UNITED DYNAMIC RELIABILITY ANALYSIS FOR PRIMARY-SECONDARY SYSTEM

GUO Wei[1] and LI Hong-nan[2]

[1] *PhD candidate, Institute of Earthquake Engineering, School of Civil and Hydraulic Engineering, Dalian University of Technology, Dalian. China*
[2] *Professor, Institute of Earthquake Engineering, School of Civil and Hydraulic Engineering, Dalian University of Technology, Dalian. China*
Email: wei.guo.86@gmail.com, Li-hong-nan@163.com

ABSTRACT :

The structures subjected to severe earthquake loading usually undergo excessive inelastic deformation and even get into destruction, especially when attached secondary system is with large mass or the ratio of secondary system frequency to post-yield frequency of primary structure distributes within a certain range. Therefore, dynamic reliability of single secondary system commonly makes no substantial sense due to destruction of primary structure, and many authors have focused on the united dynamic reliability analysis for whole primary-secondary system. The united reliability of whole primary-secondary system with yielding primary structure under gauss white noise excitation is investigated in this paper using equivalent linearization and Mone Carlo simulation, and the cross-over rate, as a equivalent measurement of expected first passage time, is computed as the analysis index for a number of combinations of parameters of primary-secondary system. Research in the paper indicates that the single secondary system reliability analysis is generally non-conservative in contrast to the whole system analysis, especially for secondary system with large mass or within certain range of the ratio of secondary system frequency to that of post-yield primary structure.

KEYWORDS: primary-secondary system, nonlinearity, united dynamic reliability, cross-over rate, first passage time

1. INTRODUCTION

The earthquake disaster researches have shown that a considerable part of loss is attributed to the failure of secondary systems, such as crucial equipments, pipes and architectural elements etc. Therefore, a research on primary-secondary system has been widely developed in the last three decades. Some researchers have focused on the nonlinearity of primary-secondary system. Villaverde introduces a simple method for calculating secondary system's dynamic response in the cases the nonlinearity of primary system or both of primary and secondary system being considered respectively. Igusa establishes the two degrees of freedom model, and analyzes the characteristic of secondary system's response based on the perturbation method. Adam investigates the effect of nonlinearity of primary system on the secondary system's response by numerical simulation and experimental study. Politopoulos studies on the characteristics of nonlinear floor response spectra.
The reliability research of secondary system has been developed form the 90's of the last century. Chen and Lutes studies the first passage problem of secondary system attached to nonlinear primary system, and analyzes the non-gauss characteristics of the response. Zhu investigates the effect of nonlinearity of primary system of single or multiple degrees of freedom on the reliability of secondary system. In the previous articles, the study on secondary system's reliability only focuses on itself, which might not agree with practical situation. In the severe earthquake excitation, structures usually collapse before secondary system's failure, which could also lead to secondary system's inoperative indirectly. Moreover, the improper setting of mass, frequency and damping ratio could also lead to the change of secondary system's reliability, and a similar example is that the setup of tuned mass damper can obviously influence primary system's response. Therefore, separate secondary system's reliability analysis is not always exact, and united dynamic reliability analysis for primary-secondary system is more reasonable.

In this paper, the cross-over rate is adopted as the index of dynamic reliability analysis based on first passage theory. For nonlinearity case, equivalent linearization is introduced to approximate the reliability of secondary system mounted on nonlinear primary system, based on which the effect of primary-secondary system's parameter variety on the united reliability is studied.

2. NONLINEAR MODEL OF PRIMARY-SECONDARY SYSTEM

A schematic model of two degrees of freedom system is given in Fig.1. The mass, initial stiffness, damping coefficient, frequency, and damping ratio of primary system are denoted by symbols m_p, k_p, c_p, ω_p, and ζ_p respectively; the mass, initial stiffness, damping coefficient, frequency, and damping ratio of secondary system are denoted by symbols m_s, k_s, c_s, ω_s, and ζ_s respectively; the responses of primary system and secondary system are x_p, x_s respectively.

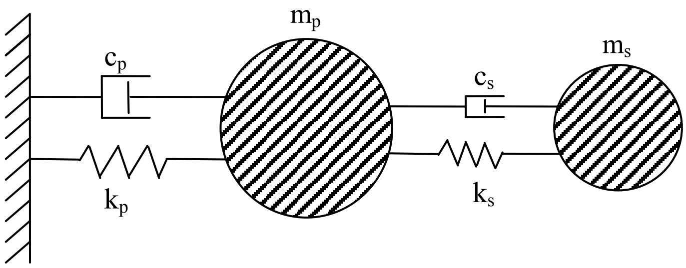


Figure1 Model of primary-secondary system

Assuming $y_1 = x_p - x_g$, $y_2 = x_s - x_p$, $\mu = m_s / m_p$, $\eta = \omega_s / \omega_p$, $\tilde{t} = \omega_p t$. The equations of motion can be written as

$$y_1'' + 2\varsigma_p y_1' + \alpha y_1 + (1-\alpha)z - 2\varsigma_s \mu\eta y_2' - \mu\eta^2 y_2 = -x_g''$$
$$y_2'' + 2\varsigma_s \eta y_2' + \eta^2 y_2 + y_1'' = -x_g'' \tag{2.1}$$

Bouc-Wen model is adopted as the nonlinear hysteretic model, which is expressed in the hysteretic force $z' = -\gamma |y_1'| z |z|^{n-1} - \beta y_1' |z|^n + A y_1'$. In this paper, the values of nonlinear hysteretic model parameters are assumed as $A = n = 1$, $\gamma = 0.95$, $\beta = 0.05$. Gauss white gauss is adopted as the earthquake excitation, and $x_g'' = \ddot{x}_g / \omega_p^2$.

3. EQUIVALENT LINEARIZATION METHOD

Equivalent linearization method is used as an approximate approach when primary system yields due to severe earthquake excitation. For the equivalent linear system excited by gauss white noise, the processes of y_1, y_1' and z is also gauss process. According to equivalent linearization method, c_e and k_e are determined by

$$k_e = -\gamma E[|y_1'|] - \beta E(y_1' \frac{\partial |z|}{\partial z}) = -\sqrt{\frac{2}{\pi}}[\gamma \sigma_{y_1'} + \beta \frac{E(y_1' z)}{\sigma_z}]$$

$$c_e = A - \gamma E(z\frac{\partial |y_1'|}{\partial y_1'}) - \beta E[|z|] = A - \sqrt{\frac{2}{\pi}}[\gamma \frac{E(y_1' z)}{\sigma_{y_1'}} + \beta \sigma_z] \tag{3.1}$$

The equations of motion of primary-secondary system ca be expressed in first order differential equations of state vector

$$Y' + GY = F(t) \tag{3.2}$$

where $Y = [y_1, y_1', z, y_2, y_2']^T$, $F(t) = [0 \quad -x_g'' \quad 0 \quad 0 \quad 0]^T$,

$$G = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ \alpha & 2\varsigma_p & 1-\alpha & -\mu\eta^2 & -2\varsigma_s\mu\eta \\ 0 & -c_e & -k_e & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ -\alpha & -2\varsigma_p & -(1-\alpha) & \mu\eta^2+\eta^2 & 2\varsigma_s\mu\eta + 2\varsigma_s\eta \end{bmatrix}.$$

Suppose $U = E(YY^T)$, Eqn.(3.2) is right multiplied by Y^T, add the expectation of equation with its transpose. Only the stationary response of equivalent linear system under gauss white noise excitation is considered, so the he covariance matrix equation can be written as

$$BU + UB^T + CC^T D = 0 \tag{3.3}$$

where $C = [0 \quad -1 \quad 0 \quad 0 \quad 0]^T$, $B = -G$, D is the intensity of the excitation x_g''. According to the above equations, the value of k_e and c_e determine the covariance matrix equation by Eqn.(3.3) while the covariance matrix equation can also determine the value of k_e and c_e by Eqn.(3.1). By a number of iteration calculation of Eqn.(3.1) and (3.3), the value of k_e, c_e and the covariance matrix $U = E(YY^T)$ of stationary response can be obtained. Suppose $x_g'' = e^{i\omega\tilde{t}}$, $y_1 = H_1 e^{i\omega\tilde{t}}$, $y_2 = H_2 e^{i\omega\tilde{t}}$, $z = H_3 e^{i\omega\tilde{t}}$, the statistic of secondary system's absolute acceleration $x_2 = y_2'' + y_1'' + x_g''$ can be written as

$$x_2 = \omega_p^2 (y_2'' + y_1'' + x_g'') = \omega_p^2 \left[H_1(-\omega^2) + H_2(-\omega^2) + 1 \right] e^{i\omega\tilde{t}} = \omega_p^2 H_{x_2} e^{i\omega\tilde{t}} \tag{3.4}$$

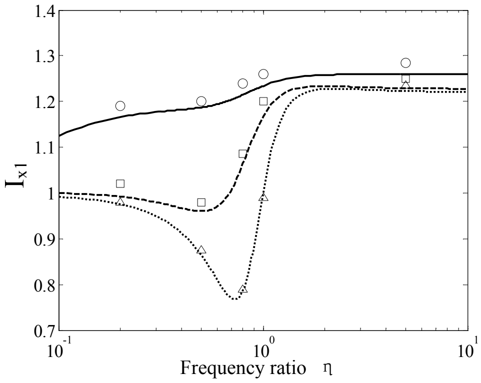
$$\sigma_{x_2} = E(x_2^2) = \frac{\omega_p^2}{\pi} \int_0^\infty |H_{x_2}|^2 D d\omega, \quad \sigma_{x_2'} = E(x_2'^2) = \frac{\omega_p^2}{\pi} \int_0^\infty \omega^2 |H_{x_2}|^2 D d\omega \tag{3.5}$$

the matrix equation of frequency response function H_1、H_2 and H_3 can be obtained by

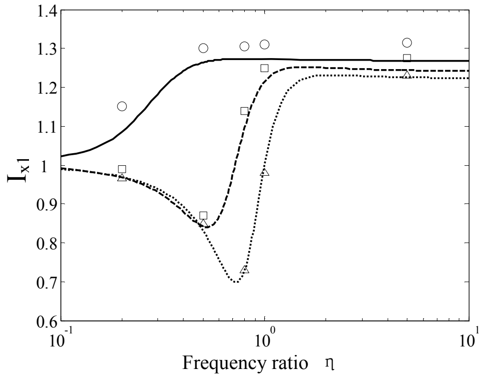
$$\begin{bmatrix} \alpha + 2\varsigma_p(i\omega) - \omega^2 & -2\varsigma_s\mu\eta(i\omega) - \mu\eta^2 & 1-\alpha \\ -\omega^2 & \eta^2 + 2\varsigma_s\eta(i\omega) - \omega^2 & 0 \\ c_e(i\omega) & 0 & k_e - i\omega \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \tag{3.6}$$

4. MONTE CARLO SIMULATION

Monte Carlo simulation offers an alternative to the equivalent linearization method, when nonlinear system is considered. In this research, samples of the gauss white noise are constructed from independent gauss random sequences, and a sample of the stochastic response is obtained by direct numerical simulation of the equations of motion. For each sample function only the stationary responses are retained. The method is proposed when the quality of an approximate solution technique such as the equivalent linearization method needs to be accessed. The integration procedure, which is used, is fourth-order Runge-Kutta integration. Define the amplification factor for variance of relative displacement of primary system $I_{x_1} = \sigma_{x_1}^2 / \sigma_{x_{1,0}}^2$, in which $\sigma_{x_1}^2$ corresponds to the situation of secondary system attaching, $\sigma_{x_1}^2$ corresponds to the situation with secondary system absent. Fig.2 gives the accuracy of the equivalent linearization method, and it can be known that the equivalent linearization method is approximately close to the exact solution.



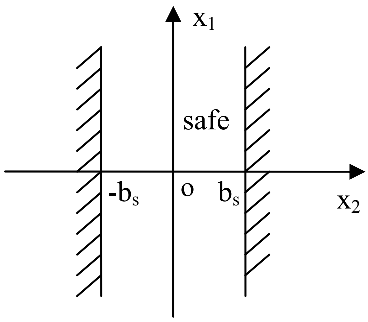
(a) D=0.8



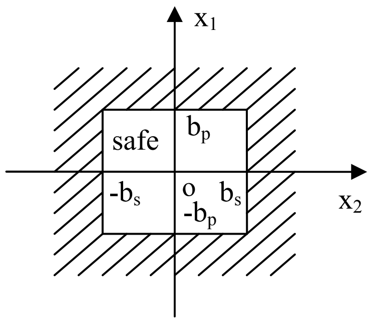
(b) D=5.0

Figure 2 Amplification factor for variance of relative displacment of primary system with respect to the ground. μ=0.1, ζ_p=0.05, ζ_s=0.05, A=n=1, β=0.05, γ=0.95. —, Equivalent linearization, ○, Moner Carlo, for α =0.1; - -, Equivalent linearization, □, Moner Carlo simulation, for α =0.5; ---, Equivalent linearization, △, Moner Carlo simulation, for α =0.8.

5. UNITED RELIABILITY THEORY OF PRIMARY-SECONDARY SYSTEM



a. secondary system



b. primary-secondary system

Figure 3 Schematic diagram of safe region

In this paper, primary system's relative displacement and secondary system's absolute acceleration is adopted as the analysis indexes. The schematic diagram of the safe region and boundary is shown in Fig.3, in which the boundary is assumed to be symmetrical. Suppose that the cross-over time of stochastic response with boundary obey the poission distribution, and the response is a stationary process, then reliability in the time region $(0,T]$ can de described by the cross-over rate with the boundary B

$$P=\exp(-\nu_B T) \tag{5.1}$$

For separate secondary system, based on the Rice formula and passion distribution, cross-over rate of stationary gauss process with zero mean can be obtained

$$\nu_B = 2\nu_{b_s}^{+} = 2\nu_{b_s}^{-} = \frac{\sigma_{x_2'}}{\pi\sigma_{x_2}}\exp(-\frac{b_s^2}{2\sigma_{x_2}^2}) \tag{5.2}$$

For primary-secondary system, suppose the safe region S is a polygon enclosed by L straight lines, $n_k=(\cos\alpha_k,\sin\alpha_k)$ is the normal vector vertical to line l_k, r_k is the distance from the origin to the line l_k, c_{1k} and c_{2k} are the distances from projection of the origin on the line l_k to the endpoints of the line l_k respectively, and $c_{2k}\ge c_{1k}$. According to the general Rice formula for multiple degrees of freedom proposed by Belyaev in 1968, the cross-over rate for the plane polygon safe region is

$$\nu_B=\frac{1}{2\pi}\sum_{k=1}^{L}[\Phi(c_{2k})-\Phi(c_{1k})]\sqrt{(\sigma_{\bar{x}_2'}^2\cos^2\alpha_k+\sigma_{\bar{x}_1'}^2\sin^2\alpha_k)}\exp(-\frac{r_k^2}{2}) \tag{5.3}$$

in which the random vector process $\bar{X}\sim N[0,I]$, $\bar{X}'\sim N[0,diag(\sigma_{\bar{x}_i'}^2)]$.
In this paper, the random vector process is $X=[x_1,x_2]$, which can be transformed into a new vector process whose covariance matrix is diagonal, then the new vector process can be convert into zero mean and one square mean. Suppose $\bar{X}=[\bar{x}_1,\bar{x}_2]=\phi(\bar{Y})=[ax_1+bx_2,dx_1+ex_2]$, then $\bar{X}'=[ax_1'+bx_2',dx_1'+ex_2']$, the constraints is: $\bar{X}\sim N[0,I]$, $\bar{X}'\sim N[0,diag(\sigma_{\bar{x}_i'}^2)]$. Therefore, a series equations with a, b, d, e as the arguments can be obtained

$$\begin{cases} E(\bar{x}_1\bar{x}_2)=0 \\ E(\bar{x}_1'\bar{x}_2')=0 \\ E(\bar{x}_1\bar{x}_1)=E(\bar{x}_2\bar{x}_2)=1 \end{cases} \tag{5.4}$$

From Eqn.(5.4), the values of a, b, d and e is calculated, and simultaneously the safe region of $\bar{X}$ can be obtained

$$\begin{aligned} S_{\bar{X}}=\{\bar{X}:-|ae-bd|b_p\le e\bar{x}_1-b\bar{x}_2\le|ae-bd|b_p, \\ -|ae-bd|b_s\le d\bar{x}_1-a\bar{x}_2\le|ae-bd|b_s\} \end{aligned} \tag{5.5}$$

In this paper, the cross-over rate is adopted as the analysis index, and is calculated according to Eqn.(5.3). Based on passion distribution, the reliability of primary-secondary system can be calculated according to the cross-over rate ν_B.

6. NUMERICAL CALCULATION

In this section, the effect of several major parameters on the united reliability of primary-secondary system is studied, including mass ratio μ, frequency ratio η, secondary system's damping ratio ς_s, nonlinear primary system's hysteretic parameter α, and intensity of earthquake excitation D. According to the mathematical model and reliability theory, the reliability of primary-secondary system under different parameter value combinations is investigated, and some valuable conclusions are obtained. The computation has been conducted for the following values of the parameters: mass ratio $\mu = 0.01$; frequency ratio $\eta = 0.1$; damping ratio f secondary system $\varsigma_s = 0.05$; hysteretic parameters of nonlinear primary system: $\alpha = 0.5$, $A = n = 1$, $\gamma = 0.95$, $\beta = 0.05$; earthquake excitation x_g'' is assumed to be gauss white noise with the intensity of D. As mentioned above, the cross-over rate is adopted as the reliability index and the failure boundary of primary system is $b_p = 3\sigma_{y_1}$ in which σ_{y_1} is the mean square of displacement under white noise excitation. Meanwhile, the failure boundary of secondary system is a certain value which represents the limit acceleration of secondary system: $b_s \le \sigma_{x_2}$ means the low boundary; $\sigma_{x_2} < b_s < 3\sigma_{x_2}$ means the medium boundary; $b_s \ge 3\sigma_{x_2}$ means high boundary. Consequently, $b_s = \sigma_{x_2}$, $b_s = 2\sigma_{x_2}$ and $b_s = 3\sigma_{x_2}$ is used as three representative values for the united reliability analysis of primary-secondary system in this paper.

Fig.4 shows that the change trend of cross-over rate of primary-secondary system with respect to mass ratio is different for three boundary case.

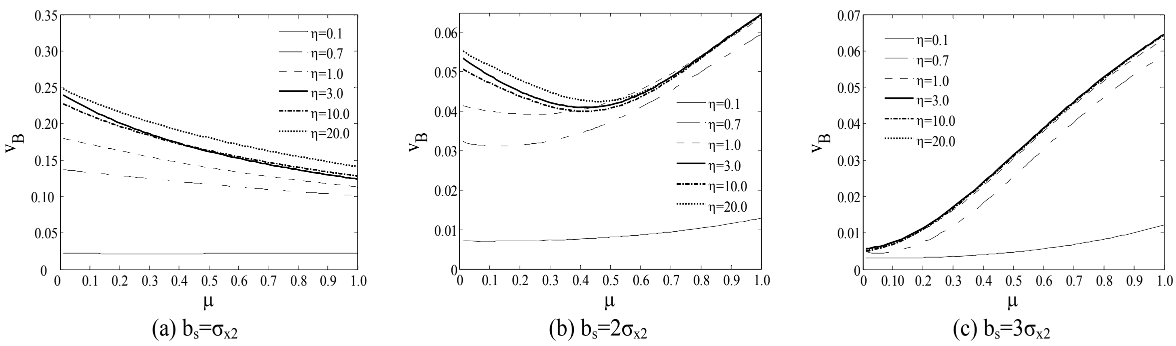


Figure 4 Effect of mass ratio on the crossing rate of primary-secondary systems

Fig.5 shows the change trend of cross-over rate of primary-secondary system with respect to frequency ratio in three different boundary case.

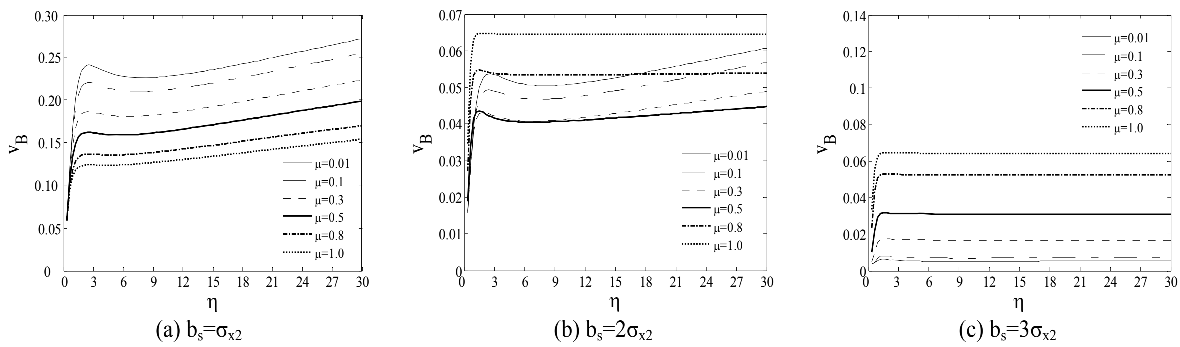
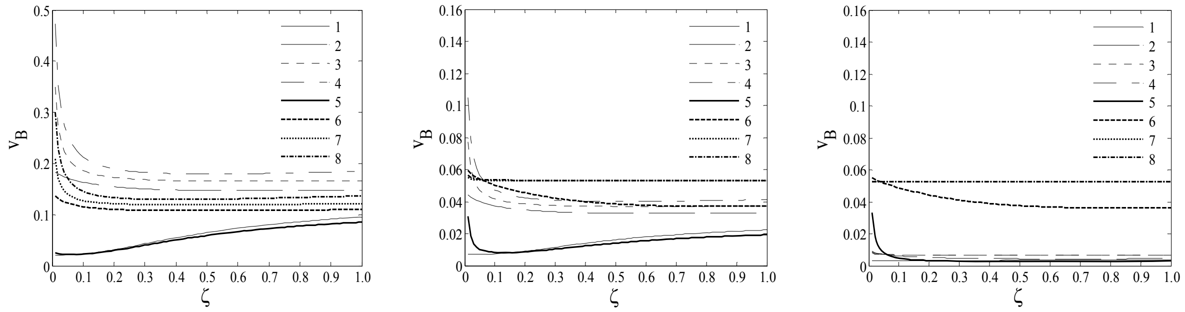


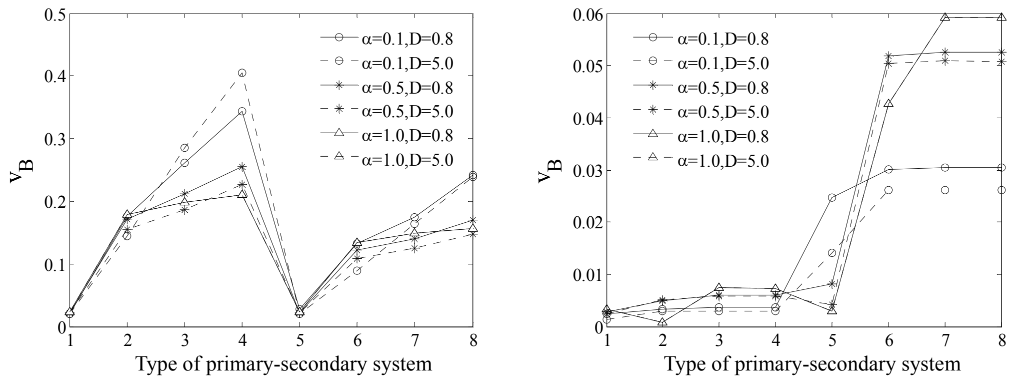
Figure 5 Effect of frequency ratio on the crossing rate of primary-secondary systems

Fig.6 shows the change trend of cross-over rate of primary-secondary system with respect to damping ratio in three different boundary case. Classify primary-secondary system by mass ratio μ and frequency ratio η and number them as shown in Table.1. The cross-over rates of primary-secondary system are calculated in view of different combination of the values of α and D, and the result is described in Fig.7.



(a) $b_s=\sigma_{x2}$ (b) $b_s=2\sigma_{x2}$ (c) $b_s=3\sigma_{x2}$

Figure 6 Effect of damping ratio on the crossing rate of primary-secondary systems



(a) $b_s=\sigma_{x2}$ (b) $b_s=3\sigma_{x2}$

Figure 7 Effect ofαand D on crossing rate of primary-secondary systems

Table.1 Several types of primary-secondary systems

$\mu\backslash\eta$	0.1	1	10	30
0.1	1	2	3	4
0.8	5	6	7	8

Note: classify primary-secondary system by the mass ratio and frequency ratio.

7. CONCLUSION

A simple two degrees of freedom primary-secondary system is established in this paper, and the united reliability of primary-secondary system is calculated based on equivalent linearization method and poission hypothesis. The cross-over rate is adopted as the index representing the reliability. some important parameters are studied such as: mass ratio μ, frequency ratio η, damping ratio ς_s, nonlinear hysteretic parameter α, intensity of earthquake excitation D, etc. Some useful conclusions are obtained by the above research in the paper:

(1) while secondary system's boundary is low, the cross-over rate decreases with the increase of mass ratio for high frequency ratio, and trends to a constant for low frequency ratio; while secondary system's boundary is medium, the cross-over rate increases first and then decreases with the increase of mass ratio, and exist a

minimum point; while secondary system's boundary is high, the cross-over rate increases with the increase of mass ratio.
(2) while secondary system's boundary is low or medium, the cross-over rate increases with the increase of frequency ratio for low and high frequency ratio, and increases first and then decreases for medium frequency ratio; while secondary system's boundary is high, the cross-over rate increases with the increase of frequency ratio for low frequency ratio, and trends to a constant for medium and high frequency ratio.
(3) while secondary system's boundary is low, the cross-over rate increases with the increase of damping ratio for low frequency ratio, and decreases for high frequency ratio; while secondary system's boundary is medium, the cross-over rate increases with the increase of damping ratio for low frequency ratio and high mass ratio, and decreases for other cases; while secondary system's boundary is high, the cross-over rate decreases with the increase of damping ratio for low frequency ratio and high mass ratio, and trends to a constant for other cases. As shown in previous sections, the cross-over rate with the change of damping ratio shifts obviously while the damping ratio is less than 0.1, and less evidently while the damping ratio is large than 0.1.
(4) the intensity of earthquake excitation has little influence on the cross-over rate of primary-secondary system except the situation that nonlinear hysteretic parameter α is small.
(5) while secondary system's boundary is low, the nonlinear hysteretic parameter α has little influence on the cross-over rate while the frequency ratio is less than 1, and the cross-over rate decreases with the increase of nonlinear hysteretic parameter α while the frequency ratio is larger than 1; while secondary system's boundary is high, the nonlinear hysteretic parameter α has obvious influence on the cross-over rate, and the cross-over rate increases with the increase of nonlinear hysteretic parameter α while the frequency ratio is larger than 1.

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