Study on Dynamic Rupture of Seismic Fault Based on Parallel Computation and Dynamics

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ABSTRACT :

Using the explicit decoupling finite element method and combing with the highly efficient processing method of artificial boundary and parallel computation, the physical and mathematical models are setup to study the dynamic rupture of fault during seismic action. Through the transmitting boundary, a way of simulating fault rupture is given and comparing with analytical solution, the methods' reliability and parallel computation programming is verified. The results show that, using the theory of dynamic fault rupture, the theoretical analysis and simulation can better reveal the rupture mechanism. Basically, the rupture beginning and development are disclosed and the surface rupture and dislocation are estimated reasonably. It is concluded that, utilizing the parallel computation of dynamic fault rupture, the dynamic dislocation is more than the static one during rupture process. Meanwhile, the surface is displaced largely and bigger area of rupture can be caused. With the same stress drop, the surface rupture can be easily formed during dynamic rupture.

KEYWORDS: seismic fault, dynamic rupture, explicit finite element, parallel computation

1. INTRODUCTION

Recently, studying the seismic rupture brings us a new enlightenment about understanding the earthquake rupture spreading and stress relaxation, and the fundamental role of friction is approved. These models showed that the complexity of rupture process was not simply illustrated by uniform loading medium with 2D or 3D model. Boundary integral equations, an efficiently numerical method studying dynamic rupture, was improved through elimination of strong singularity, which is not only relatively effective in the earthquake recurrence and transiently accelerated fault creep but in the fully dynamic rupture propagation. However, it can not be used for heterogeneous fault. Finite difference method of seismic rupture is used to study the rupture propagation of heterogeneous and elastic medium. It is very effective but the boundary conditions, especially the free surface conditions, must be used cautiously. Using the comprehensive boundary conditions, staggered grid speed-stress finite difference method can be used to deduce the surrounding wave propagation formula in large-scale 3D models with effective kinematic definition. Therefore, a new approach is introduced to attach the mixed boundary conditions with an identical plane in the mesh, which can simulate more complicatedly geometric rupture.

Serious earthquake fault rupture is under the control of fault friction which influences the initial fracture, development and healing. Through low sliding rate of testing, friction model is raised in the light of rate and state being explained by simply slight sliding friction law which is about the plane rupture and dynamic three-dimensional model of fault rupture simulation. The regularity of numerical model adjusts the concentration of stress distribution and sliding along the controlling length of friction distance. Appropriate changes to sliding rate fields on the implementation of friction are essential to resolve the sliding rate near the rupture where a large number of grids are divided.

In recent years, several major earthquakes, especially recent Wenchuan earthquake in Sichuan Province, 2008, China, indicate that there are great impacts on the near-field plate movement by seismic fault rupture and interaction between seismic wave and fault dynamics. Using numerical methods, many scholars studied the dynamic rupture and the ground movement, through which the fault dislocation, ground movement and the scale between ground surface and its rupture were studied. FEM is the effective tool to simulate the dynamic rupture, which can not only handle complex boundary but calculate the nonlinear problems. Since the fault rupture is a dynamic 3D problem, so the physical region and mesh division are great. However, it is difficult to achieve and parallel computing algorithm is necessary. Presently, parallel computation technology is matured with the help of local LAN, through which the complicated calculations can be realized. On the basis of FEM and parallel computing algorithm using LAN, explicit parallel FEM is to simulate dynamic fault rupture in the light of friction model.

2. DYNAMIC RESPONSE OF DEFORMATION UNDER SEISMIC FAULT RUPTURE

In order to assess the instable destruction under earthquake, it is necessary to analyze the dynamic response of large fault deformation. In 3D problems, relaxation modulus is expressed as $E(t)=\sum_{k=1}^{M} E_k e^{-a_k t}$ in order to study multi-axis stress. At time t_n, Kirchhoff stress, S_{ij}, Green strain, ε_{ij} and its strain rate, $\dot{\varepsilon}_{ij}$ can be expressed as $\left\{S_{ij}(t_n)\right\}=\phi(\varepsilon_{ij})\sum_{k=1}^{M}\left\{\square_{ij}\right\}_{n,k}$. Where, $\left\{\square_{ij}\right\}_{n,k}$ is the contribution to stress by k^{th} Prony series. Considering the time step of dynamic rupture, the constitutive model of dynamic increment can be deduced (LIU2005).

$$\left\{\Delta S_{ij}\right\}_n=\phi_n\sum_{k=1}^{M}\left\{\square_{ij}\right\}_{n,k}-\phi_{n-1}\sum_{k=1}^{M}\left\{\square_{ij}\right\}_{n-1,k}=\left\{\Delta T_{ij}^{'}\right\}_{n-1}+G^{'}\left[A\right]\left\{\Delta\varepsilon_{ij}\right\}_n \tag{2.1}$$

Where $\left\{\Delta S_{ij}\right\}_n=\phi_n\sum_{k=1}^{M}(-Q_k)\left\{\square_{ij}\right\}_{n-1,k}+\Delta\phi_n\sum_{k=1}^{M}R_k\left\{\square_{ij}\right\}_{n-1,k}+\phi_n\sum_{k=1}^{M}\left\{PA_{ij}\right\}_{n,k}+\phi_n\sum_{k=1}^{M}R_k\left\{Z_{ij}\right\}_{n-1,k}$, $G^{'}=\phi_n\sum_{k=1}^{M}G_k^{'}$, $Q_k=1-R_k, R_k=e^{-a_k B_n \Delta t_n}, \left\{Z_{ij}\right\}_{n-1,k}=\beta\left\{\square\square_{ij}\right\}_{n-1,k}-\beta\left\{\square_{ij}\right\}_{n-1,k}$. As to dynamic fault rupture, the variation equation is deduced based on Total Lagrangian method and virtual work (LIU2005).

$$\iiint_{\Omega}\left\{A-B+\square\right\}dV^{(0)}=\iint_{S_\sigma}\left(\overline{T}_i+\Delta\overline{T}_i\right)\delta\Delta u_i dS^{(0)} \tag{2.2}$$

Where $A=\Delta S_{ij}\delta\Delta\varepsilon_{ij}^{L}+S_{ij}\delta\Delta\varepsilon_{ij}^{N}$, $B=\Delta\overline{P}_i\delta\Delta u_i-\rho\Delta\ddot{u}_i\delta\Delta u_i$, $\square=S_{ij}\delta\Delta\varepsilon_{ij}^{L}-\overline{P}_i\delta\Delta u_i+\rho\Delta\dot{u}_i\delta\Delta u_i$. $\Delta\varepsilon_{ij}^{L}$ and $\Delta\varepsilon_{ij}^{N}$ are the linear and nonlinear tensor increment of Green strain respectively. $\overline{P}_i$ (or $\Delta\overline{P}_i$) and $\overline{T}_i$ (or $\Delta\overline{T}_i$) are the body force (or increment) and surface force (or increment) on rupture structure. As to 3D rupture, the vector of equation 2.2, B^L, is deduced and expressed as the differential operators. Substituting equations 2.1 and B^L into variation equation, the shape function is introduced in FEM and Gauss integration. And the FEM equation can be expressed as $K^e\Delta^e=F^e$. Where, K^e is the stiffness matrix of equivalent element and F^e is the equivalent node load.

3. DYNAMIC ANALYSIS OF RAPID FAULT DISLOCATION

After initial expansion, fault may extend over a slow period of growth and then stop. Only the rapid expansion may stop crack but it is more likely to lead to earthquake. Therefore, studying rapid fault dislocation is very important, which is moving boundary problem of wave equation but not easy to be solved (FAN1992). While the fault is rapidly dislocated, the governing equations are wave equations $\nabla^2\hbar=\frac{1}{c_1^2}\frac{\partial^2\hbar}{\partial t^2},\nabla^2\lambda=\frac{1}{c_2^2}\frac{\partial^2\lambda}{\partial t^2}$. Where, c_1 and c_2 are *P*-wave and *S*-wave velocity respectively. $\hbar$ and λ are Lamé potentials which are related to u_{x_1} and u_y. $u_{x_1}=\frac{\partial\hbar}{\partial x_1}+\frac{\partial\lambda}{\partial y}$, $u_y=\frac{\partial\hbar}{\partial y}-\frac{\partial\lambda}{\partial x_1}$. Assuming the fault is expanded at constant speed, *V*, along the direction of x_1, the Galileo transforms, $x=x_1-Vt$ and $y=y$, are used. Wave equations are transformed into Laplace equation with implicit time.

$$\nabla_1^2\hbar(x,y_1)=0,\nabla_2^2\lambda(x,y_2)=0 \tag{3.1}$$

In the moving coordinate system, the boundary value problems of equations 3.1 uses $z_1/a_1, z_2/a_2=\omega(\gamma)=\frac{H}{\pi}\ln\left[1+\left(\frac{1+\gamma}{1-\gamma}\right)^2\right]$ as the conformal mapping. The physical plane of z_1 and z_2 with rupture is changed into the inner domain of unit circle in γ plane, and analytic functions $F_1(z_1)$ and $F_2(z_2)$ are transformed as

$$F_1(z_1)=F_1\left[a_1\omega(\gamma)\right]=f_1(\gamma),F_2(z_2)=F_2\left[a_2\omega(\gamma)\right]=f_2(\gamma) \tag{3.2}$$

Equation 3.2 can be obtained by Cauchy integration and the dynamic stress intensity factor (DSIF) is deduced (FAN1992). As to $-\left(\tau-\tau_f\right)$, DSIF is $\square^{\left(\tau-\tau_f\right)}(V)=\frac{\sqrt{2a_1H}\left(\tau-\tau_f\right)}{2\pi}F\left(a/a_1H\right)$. As to $\left(\tau_b-\tau_f\right)$, DSIF is $\square^{\left(\tau_b-\tau_f\right)}(V)=\frac{\sqrt{2a_1H}\left(\tau_f-\tau_b\right)}{2\pi}F\left(R/a_1H\right)$. Obviously, not only is DSIF related to *V* but also to the wave velocity of flexible medium wave. Based on stress limit on the top of the fault, $\square^{\left(\tau-\tau_f\right)}(V)+\square^{\left(\tau_b-\tau_f\right)}(V)=0$. The dislocation formed by rapid propagation of a fault is

$$\delta(V)=\frac{a_2\left(1-a_2^2\right)}{(1-v)\left[4a_1a_2-\left(1+a_2^2\right)^2\right]}\left\{\frac{(1-v)^2\left[\square^{\left(\tau-\tau_f\right)}(V)\right]^2}{E\left(\tau_f-\tau_b\right)}\right\} \tag{3.3}$$

Where, $\delta(V)$ is related to the propagation speed of rupture and the elastic wave speed. The unsteady judging criteria can be established, $\delta(V)\le\delta_c^D$. Where, δ_c^D is a material constant, the critical value of dynamic dislocation.

4. COMBINATION OF EXPLICIT FEM AND PARALLEL COMPUTION TECHNOLOGY

Tetrahedral units are used to divide the calculation region with finite element, which is relatively easy to divide complex fields into limited units. With the exception of internal nodes outside of artificial boundary, vibration of nodes in earthquake is simulated using explicit FEM. And artificial boundary nodes are calculated by multi-transmitting boundary. According to *N*-order multiple transmission formula, the displacement, velocity or acceleration in artificial boundary points at selected time can be solved through the parameters in the first few time points' nodes. Random vibration theory is used in high-frequency seismic action to synthesize explicit decoupling finite element technology, which can greatly improve calculation efficiency and reduce the demand for computer memory. However, using finite element to simulate the low-frequency vibration of complex fields, a large number of units are needed. In order to ensure the stability of numerical calculation, time step is small, which requires much computer memory, long calculation time. It is very difficult for a single computer but a parallel computing technology to achieve the goal.

In recent years, high-performance parallel computing technology is broadly applied in scientific research, engineering and other technical and military fields. Parallel computing technology can be divided into SIMD (Single-instruction and multi-data) and MIMD (Multi - instruction and multi-data). SIMD, based on Message passing interface (MPI), is recommended. MPI is library function processing communications between processors and computer, which can be called by Visual Fortran and Visual C. While simulating near-fault low-frequency seismic action, the calculation region will be divided into *M* sub-region, and each can complete a single calculation. At every calculation time, a sub-region needs to exchange data information with other sub-regions. Explicit FEM takes full advantage of the local characteristics of space-time fluctuations. Because the wave propagation speed is limited, so a point movement at any time is only related to the approaching time of a number of neighboring regions' points. Thus, only sub-regions exchange interface information with its adjacent sub-regions, which greatly reduces the extent of data exchange and saves communication time.

With the help of VF and VC++, the library function, MPI_SEND can be called to send information and MPIes RECV is used to receive information. For large amounts of data exchange, the MPI group communications and non-blocking communication library functions can be used. There are no limits in dividing calculation regions using parallel computing technology based on the MPI as long as any two calculating regions can exchange through MPI library functions. However, different division methods of calculating region need different information exchange. According to the specific model, user can select the appropriate regional segmentation method.

5. SIMULATION OF FAULT DYNAMIC RUPTURE

Simulating the dynamic fault rupture, the rupture area must be included in the scope of calculation region. The fault by M7 earthquake is up to 60 km long, 15 km wide. Considering the study interest on earthquake engineering and in accordance with the discrete criteria, 7-9 units with the smallest wavelength should be included in calculation band when simulating fault rupture and seismic fields. Assuming that shear wave velocity is 3km/s and considering the highest frequency of 5 Hz (actually higher than 5 Hz), the shortest wavelength is 700 meters, and the unit should be divided between 70 and 90 meters or less. 700-900 units are divided only in fault itself along the direction of length. Taking into account the distance the observation points away from cross section of fault, for example, l0km, the number of units of 10^7-10^8 because discrete units of three-dimensional problem are cubic power of that of one-dimensional. If the surface soil is considered, due to lower wave velocity, then the units will be up to 10^{10}. Therefore, the calculating region and its calculation are very amazing and an efficient algorithm is the key to simulate.

To achieve efficient simulation, computation technology, such as the explicit decoupling FEM needs to be improved while combined with efficient artificial boundary. Due to great amount of computation, a parallel computing technology is necessary. Using explicit finite element and parallel computing technologies and combining with boundary transmission, a way of simulating dynamic fault rupture is given. Comparing with analytical solutions, methods and procedures for calculation are verified.

5.1 Model of Dynamic Fault Rupture

The earthquake fault rupture is generally the shear stress under the tectonic stress. In dynamic rupture, it is assumed that the fault rupture is plane and there is stress and no relative normal displacement. Shear rupture can be simply described. Under tectonic stress, the rupture is concentrated to a point in a small area. That tectonic stress is greater than the rupture strength will lead to initial rupture. While stress is increased near the end of rupture, stress concentration will make rapid rupture extension. After that, shear stress is declined as the dynamic friction stress. When the sliding stops, the dynamic friction stress is transformed to the static friction stress. Therefore, two problems are concerned with building dynamic fault rupture model. One is how to judge the rupture beginning, developing and stopping. Another is to describe faults interaction in the process of rupture and sliding. The critical stress criteria are used in simulation and slip-and-rate-weakening-model was given to describe dynamic fault rupture accurately.

5.2 Basic Equations and Boundary Conditions

Taking an inclining fault as an example, basic equations and boundary conditions of fault rupture are illustrated. Fig.1 shows the physical model of dynamic fault rupture (LIU2005). The earthquake fault can also be ruptured to the surface or not. Left and right side, the base is artificial boundary. It is assumed that the initial tectonic stress direction parallels to the surface. *X*-axis in overall coordinates parallels to the direction of the initial tectonic stress. *Y* and *Z* axis are perpendicular to the fault plane and the surface respectively. The initial point *O'* is the coordinate origin, local coordinate system is established and the rupture stops while arrived at the edge of the fault.

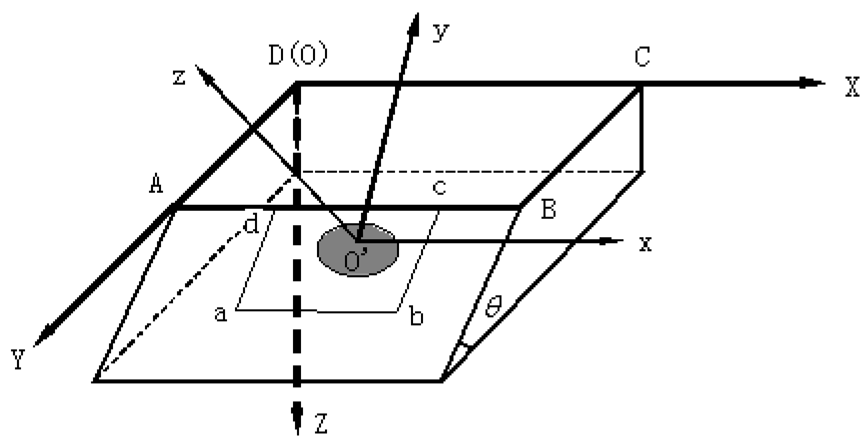

Fig.1 Physical model of dynamic fault rupture

5.2.1 Basic governing equations

In the elastic and isotropic continuous medium, the motion equations are expressed in equation 5.1.

5.2.2 Boundary conditions

The boundary conditions mean the damaged fault section and artificial boundary at t in the surface.

Artificial boundary is introduced in order to transform the infinite field such as dynamic fault rupture into the finite field. In destroyed fault surface, the normal displacement of two parts of fault is continuous. At the same time, normal and tangential stresses are continuous. From fig.1, in local coordinate system of a damaged fault surface, $w_1^{'} = w_2^{'}, \tau_{z'z'1} = \tau_{z'z'2}, \tau_{z'x'1} = \tau_{z'x'2}, \tau_{z'y'1} = \tau_{z'y'2}$. $\tau_{z'x'}$ and $\tau_{z'y'}$ are along the fault plane, which is calculated by the fault plane friction model (LIU2005). At time *t*, the equation $\tau_{zx} = \tau_{zy} = \tau_{zz} = 0$ is expressed in the damaged fault plane.

$$\left.\begin{aligned}
(\lambda+\mu)\frac{\partial^2 u}{\partial x^2} + \mu\nabla^2 u + (\lambda+\mu)\frac{\partial^2 v}{\partial x \partial y} + (\lambda+\mu)\frac{\partial^2 w}{\partial x \partial z} = \rho\ddot{u} \\
(\lambda+\mu)\frac{\partial^2 u}{\partial y \partial x} + \mu\nabla^2 v + (\lambda+\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda+\mu)\frac{\partial^2 w}{\partial y \partial z} = \rho\ddot{v} \\
(\lambda+\mu)\frac{\partial^2 u}{\partial z \partial x} + \mu\nabla^2 w + (\lambda+\mu)\frac{\partial^2 v}{\partial z \partial y} + (\lambda+\mu)\frac{\partial^2 w}{\partial z^2} = \rho\ddot{w}
\end{aligned}\right\} \tag{5.1}$$

Where, u, v, w are corresponding to the displacement of *X*, *Y*, *Z* direction respectively.

5.3 Simulation and Analysis of Rupture and Dislocation

Fig.2 shows Dynamic fault rupture and dislocation at different times such as 1s, 2s, 5s and 10s. From simulation process, the specific source of earthquake energy is the releasing of accumulated strain energy in the crust, which is caused by a sudden shear rupture. When the stress and strain by continuous deformation of geological structure are greater than material strength, some critical points in fault are destroyed and rapidly propagated along the whole length of high stress material.

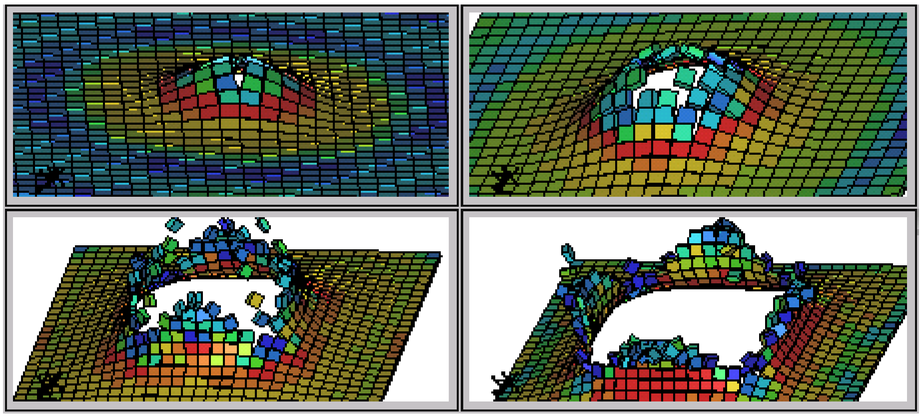

Fig.2 Dynamic fault rupture and dislocation at different time

The stress takes the advantages in crust. With the increasing of rupture, stress is increased. The surfaces of shear rupture under pressure are closed each other and interacted, which can be expressed by the normal stress and shear stress. Due to the relative sliding movement and speed, friction and crack are controlled and coupled each other, which makes the problem with an intense non-linearity. Because of the faults, joints, cracks and holes of the crust, multi-phase, non-elasticity and prominent rheological property, so the time factor is inevitably arisen. Therefore, the earthquake fault movement can be simulated by dynamic fracture model.

5.4 Influential Factors of Rupture Rate

5.4.1 Effect of critical rupture stress and slip-weakening distance on rupture rate

Rupture rate is the key of fault rupture because the rupture rate change is the main reason that the fault rupture stimulates high-frequency earthquake. A large number of seismic inversion analyses also show that the fault rupture rate is uneven. The rupture rate and the difference between critical stress and initial stress are related to stress drop.

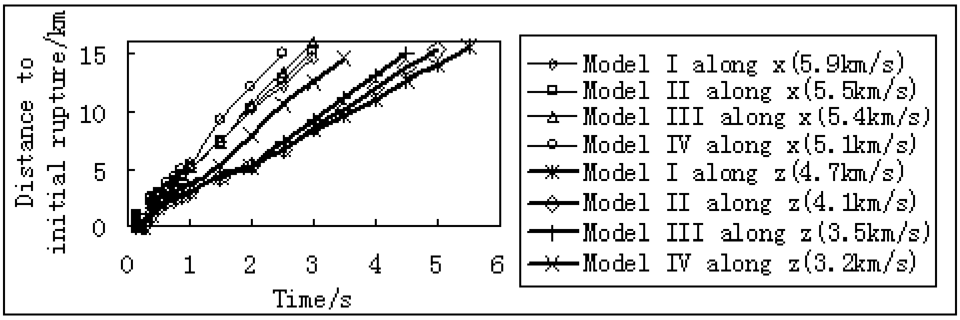

Fig.3 Trace of rupture front with slip-weakening distance and critical rupture stress

As to the even rupture fault and without considering sliding weakening, while the initial stress is almost equal to the critical fracture stress, fracture rate along the direction parallel to and the direction of the initial stress in the fault plane is more than shear wave velocity, shown in fig.3. Fracture rate along the fault is also more than shear velocity when the critical stress is greater than a certain value, but the rate perpendicular to the direction of initial stress is lower than shear velocity. The basic rupture shape is oval. With the increasing of critical rupture stress, the rupture rate along the two directions will be lower than the shear wave velocity and the basic shape is circular.

Slip-weakening distance has important impact on the development of rupture rate. As to a fixed stress drop and burst strength, when slip-weakening distance is increased, the time that the rupture rate tends to be steady is increased. For the same slip-weakening distance and stress drop, the time that the rupture rate achieves steady is also increased when the critical rupture stress is increased, especially slip-weakening distance is 0.8m. Though the time of slip-weakening distance influencing steady rupture rate is required, it doesn't almost influence the rate unless the critical rupture stress is equal to the initial rupture stress. At the same time, slip-weakening distance is increased, which will reduce the steady rupture velocity.

5.4.2 Impact of Asperity on rupture rate

Actual earthquake rupture is extremely uneven. Dynamic model to the earthquake inversion shows that the irregular Asperity and the Barrier are distributed in the fault surface shown in fig.4. The dynamic inversion of actual fault rupture process also illustrates that the rupture rate can exceed the shear wave velocity. Its rupture rate will be increased when the rupture front passes through larger Asperity, which depends on the difference between the critical fracture stress and the initial stress of Asperity. From the complex rupture model, the rupture is very complicated and the Barrier distribution and Asperity on the actual earthquake fault may be more complicated. Therefore, whether the definite model can be used to simulate and predict or not will be further studied.

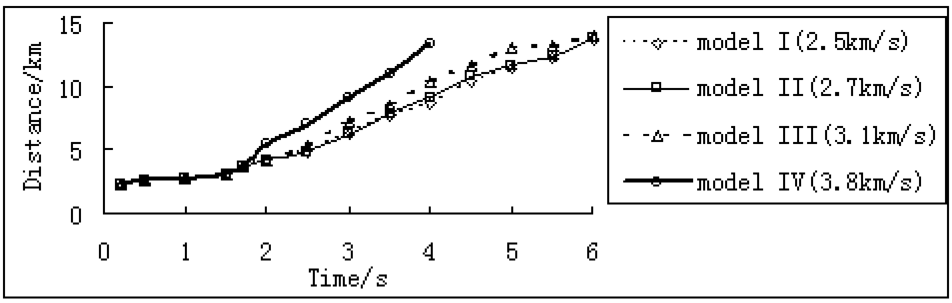

Fig.4 Trace of rupture front of even rupture and three Asperity models along X axis

6. CONCLUSIONS

On the basis of near field finite element, according to friction model, the explicit decoupling calculation formulas of nodes in fault plane are deduced. Combining with transmission boundary and parallel computing technology, an explicit decoupling FEM simulating the dynamic fault rupture is given. Using this technology, the rupture process under the condition of complex fields, the ground movement and surface fracture can be simulated.

Using the explicit decoupling FEM, the dynamic fault rupture is solved, which mainly analyzes the currently widespread slip-weakening model. The model parameters will influence the rupture rate, the fault dislocation and distribution, the function of seismic source time and the near-fault seismic action. At the same time, the impact of Asperity on rupture rate and the difference between surface rupture and non-surface rupture is analyzed.

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