

FEM Analysis of Composite Soil Nailed Wall on the Dynamic Response of Earthquake

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Abstract: Finite Element Analysis method of composite soil nailed walls are regarded as a plane problem or a partial spatial one, but the whole three dimensions analysis method is seldom found. Especially the FEM analysis of the dynamic response of the composite soil nailed wall which subjected to earthquake pore pressure coupled is studied rarely. In this paper, with the help of the finite element software ADINA, an actual composite soil nailed wall was solved. The dynamic response of the composite soil nailed wall is analyzed and discussed under the EL-Centro and man-made Lanzhou accelerogram. And the variation principles of the soil nailed wall which subjected to the earthquake, and the earthquake coupled with pore pressure, are demonstrated respectively. The results of the FEM dynamic analysis can be a useful reference for engineers of the design and construction of the composite soil nailed wall

Key words: Composite soil nailed wall; 3-D FEM; accelerogram; earthquake action; dynamic response; analysis

1 INTRODUCTION

The composite supporting structure consists of the prestressed anchor and soil nails which can be constructed all together. The installation of the soil nails and the prestressed anchor can be arranged alternated arrangement or using prestressed anchors replace partially the soil nails (Figure 1). The prestressed anchor also can be installed after soil nails are constructed in order to reinforcing the soil nailed wall. The arrangement location, number and length of the pestressed anchors are determined in accordance with the equilibrium condition and the deformation of composite soil nailed wall.

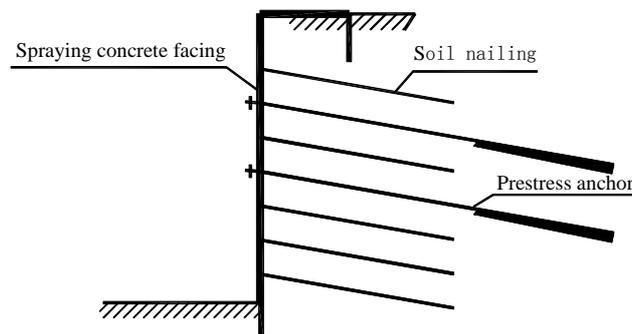


Fig.1 The composite soil nailed wall

The analysis of composite soil nailed wall is very complicated because it concerns the interaction of anchoring system, retaining structure and soil. The limit equilibrium method in common use can only calculate static axial force of the soil nails and the anchors and the safety factor of the slope stability but it can not provide the deformation and internal force distribution of supporting structure. Moreover, its characteristic of vibration is much more complicated than the static one, so it is more difficult to obtain the dynamic response of the structure. The finite element method is certainly the most comprehensive approaches to analyze the performance of soil and structures subjected to seismic loading. It can overcome the shortcoming of the limit equilibrium method. The finite element method not only provide axial force of soil nails, the stress-strain relationship of soil, simulating construction process of the wall, but also it considers inhomogeneous and aeolotropism property of the soil. In the past twenties years the finite-element method has been used to analyze a large number of soil nailed wall. C.K.Shen et al.(1975) uses finite element method to carry out parameters analysis of soil nailed walls. Song Erxiang et al.(1996) have analyzed the deformation characteristic of soil nailed wall by two and three-dimensional finite element model.Until now, nobody has studied the seismic analysis of the composite soil nailed wall using the finite element method. In this paper the finite element method is employed to carry out seismic response

analysis of composite soil nailed wall. The study content includes the axial force response of the soil nails and anchors, the acceleration response of soils and the displacement response of the supporting structure under earthquake process.

2 DYNAMIC FINITE ELEMENT METHOD

2.1 Dynamic finite element theory

The finite element software ADINA (2004) is adopted to analyze the dynamic behavior of the composite soil nailed wall in this paper. This software can provide a lot of material models including Mohr-Coulomb, Cam-Clay, Drucker-Prager, Concrete model and so on . It also can provide a number of structure elements, for example, the truss and cable element, beam element, shell element, pipe element. So ADINA will be capable of accounting for nonlinear, inelastic dynamic behavior of the soil and of the soil-structure interaction.

Dynamics finite element method is same with static one. The continuous body will be separated into the limited number elements. However, the inertial force and the damping force must be considered when the dynamic analysis is carried out; the total body force, $\{p\}$, can then be expressed as

$$\{p\} = \{p_s\} - \rho \frac{\partial^2}{\partial t^2} \{u\} - \nu \frac{\partial}{\partial t} \{u\} \quad (1)$$

Where $\{p_s\}$ is the static body force; $\{u\}$ is the displacement; $-\rho \frac{\partial^2}{\partial t^2} \{u\}$ is inertial force; $-\nu \frac{\partial}{\partial t} \{u\}$ is the damping force; ρ is the material density; ν is the damping coefficient.

When finite element method is used to solve the dynamic problem, the displacement function adopted is shown as follows:

$$\{f\} = [N]\{u\}^e \quad (2)$$

Where $[N]$ is the shape function matrix; $\{u\}^e$ is the element node displacement matrix.

Element stiffness matrix, mass matrix and damp matrix are respectively:

$$[K^e] = \int [B]^T [D] [B] dV \quad (3)$$

$$[M^e] = \int [N]^T \rho [N] dV \quad (4)$$

$$[C^e] = \int [N]^T \nu [N] dV \quad (5)$$

Where $[B]$ is the strain matrix; $[D]$ is the elasticity matrix.

Through the node force equilibrium, the dynamic motion equation can be obtained:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \quad (6)$$

Where $[K]$ is the total stiffness matrix; $\{u\}$ is the node displacement matrix, $\{\dot{u}\}$ is the node velocity matrix, $\{\ddot{u}\}$ is the node acceleration matrix, $\{F\}$ is the node load vector, $[C]$ is the total damp matrix, $[M]$ is the total mass matrix.

The key to solve motion equation is the dynamic response of the wall which includes displacement, velocity, acceleration, stress and strain of the soil and retaining structure. There are some difference between dynamic equation and static equation apparently: one is that dynamic equation contains inertial force and damping force, and the other is the load variable along with the time.

The equation (6) can be solved by the direct integration. In this paper Newmark method is adopted in the dynamic analysis which is generalized linear acceleration method. Its basic theory is to assume that the acceleration changes linearly within time interval Δt , according to this assumption the displacement vector and velocity vector are obtained by integrating at time $t + \Delta t$, then the velocity and acceleration vector function will be expressed by the displacement function. Substituting the velocity and acceleration vector into motion equation, the recursion formula will be written as:

$$\{\dot{u}(t + \Delta t)\} = \{\dot{u}(t)\} + [(1-r)\{\ddot{u}(t)\} + r\{\ddot{u}(t + \Delta t)\}]\Delta t \quad (7)$$

$$\{u(t + \Delta t)\} = \{u(t)\} + \Delta t\{\dot{u}(t)\} + [(\frac{1}{2} - \beta)\{\ddot{u}(t)\} + \beta\{\ddot{u}(t + \Delta t)\}]\Delta t^2 \quad (8)$$

where r , β are the parameter which is determined by the integrating accuracy and stability, when $r = 1/2$, $\beta = 1/6$, it is called linear acceleration method. Equation (7) and (8) can be written as:

$$\{\ddot{u}(t + \Delta t)\} = \frac{1}{\beta\Delta t^2}[\{u(t + \Delta t)\} - \{u(t)\}] - \frac{1}{\beta\Delta t}\{\dot{u}(t)\} - (\frac{1}{2\beta} - 1)\{\ddot{u}(t)\} \quad (9)$$

$$\{\dot{u}(t + \Delta t)\} = \frac{r}{\beta\Delta t}[\{u(t + \Delta t)\} - \{u(t)\}] - (1 - \frac{r}{\beta})\{\dot{u}(t)\} + (1 - \frac{r}{2\beta})\Delta t\{\ddot{u}(t)\} \quad (10)$$

Substituting the equation (9)~(10) into the motion equation (6)

$$[K]^* \{u(t + \Delta t)\} = \{F(t)\}^* \quad (11)$$

Where $[K]^* = [K] + \frac{1}{\beta(\Delta t)^2}[M] + \frac{r}{\beta\Delta t}[C]$ (12)

$$\{F(t)\}^* = \{F(t + \Delta t)\} + [M] \left\{ \frac{1}{\beta\Delta t^2} \left\{ u(t) + \frac{1}{\beta\Delta t} \dot{u}(t) \right\} + \left(\frac{1}{2\beta} - 1 \right) \ddot{u}(t) \right\} +$$

$$[C] \left\{ \frac{r}{\beta\Delta t} \dot{u}(t) \right\} + \left(\frac{r}{\beta} - 1 \right) \dot{u}(t) + \left(\frac{r}{2\beta} - 1 \right) \Delta t \ddot{u}(t)$$

The study illustrates that the equation is convergent without condition when $r \geq 0.5$, $\beta \geq 0.25(r + 0.5)^2$. With regard to the dynamic analysis, if $r = 0.5$, $\beta = 0.25$, the fine results can be got.

The main advantage of the method is one kind of unconditional stable algorithm, the stability of the solution can not be affected by the step size of the time, Δt , which is determined by the accuracy of the solution. Moreover, the bigger time step adopted may filter away the influence of the high order inexactitude eigenvector of the solution, but the method needs to inverse the stiffness matrix and the time of the calculation costs longer.

2.2 Damping calculation

The damping is to make attenuation of shaking or energy dissipation of shaking. Rayleigh damping is used in this paper, which can be broken into a component proportional to the mass matrix and a component proportional to the stiffness matrix. It can be written in the matrix form

$$C = \alpha M + \beta K \quad (14)$$

Where α is damping coefficient related to the mass, β is damping coefficient related to the stiffness, their value can be calculated as follows:

$$\alpha = \frac{2\omega_i\omega_j(\xi_i\omega_j - \xi_j\omega_i)}{\omega_j^2 - \omega_i^2} \quad (15)$$

$$\beta = \frac{2(\xi_j \omega_j - \xi_i \omega_i)}{\omega_j^2 - \omega_i^2} \quad (16)$$

Where ω_i and ω_j are the natural frequency of mode i and mode j in the calculation model, ξ_i and ξ_j are the damping ratio of the corresponding natural frequency, which can be obtained through test. The damping ratio ranges from 0.02 to 0.2, here the soil damping ratio is $\xi_1 = \xi_2 = 0.06$.

3 COMPOSITE SOIL NAILING RETAINING WALL SEISMIC SIMULATION

ADINA can analyse seismic problems on the slope with the supporting structure. It is difficult to carry out seismic response analysis of slope with the supporting structure and choose reasonable retaining wall, especially when the supporting structures are on the bad environment. All the problems will be a challenge for some commercialized finite element software. ADINA can solve the problems because it provides some advanced technology, for example: ①Energy radiation can be simulated exactly and efficiently on non-reflecting infinite boundary, and plane wave, sphere wave, cylindrical wave, etc can be filtered. Users also can use damping and mass combining non-linear spring to simulate such kind boundary; ② Wilson-Theta dynamic method can be used for non-linear time history dynamic analysis; ③the complex geological characteristics which are soft soils and fragmentation of fault in the bad area can be dealt with.

3.1 Engineering investigation

The project located in a road of Lanzhou city of China, the length of slope is about 320m, the depth is about 8.00m, the site soil layers are composed of pleistocene filling soils, and they are described from up to down: (1) filling layer; (2) clay layer; (3) medium sand layer; (4) sandstone layer; (5) bedrock layer. Physical parameters of all layers and thickness are listed at table 1.

Table 1 Soil profile at the site

Soil Type	Unit weight /($\text{kN}\cdot\text{m}^{-3}$)	Cohesion /kPa	Internal friction/($^\circ$)	Thickness of soil layer/(m)
fill	17.7	10	8	0.4
clay	19.9	31	15	3.8
medium sand	20.2	37	10	3.9
Sandrock	21.0	60	30	21.9

3.2 FEM model

The partial 3-D FEM model is established. The width of model is the nail spacing. In the underground structural dynamic analysis, because of vibration or fluctuation infinitely propagating, the research object is a half spatial problem. However, finite element method can only solve finite boundary problem, so this needs to approximate treatment. Theoretically, the further the distance is from the structure, the less the reflection of wave is on boundary and the less the influence on underground dynamic response from artificial boundary is. So, the determining principle of the FEM calculating boundary is that the influence of reflection wave along the boundary can decrease small as much as possible. According to experimental method, usually the width which is 5~10 times to the width of underground structure treats as the calculated field. In the paper the width is 8 times of slope height, namely the length of the model is 75m. The dynamic analysis model is shown in Figure 2 and 3.

The calculated parameters are as follows: (1) the height of slope is 8.10m, slope is vertical; (2) soil: in the FEM analysis, the model of soil uses Mohr-coulomb model, all parameters are listed at table 1. (3) arrangement of soil nails: they are composed of soil nails and anchors; The first, third, fifth, and sixth rows are soil nails; the second and fourth are prestressed anchors which length are 18m. Incline angle is 10° , horizontal spacing is 1.40m, vertical spacing is 1.30m (except the first row is 1.6m). The rebar element is used for soil nail, and its material is

considered as elastic-plastic material, drilled borehole diameter is 130mm, diameter of reinforcement is 25mm, elastic modulus can be obtained according to area ratio which is $E=2.0 \times 10^{11}$ Pa, Poisson's ratio=0.25, mortar is M20, elastic modulus $E=2.0 \times 10^{10}$ Pa, Poisson's ratio $\mu=0.2$, bonding strength between nails and soil is 50kPa, soil nail elastic modulus is $E=2.67 \times 10^{10}$ Pa and Poisson's ratio is $\mu=0.202$; (4) the second and fourth rows are prestressed anchors, and free length is 3m, the anchored length is 15m, prestressing force is 100kN. Its material is considered as elastic material, drilled borehole diameter is 150mm, diameter of reinforcement is 28mm, elastic modulus is $E=2.0 \times 10^{10}$ Pa, Poisson's ratio=0.25, mortar is M20, elastic modulus is $E=2.0 \times 10^{10}$ Pa. anchor elastic modulus is $E=2.67 \times 10^{10}$ Pa and Poisson's ratio $\mu=0.202$; (5) free section of anchor: its material is considered as elastic material, and the steel bar is only considered, its elastic modulus is $E=2.0 \times 10^{10}$ Pa, poisson ratio $\mu=0.25$; (6) interface: the material of interface is considered as linear relation, tangential stiffness on the interface between pile and soil is $K_s=40000\text{kN/m}^3$, normal stiffness is $K_n=50000\text{kN/m}^3$; (7) facing: thickness of facing is 100mm, because there are usually reinforced concrete in the facing, so it can be seen as elastic material. The elastic modulus is $E=2.0 \times 10^4\text{MPa}$, Poisson's ratio is $\mu=0.20$.

FEM model of the composite soil nailed wall and the prestressed anchors are shown in Fig.2 and Fig.3.

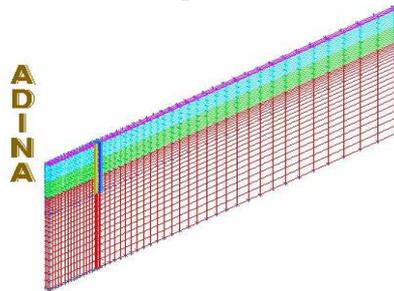


Fig.2 The finite element mesh of computing domain

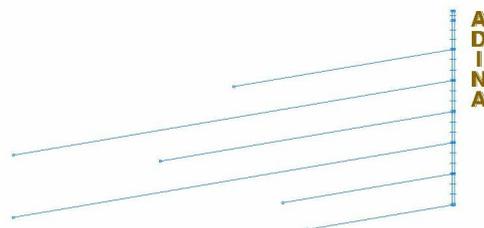


Fig.3 The scheme plan of soil nailing with anchor

3.3 Input earthquake motions

The accuracy of seismic response analysis depends on the accuracy of the input motion applied to the FEM model. Acceleration peak of seismic record should be amplified or reduced properly to make acceleration peak be equal to the one of local earthquake intensity. Acceleration peak can modify according to the equation (17) as follows:

$$\alpha'(t) = \frac{\alpha'_{\max}}{\alpha_{\max}} \alpha(t) \quad (17)$$

where $\alpha(t)$ 、 α_{\max} are seismic acceleration and peak acceleration of original record, $\alpha'(t)$ 、 α'_{\max} are seismic acceleration and peak acceleration after adjustment. The peak acceleration value modified is selected according to seismic code (GB50011—2001,China Building Seismic Design Code,2001). The seismic design intensity of Lanzhou city is 8 degree, and basic seismic design acceleration is 0.20g.

America EL—Centro and Lanzhou seismic excitations are adopted for the seismic dynamic analysis of the composite soil nailed wall. In order to save calculating time, only 7.6 sec. duration of earthquake excitation including the peak acceleration is intercepted, time interval is 0.005 sec., the total 1520 acceleration record points

are regarded as the input seismic acceleration records, and the steps of calculating are 0.02 sec., the total 450 steps, end 9s. EL—Centro seismic horizontal and vertical excitations are shown in Figure 4 and 5. (after amplitude adjustment). 9.6 sec. The duration of Lanzhou earthquake excitation is intercepted including the peak acceleration, time interval 0.024 sec. The total 402 acceleration records points are as the input seismic acceleration records, and stepsof calculating are 0.024 sec., the total 400 steps, end 9s. Lanzhou man-made seismic acceleration excitation is shown in Fig.6 (after amplitude adjustment).

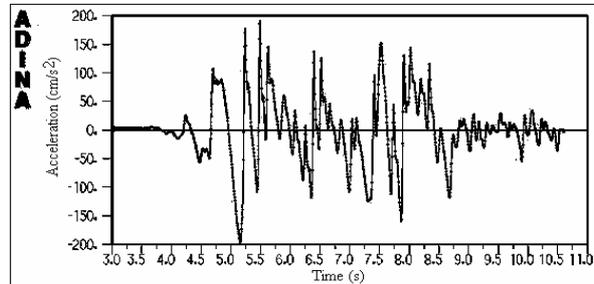


Fig.4 EL—Centro horizontal earthquake excitations

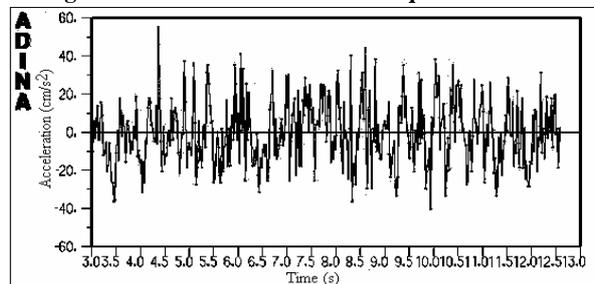


Fig.5 EL—Centro vertical earthquake excitations

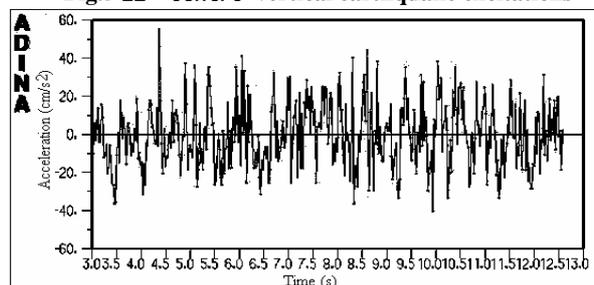


Fig.6 Lanzhou man-made earthquake excitations

For studying the influence of the input different seismic excitations on dynamic response, EL-Centro and Lanzhou seismic excitations are used to carry out the analysis of the composite soil nailed wall. The study objects include dynamic response time-history of soil nails and anchor axial force, axial force variation, dynamic displacement, dynamic acceleration and peak displacement after earthquake etc.

4 CALCULATION RESULTS ANALYSIS

4.1 Comparing the axial force of soil nails and anchor after construction and under earthquake

The variations of every soil nail and anchor axial force under EL-Centro and Lanzhou seismic excitations and after construction are shown in Figure 7 to 12. From the Figures, after earthquake, soil nail axial force increases comparing with post-construction. The increasing extent of upside rows(the first, second, third rows) is obvious especially, and the soil nail axial force increases more under EL—Centro seismic excitations than under Lanzhou artificial seismic excitations. From the results, the axial force increments of the first, third rows soil nails and second rows anchor are 42.7%、29.4%、20.9% respectively (under EL-Centro seismic excitations); 8.2%、6.4%、1.3% respectively (under Lan Zhou artificial seismic excitations) .

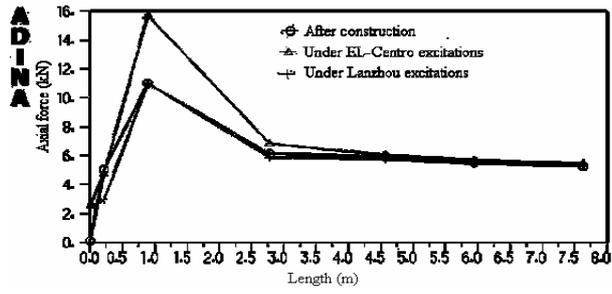


Fig.7 Comparing the axial force of the first row soil nail after construction and under earthquake

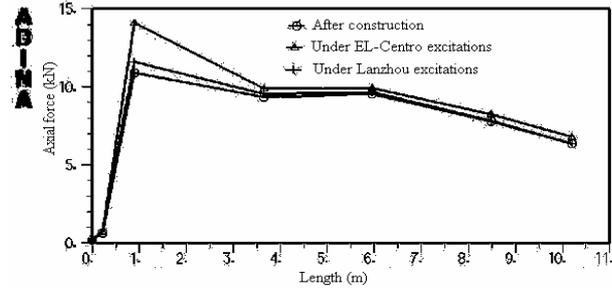


Fig.8 Comparing the axial force of the second row anchor after construction and under earthquake

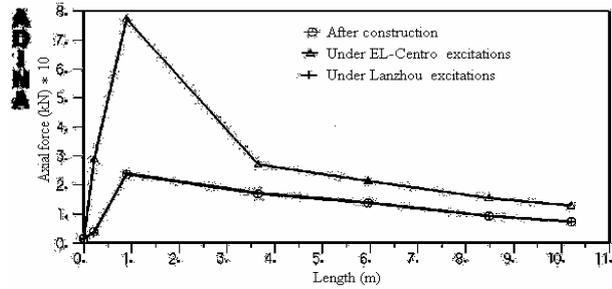


Fig.9 Comparing the axial force of the third row soil nail after construction and under earthquake

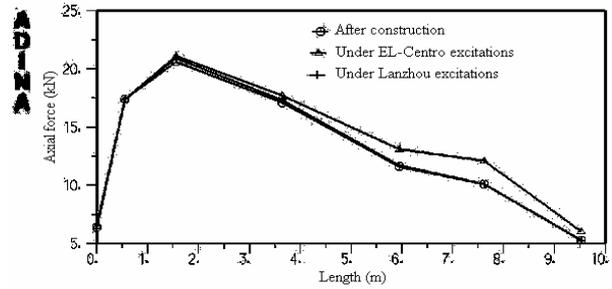


Fig.10 Comparing the axial force of the fourth row anchor after construction and under earthquake

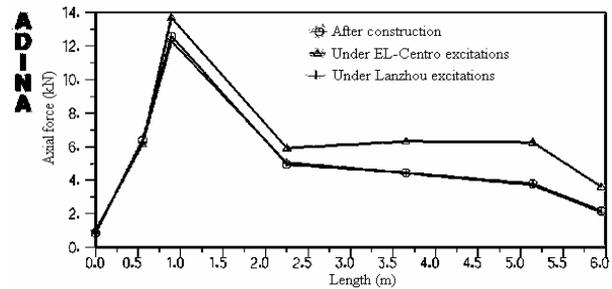


Fig.11 Comparing the axial force of the fifth row soil nail after construction and under earthquake

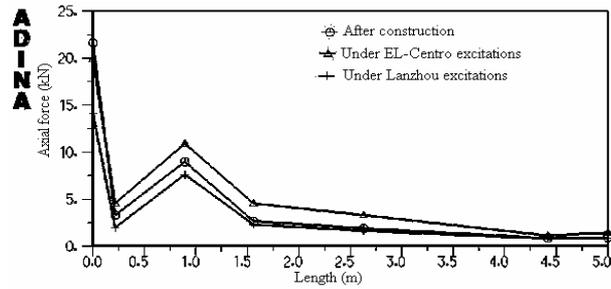


Fig.12 Comparing the axial force of the sixth row soil nail after construction and under earthquake

4.2 Seismic response on axial force of composite soil nailing

In order to investigate the dynamic response of soil nail axial force during earthquake, the dynamic response time history of an element or node of soil nails and anchors must be obtained. In the ADINA software, if element groups of rebar are defined, element can be meshed automatically by constraint equation of the system. Soil nail elements meshed are shown in Figure 13 and the nail is composed of 14 elements. According to calculating results, after construction, the biggest axial force element of every row is number 4 in composite soil nailed wall.

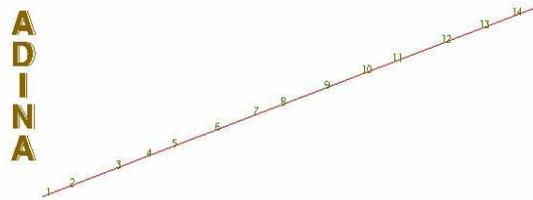


Fig.13 Meshing elements of soil nail

The axial force time history of composite soil nailed wall is shown in Figure 14~19. From the results:

(1) Under EL—Centro earthquake excitations, the fluctuation trend of the axial force of the composite soil nailed wall is more obvious than one of the pure soil nailed wall with the variety of the acceleration excitations. The increment of the axial force of the soil nail gradually reduces from the first rows to fifth rows (this means slide between soil and soil nails is decreasing gradually from the first row to the fifth row), but the increment of the axial force of the sixth rows obviously increases. The total axial force of the soil nail gradually reduces from the first rows to sixth rows. However, under Lanzhou man-made earthquake excitations, the fluctuation trend of the axial force of the composite soil nailed wall is comparatively stable, the decrease of the total axial force is more and more obvious from the first rows to sixth rows. For example, under EL-Centro seismic excitations, in each row soil nail, the biggest axial forces are increased respectively from earthquake beginning 11.0kN, 10.8 kN, 23.6 kN, 27.7 kN, 12.5 kN, 8.84 kN to 15.7 kN, 14.1 kN, 25.6 kN, 29.0 kN, 13.7kN, 10.9 kN of earthquake end, and the increments are 4.7 kN, 3.3 kN, 2.0 kN, 1.3 kN, 1.2 kN, 2.06 kN respectively. Under Lan Zhou artificial seismic excitations, from earthquake beginning 10.9kN, 10.8kN, 23.6kN, 27.6 kN, 12.5 kN, 8.80 kN to 11.9 kN, 11.6 kN, 24.0kN, 27.9 kN, 12.3 kN, 7.61 kN of earthquake end, the increments as 1.0 kN, 0.8 kN, 0.4 kN, 0.3 kN, -0.2 kN, -1.19 kN respectively.

(2) The peak axial force of soil nail and anchor in the composite soil nailed wall varies with the input seismic excitations largely, but final axial force is nearly the same under EL-Centro and Lanzhou earthquake excitations. This means seismic response of soil nail and anchor is different largely under different seismic excitations, so soil nail and anchor design should be different in different area despite the same seismic intensity.

(3) Under different seismic excitations, appearance time of peak axial force of soil nail and anchor is different, for example, under EL-Centro seismic excitations, the appearance time of peak axial force is $t=5.86s$, and $t=6.31s$, $7.13s$, under Lanzhou seismic excitations.

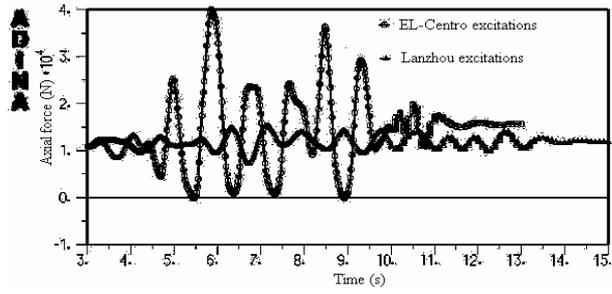


Fig.14 The axial force time history of the first row soil nail

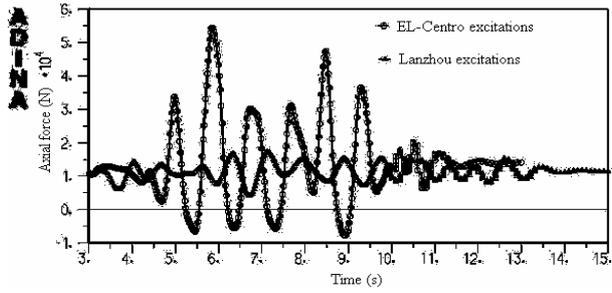


Fig.15 The axial force time history of the second row anchor

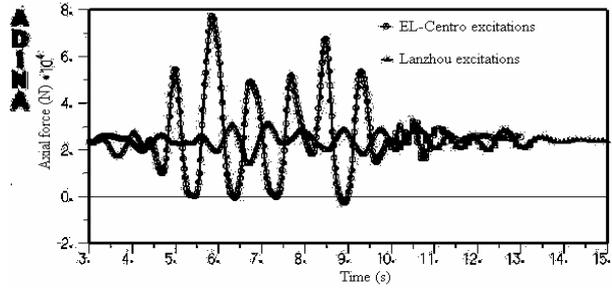


Fig.16 The axial force time history of the third row soil nail

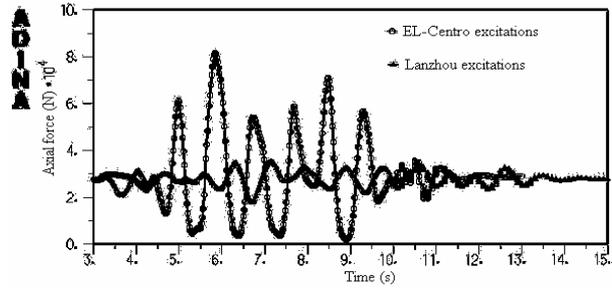


Fig.17 The axial force time history of the fourth row anchor

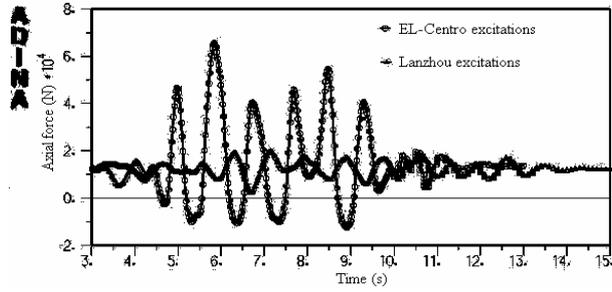


Fig.18 The axial force time history of the fifth row soil nail

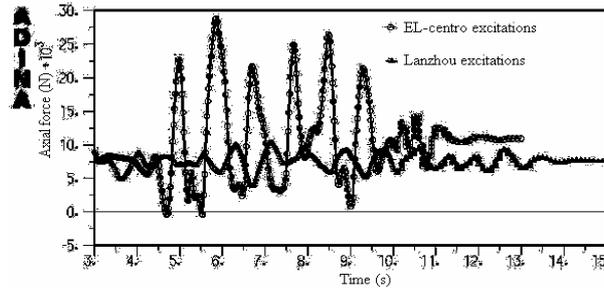


Fig.19 The axial force time history of the sixth row soil nail

4.3 Seismic response of displacements

The horizontal and vertical displacement responses of the composite soil nailed wall along slope height are shown in the Figure 20 ~ 25 under earthquake. From the results:

- (1) The horizontal displacements are much more than the vertical ones;
- (2) The permanent displacements occur in the composite soil nailed wall after earthquake.

Under EL-Centro earthquake excitation, the horizontal and vertical displacements in node 210 which is the top surface of the slope supported by the composite soil nailed wall are 2.60mm and 1.99mm.

Under Lanzhou man-made earthquake excitation, the horizontal and vertical displacements in node 210 which is the top surface of the slope supported by the composite soil nailed wall are 0.7mm and 1.19mm.

For the same structure, the permanent displacement produced by EL-Centro earthquake excitation is much more than one produced by Lanzhou man-made earthquake excitation.

- (3) Under two kinds of earthquake excitations, the time history of the horizontal and vertical displacements are similar.

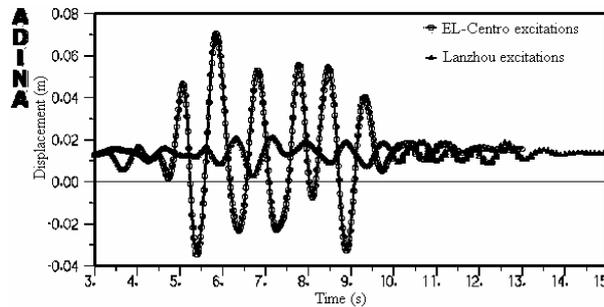


Fig.20 The node 210 horizontal displacement time history

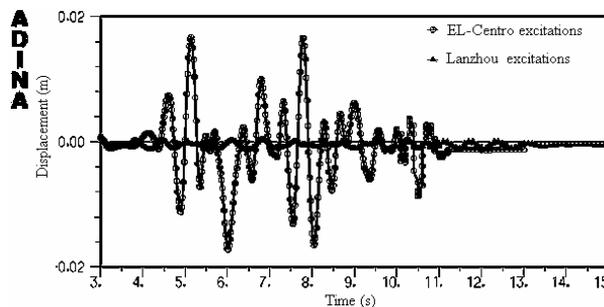


Fig.21 The node 210 vertical displacement time history

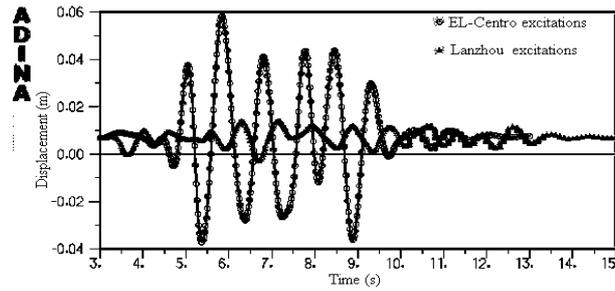


Fig.22 The node 200 horizontal displacement time history

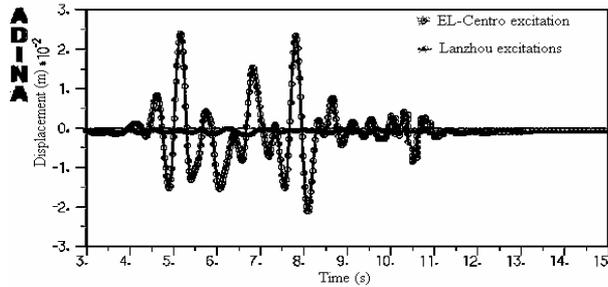


Fig.23 The node 200 vertical displacement time history

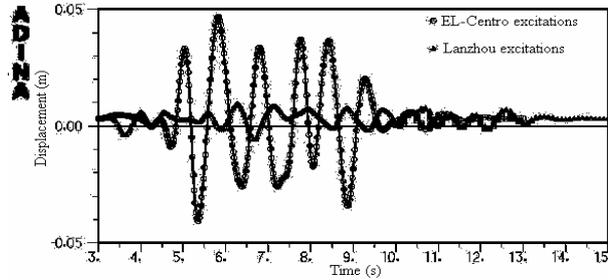


Fig.24 The node 190 horizontal displacement time history

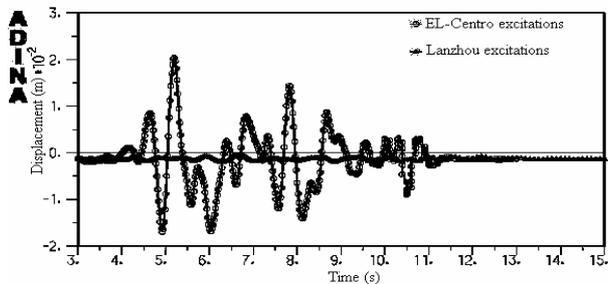


Fig.25 The node 190 vertical displacement time history

5 CONCLUSIONS

When the geology condition or the environment is comparatively complicated, moreover, the deformation of the slope is controlled strictly, using simple soil nailed wall is very difficult to satisfy the project demand, so the composite soil nailed technology which is very effective to control displacement should be adopted. In this paper referring to the former research achievement, the finite element method (ADINA) is adopted to carry out the 3-D nonlinear analysis of the composite soil nailed wall under earthquake, and the following conclusions can be obtained:

(1) Under EL—Centro earthquake excitation, the fluctuation trend of the axial force of the composite soil nailed wall is more obvious. The increment of the axial force of the soil nail gradually reduces from the first rows to fifth rows, but the increment of the axial force of the sixth rows obviously increases. The total axial force of the

soil nail gradually reduces from the first rows to sixth rows. Under Lanzhou man-made earthquake wave, the fluctuation trend of the axial force of the composite soil nail is comparatively stable, and the decrease of the total axial force is more and more obvious from the first rows to sixth rows.

(2) Under earthquake the amplitude of every node is unequal from top to bottom along the slope height, so the relative displacement among soil layers will be produced, such displacement may make the supporting structure produce the shearing deformation until the failure of the wall. The seismic response of soil layers in the vicinity of the slope top are the fiercest and the easiest to damage.

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