relation between the dimensionless continuous-time frequency $\overline{\omega}$ and the dimensionless discrete-time frequency $\overline{\omega}_D$ is obtained as

$$\overline{\omega} = \frac{2}{\Delta \bar{t}} \tan\left(\frac{\overline{\omega}_D \Delta \bar{t}}{2}\right) \tag{2.6}$$

It is clear that $\overline{\omega}$ is a periodic function of $\overline{\omega}_D$, where the period is just that of DRA, i.e. $\overline{\omega}_{\max} = 2\pi/\Delta \bar{t}$. Thus, $-\infty < \overline{\omega} < \infty$ is compressed into $-\overline{\omega}_{\max}/2 \le \overline{\omega}_D \le \overline{\omega}_{\max}/2$ without any lost of accuracy and then the periodic extension is performed. Therefore, the aliasing and high-frequency lost are avoided.

Substituting Eqn. 2.5 into Eqn. 2.3, SDRA without singularity is obtained as follows

$$\hat{S}_D(z) = S_0 \frac{\hat{b}_0 + \hat{b}_1 z^{-1} + \ldots + \hat{b}_N z^{-N}}{1 + \hat{a}_1 z^{-1} + \ldots + \hat{a}_N z^{-N}} \tag{2.7}$$

where the parameters $\hat{a}_j$ and $\hat{b}_j$ can be obtained from $\hat{p}_j$ and $q_j$ by simple computation. Note that $\hat{a}_j$ and $\hat{b}_j$ are dependent of the time-step size. Correspondingly, substituting Eqn. 2.5 into Eqn. 2.4, FDRA is obtained as follows

$$F_D(z) = F_0 \frac{b_0 + b_1 z^{-1} + \ldots + b_{N+1} z^{-(N+1)}}{1 + a_1 z^{-1} + \ldots + a_{N+1} z^{-(N+1)}} \tag{2.8}$$

where the parameters $a_j$ and $b_j$ can be obtained from $p_j$ and $q_j$ by simple computation, and are also dependent of the time-step size.

### *2.3. Partial-Fraction Expansion*

The real-parameter partial-fraction expansion of SDRA without singularity can be written as

$$\frac{\hat{S}_D(z)}{S_0} = \hat{b}_0 + \sum_{j=1}^{N-2L_1} \frac{\hat{A}_j z^{-1}}{1 - \hat{s}_j z^{-1}} + \sum_{j=1}^{L_1} \frac{\hat{\beta}_{j1} z^{-1} + \hat{\beta}_{j2} z^{-2}}{1 + \hat{\alpha}_{j1} z^{-1} + \hat{\alpha}_{j2} z^{-2}} \tag{2.9}$$

where $\hat{s}_j$ and $\hat{A}_j$ are the poles of $\hat{S}_D(z)/S_0$ and the corresponding residues, respectively, $L_1$ denotes the number of pairs of complex conjugate poles, and $\hat{\beta}_{j1} = 2\hat{A}_{j1}$, $\hat{\beta}_{j2} = -2\left(\hat{A}_{j1}\hat{s}_{j1} + \hat{A}_{j2}\hat{s}_{j1}\right)$, $\hat{\alpha}_{j1} = -2\hat{s}_{j1}$ and $\hat{\alpha}_{j2} = \hat{s}_{j1}^2 + \hat{s}_{j2}^2$ with the complex conjugate poles $\hat{s}_j^S = \hat{s}_{j1}^S \pm i\hat{s}_{j2}^S$ and their residues $\hat{A}_j^S = \hat{A}_{j1}^S \pm i\hat{A}_{j2}^S$. Similarly, the real-parameter partial-fraction expansion of FDRA can be written as

$$\frac{F_D(z)}{F_0} = b_0 + \sum_{j=1}^{N+1-2L_{21}} \frac{A_j z^{-1}}{1 - s_j z^{-1}} + \sum_{j=1}^{L_2} \frac{\beta_{j1} z^{-1} + \beta_{j2} z^{-2}}{1 + \alpha_{j1} z^{-1} + \alpha_{j2} z^{-2}} \tag{2.10}$$

where $s_j$ and $A_j$ are the poles of $F_D(z)/F_0$ and the corresponding residues, respectively, $L_2$ denotes the number of pairs of complex conjugate poles, and $\beta_{j1} = 2A_{j1}$, $\beta_{j2} = -2\left(A_{j1}s_{j1} + A_{j2}s_{j1}\right)$, $\alpha_{j1} = -2s_{j1}$ and $\alpha_{j2} = s_{j1}^2 + s_{j2}^2$ with the complex conjugate poles $s_j^S = s_{j1}^S \pm is_{j2}^S$ and their residues $A_j^S = A_{j1}^S \pm iA_{j2}^S$. Note that as all existing works and continuous-time case do, the repeated poles are not considered in Eqn. 2.9 and 2.10 due to no appearance nearly.

### *2.4. Stability*

The stability of the resulting DRA based on bilinear transform is identical with that of CRA. The equivalent stability conditions can be restated in terms of the location of the poles of DRA in *z*-plane, as follows:
(1) A TDRE of impedance force is dynamically stable if and only if all poles of its SDRA without singularity lie inside a unit circle in *z*-plane, i.e. $|\hat{s}_j| < 1$ for $j = 1,..., N$ .
(2) A TDRE of response displacement is dynamically stable if and only if all poles of its FDRA lie inside a unit circle in *z*-plane, i.e. $|s_j| < 1$ for $j = 1,..., N+1$.

## 3. TIME-DOMAIN RECURSIVE EVALUATIONS

### *3.1. Direct-Form TDREs*

Applying the inverse *z*-transform to Eqn. 2.7, the TDRE of impedance force is obtained as

$$\hat{f}^n = S_0 \sum_{j=0}^{N} \hat{b}_j u^{n-j} - \sum_{j=1}^{N} \hat{a}_j \hat{f}^{n-j} \tag{3.1}$$

where the superscript $n$ denotes the instant $n\Delta t$ or $n\Delta\bar{t}$ . The foundation impedance force can be further obtained by discretizing the inverse Fourier transform of Eqn. 2.2. Correspondingly, applying the inverse *z*-transform to Eqn. 2.8, the TDRE of response displacement is obtained as

$$u^n = F_0 \sum_{j=0}^{N+1} b_j f^{n-j} - \sum_{j=1}^{N+1} a_j u^{n-j} \tag{3.2}$$

### *3.2. Parallel-Form TDREs*

Addressing each term of Eqn. 2.9 identified by *j* separately and applying the corresponding inverse *z*-transform, the TDRE of impedance force is obtained as

$$\hat{f}^n = S_0 \left( \hat{b}_0 u^n + \sum_{j=1}^{N-2L_1} u_{1j}^n + \sum_{j=1}^{L_1} u_{2j}^n \right) \tag{3.3}$$

with the formulas for first- and second-order terms, respectively, as

$$u_{1j}^n = \hat{A}_j u^{n-1} + \hat{s}_j u_{1j}^{n-1} \quad \text{for} \quad j = 1,..., N - 2L_1 \tag{3.4}$$

$$u_{2j}^n = \hat{\beta}_{j1} u^{n-1} + \hat{\beta}_{j2} u^{n-2} - \hat{\alpha}_{j1} u_{2j}^{n-1} - \hat{\alpha}_{j2} u_{2j}^{n-2} \quad \text{for} \quad j = 1,..., L_1 \tag{3.5}$$

where $u_{1j}^n$ and $u_{2j}^n$ are the auxiliary variables introduced. Correspondingly, for Eqn. 2.10 the TDRE of response displacement is

$$u^n = F_0\left(b_0 f^n + \sum_{j=1}^{N+1-2L_2} f_{1j}^n + \sum_{j=1}^{L_2} f_{2j}^n\right) \tag{3.6}$$

with the formulas for first- and second-order terms, respectively, as

$$f_{1j}^n = A_j f^{n-1} + s_j f_{1j}^{n-1} \quad \text{for} \quad j = 1,\ldots, N+1-2L_2 \tag{3.7}$$

$$f_{2j}^n = \beta_{j1} f^{n-1} + \beta_{j2} f^{n-2} - \alpha_{j1} f_{2j}^{n-1} - \alpha_{j2} f_{2j}^{n-2} \quad \text{for} \quad j = 1,\ldots, L_2 \tag{3.8}$$

where $f_{1j}^n$ and $f_{2j}^n$ are the auxiliary variables introduced.

## 4. NUMERICAL TESTS

The effectiveness of bilinear transform is verified by numerical tests in this section. According to the concept of system realization, the stability and accuracy of a TDRE are identical with those of its DRA, and a LPM identical with its CRA which has been also verified via numerical tests in paper [1]. On the other hand, the theoretical analysis for the bilinear transform indicates that the stability and accuracy of the resulting DRA are identical with the CRA. Therefore, we need only verify the identity of time-domain results of TDRE and LPM here. The examples in paper [1] are re-analyzed by using TDREs in time domain, but here the explicit central difference method is used for soil-structure-interaction analysis.

The example of rocking circular foundation on half-space elastic soil has been used to verify the stability theory of CRA in [1]. For the case of $N$=2, The TDREs of impedance force and response displacement corresponding to CRAs in Table 1 of [1] are listed in Table 1 and 2, respectively. Their time-domain results are shown in Figure 1 and 2, respectively. For the case of $N$=3, the results see Table 3 and 4, and Figure 3 and 4 based on Table 2 of [1]. It can be seen that in each case the time-domain results of direct- and parallel-form TDREs are identical and also identical with the result of Wu-Lee LPM, which indicates effectiveness of bilinear transform.

The example of semi-infinite rod on elastic foundation has been used to verify the accuracy of identification for CRA in [1]. For performing time-domain soil-structure-interaction analysis, TDREs of impedance force based on CRAs in Table 3 and 4 of [1] are listed in Table 5. The time-domain results are shown in Figure 5. It can be seen that in each case the time-domain results of TDREs are identical with that of Wu-Lee LPM, which indicates effectiveness of bilinear transform.

## 5. CONCLUSIONS

The stability and identification of DRA of foundation frequency response realized as TDREs are studied in this paper. Some conclusions are summarized as follows:

(1) The accuracy and stability of DRA determine those of the resulting TDREs.
(2) The interinvertible systems of TDRE are: the TDRE of impedance force from foundation displacement with frequency response SDRA and the TDRE of response displacement from foundation force with frequency response FDRA.
(3) DRA can be obtained from a CRA by bilinear transform. The bilinear transform guarantees the identical stability and accuracy between the resulting DRA and corresponding CRA, so that avoids the high-frequency lost and the aliasing. The stability conditions for CRA are restated in terms of the resulting DRA in $z$-plane.

Table 1 TDREs of impedance force based on CRAs in Table 1 of [1] ($N$=2, $\Delta\bar{t}$ = 0.005)

| | ( I ) | ( II , III, IV) |
|---|---|---|
| $\hat{c}_\infty$ | 0.29452431 | 0.29452431 |
| $\hat{a}_1$ | −2.01049531 | 0.32186796 |
| $\hat{a}_2$ | 1.00881397 | −0.67605491 |
| $\hat{b}_0$ | −3.33810650 | 993.07365796 |
| $\hat{b}_1$ | 6.68017937 | −1987.07789059 |
| $\hat{b}_2$ | −3.34375426 | 994.65004568 |
| $\hat{s}_j$<br>( $\lvert\hat{s}_j\rvert$ ) | 1.04658622<br>(1.04658622)<br>0.96390909<br>(0.96390909) | −0.99876132<br>(0.99876132)<br>0.67689337<br>(0.67689337) |
| $\hat{s}_1$ | 1.04658622 | −0.99876132 |
| $\hat{A}_1$ | −0.10572660 | −2369.15254717 |
| $\hat{s}_2$ | 0.96390909 | 0.67689337 |
| $\hat{A}_2$ | 0.07465853 | 62.43606775 |

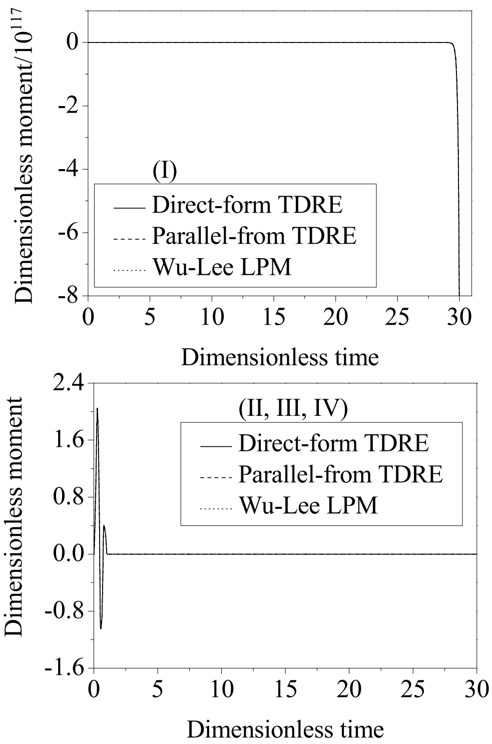


Figure 1 Time-domain results of TDREs in Table 1

Table 2 TDREs of response displacement based on CRAs in Table 1 of [1] ($N$=2, $\Delta\bar{t}$ = 0.02)

| | ( I ) | ( II , III, IV) |
|---|---|---|
| $a_1$ | −3.33306212 | −0.86629389 |
| $a_2$ | 3.63371275 | −0.99110054 |
| $a_3$ | −1.30275713 | 0.87508860 |
| $b_0$ | 0.03838763 | 0.00508228 |
| $b_1$ | −0.04082122 | 0.00950493 |
| $b_2$ | −0.03944088 | 0.00376480 |
| $b_3$ | 0.03976797 | −0.00065785 |
| $s_j$<br>( $\lvert s_j\rvert$ ) | 1.42175388<br>(1.42175388)<br>0.95565412 ± 0.05502773 i<br>(0.95723709) | −0.99997200<br>(0.99997200)<br>0.93313295 ± 0.06615135 i<br>0.93547480 |
| $s_1$ | 1.42175388 | −0.99997200 |
| $A_1$ | 0.05220702 | 0.00001211 e−6 |
| $\alpha_{11}$ | −1.91130824 | −1.86626589 |
| $\alpha_{12}$ | 0.91630285 | 0.87511310 |
| $\beta_{11}$ | 0.03492012 | 0.01390767 |
| $\beta_{12}$ | −0.02949898 | −0.00510544 |

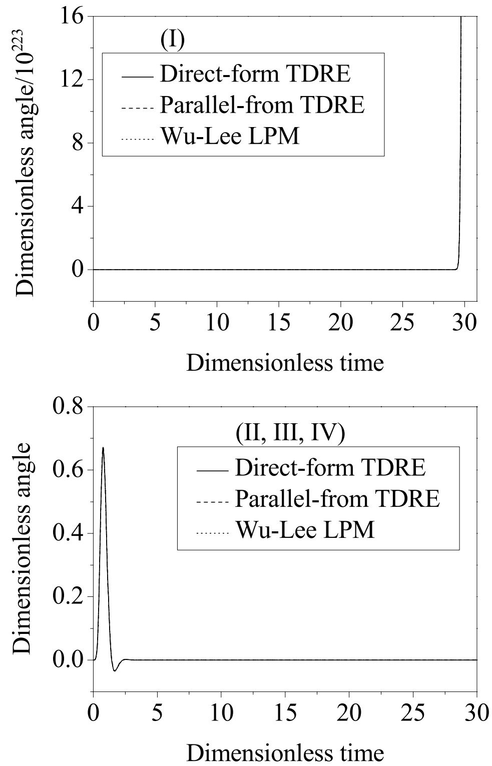


Figure 2 Time-domain results of TDREs in Table 2

Table 3 TDREs of impedance force based on CRAs in Table 2 of [1] ($N$=3, $\Delta\bar{t}=0.005$)

| | (Ⅰ, Ⅲ) | (Ⅱ, Ⅳ and $e=-1$) |
|---|---|---|
| $\hat{c}_\infty$ | 0.29452431 | 0.29452431 |
| $\hat{a}_1$ | −3.02038515 | −2.97521235 |
| $\hat{a}_2$ | 3.04927349 | 2.96039767 |
| $\hat{a}_3$ | −1.02883777 | −0.98513607 |
| $\hat{b}_0$ | 8.88621591 | 10.60772285 |
| $\hat{b}_1$ | −26.55627132 | −31.74653975 |
| $\hat{b}_2$ | 26.45918882 | 31.67624144 |
| $\hat{b}_3$ | −8.78908283 | −10.53737530 |
| $\hat{s}_j$ ($\lvert\hat{s}_j\rvert$) | 0.99415752<br>(0.99415752)<br>1.01311381 ± 0.09211116 i<br>(1.01729252) | 0.99501227<br>(0.99501227)<br>0.99010005 ± 0.09887467 i<br>(0.99502477) |
| $\hat{s}_1$ | 0.99415752 | 0.99501227 |
| $\hat{A}_1$ | 0.00241615 | 0.00186373 |
| $\hat{\alpha}_{11}$ | −2.02622763 | −1.98020009 |
| $\hat{\alpha}_{12}$ | 1.03488407 | 0.99007429 |
| $\hat{\beta}_{11}$ | 0.28110706 | −0.18817539 |
| $\hat{\beta}_{12}$ | −0.35295346 | 0.08961711 |

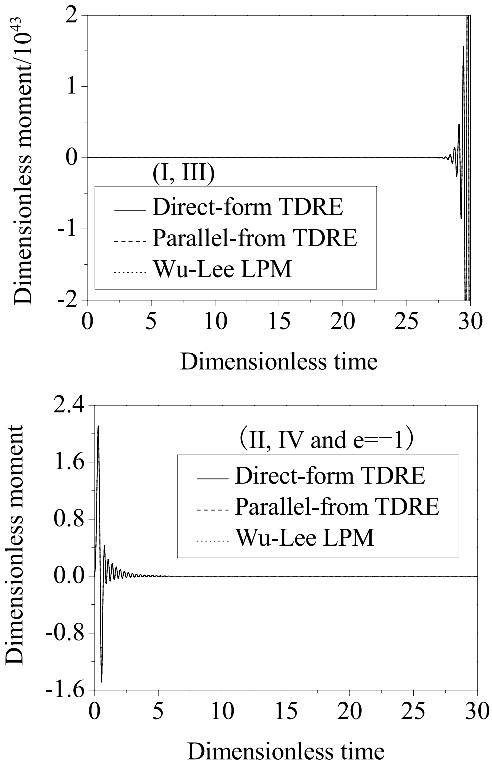


Figure 3 Time-domain results of TDREs in Table 3

Table 4 TDREs of response displacement based on CRAs in Table 2 of [1] ($N$=3, $\Delta\bar{t}=0.02$)

| | (Ⅰ, Ⅲ) | (Ⅱ, Ⅳ and $e=-1$) |
|---|---|---|
| $a_1$ | −3.49643872 | −3.32439008 |
| $a_2$ | 4.62426846 | 4.10848592 |
| $a_3$ | −2.75200984 | −2.23651551 |
| $a_4$ | 0.62434969 | 0.45256995 |
| $b_0$ | 0.02590169 | 0.02526287 |
| $b_1$ | −0.05123063 | −0.04527773 |
| $b_2$ | 0.00308678 | −0.00135976 |
| $b_3$ | 0.05131542 | 0.04535287 |
| $b_4$ | −0.02890368 | −0.02382797 |
| $s_j$ ($\lvert s_j\rvert$) | 0.96939715 ± 0.02567340 i<br>(0.96973706)<br>0.77882221 ± 0.23950435 i<br>(0.81481677) | 0.96586174 ± 0.02154267 i<br>(0.96610195)<br>0.69633330 ± 0.00246859 i<br>(0.69633768) |
| $\alpha_{11}$ | −1.93879430 | −1.93172347 |
| $\alpha_{12}$ | 0.94038996 | 0.93335298 |
| $\beta_{11}$ | 0.07913753 | 0.10583124 |
| $\beta_{12}$ | −0.07708302 | −0.10333786 |
| $\alpha_{21}$ | −1.55764442 | −1.39266661 |
| $\alpha_{22}$ | 0.66392637 | 0.48488616 |
| $\beta_{21}$ | −0.03980448 | −0.06712534 |
| $\beta_{22}$ | 0.00648886 | 0.01590600 |

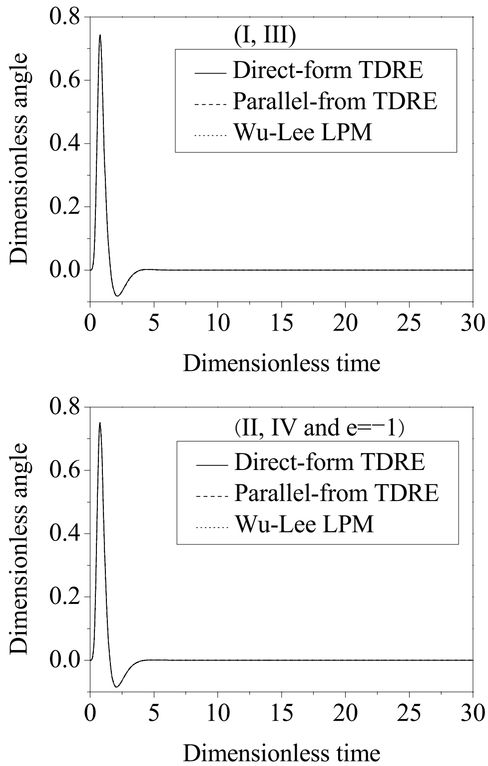


Figure 4 Time-domain results of TDREs in Table 4

Table 5 TDREs of impedance force based on CRAs in Table 3 and 4 of [1] ( $\Delta\bar{t} = 0.005$ )

| | $N$=3 | $N$=4 |
|---|---|---|
| $\hat{c}_\infty$ | 1.00000000 | 1.00000000 |
| $\hat{a}_1$ | −2.99342899 | −3.99215512 |
| $\hat{a}_2$ | 2.98689097 | 5.97651677 |
| $\hat{a}_3$ | −0.99346189 | −3.97656801 |
| $\hat{a}_4$ | – | 0.99220636 |
| $\hat{b}_0$ | −0.00781142 | 0.00565137 |
| $\hat{b}_1$ | 0.02612108 | −0.02018687 |
| $\hat{b}_2$ | −0.02879454 | 0.02667384 |
| $\hat{b}_3$ | 0.01048497 | −0.01539246 |
| $\hat{b}_4$ | – | 0.00325412 |
| $\hat{s}_j$ ( $\lvert\hat{s}_j\rvert$ ) | 0.99603249<br>(0.99603249)<br>0.99869825 ± 0.00457793 i<br>(0.99870875) | 0.99934130 ± 0.00487492 i<br>(0.99935319)<br>0.99673626 ± 0.00282354 i<br>(0.99674026) |
| $\hat{s}_1$ | 0.99603249 | – |
| $\hat{A}_1$ | 0.00283852 | – |
| $\hat{\alpha}_{11}$ | −1.99739651 | −1.99868260 |
| $\hat{\alpha}_{12}$ | 0.99741916 | 0.99870680 |
| $\hat{\beta}_{11}$ | −1.00367943 e−4 | −1.89602906 e−4 |
| $\hat{\beta}_{12}$ | 1.06990733 e−4 | 1.91804594 e−4 |
| $\hat{\alpha}_{21}$ | – | −1.99347252 |
| $\hat{\alpha}_{22}$ | – | 0.99349114 |
| $\hat{\beta}_{21}$ | – | 0.00256389 |
| $\hat{\beta}_{22}$ | – | −0.00254706 |

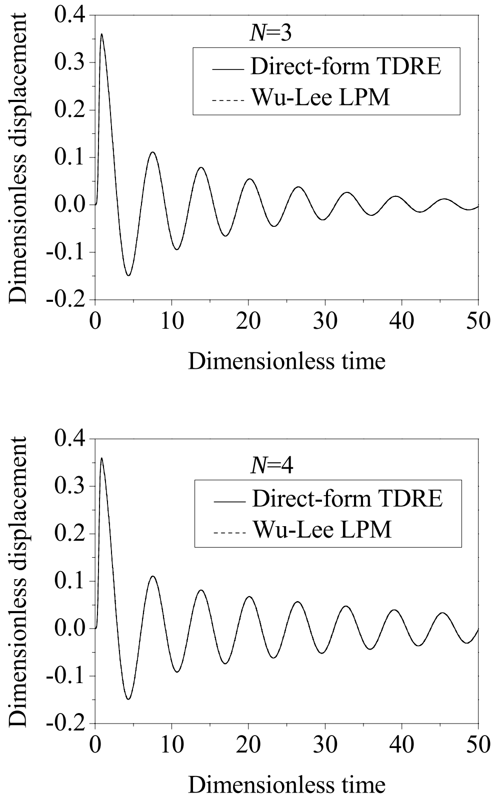


Figure 14 Time-domain soil-structure-interaction results


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