QUANTIFICATION OF STRUCTURAL PROBABILITY OF FAILURE DUE TO EXTREME EVENTS

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ABSTRACT:

This paper tries to probe the composition of probability of failure (POF) in reliability-based structural design. The focus is on extreme event (EE) related risk of structural failure. Extreme events that may cause structural failure are often described by a severity-based threshold which can be a hazard intensity measure or return period. Such definition of EE is not directly connected to the POF. In order to quantify its portion of risk in the overall POF, a different definition is formulated. It is shown that this new definition of EE delineates a subset of failure events which not only coincides with the severity-based definition in terms of probability value but also represents the tail properties of the demand and capacity distribution models. Examples are given to illustrate the use of this approach to evaluate EE in hazard load comparison for structural design. It is hoped that this effort will help to put some EE related structural design issues (e.g. uncertainty of hazard loading, different demand and capacity models, the influence of tail property of probabilistic distribution models, extreme event limit states, and load combinations etc.) in proper perspective. .

KEYWORDS: probability of failure, extreme events, tail probability, upper tail of demand, lower tail of capacity

1. INTRODUCTION

In the reliability-based structural design, the overall probability of failure POF, denoted as (P_F), plays an important role as it is the measurement for uniform reliability of a designed structural system. While the accuracy of this probability is highly desirable, the structural design community has long recognized that the true risk of structural failure can be largely influenced by many factors. One of these is the probabilistic distribution model used to represent design variables. Even under the same statistical moments such as mean and standard deviation, Ferry Borges and Castenheta (1983) and Melchers (1999) showed that different models may produce significantly different probability values for the two-variable (load and resistance) reliability problem. The reason is found to be mainly due to the different tail property of the distribution models and it is denoted as "tail sensitivity" issue. This issue is in turn closely related to the extreme events (EE).

AASHTO (2004) describes the frequency of extreme event as "event with return period in excess of the design life of the bridge". Since severe natural hazards or strength deterioration in structures can be expected to have very low chance to occur (large return period), extreme events can be regarded as associated with either excessive demand or very weak capacity. Reflecting past data and recent events of bridge structure's damage/ failure due to earthquake, hurricane, collision or weakened structural connections, a concern has been raised on the appropriateness of current design load (demand) and capacity for these extreme events. This concern eventually comes down to the question whether current probabilistic models for the design variables have properly represented the EE in the upper tail of demand distribution and lower tail of capacity distribution. It would be advantageous to address the above question by quantifying how extreme events are attributed to probability of structural failure.

Several related works have been developed in this area. Cornell (1996) showed that the probability of structure failure subjected to earthquake ground motion is dominated by the seismic hazard curve – annual frequency of hazard's exceeding structure's median capacity. He provided an explicit method to estimate P_F using the hazard exceeding probability multiplied by a correction factor which depends on the dispersion of hazard curve and capacity's probability distribution. This method has been widely applied to seismic performance evaluation of structures (Ellingwood, 2001). A basic assumption of the method is that seismic hazard curve can be logarithmically linearized in the interest region of exceedance rate. Extended from such specific case for seismic hazard with approximated demand model, it can be considered for general application to extreme events in addition to earthquake, not only with demand (hazard) side but also with capacity side as extreme event affecting P_F. A probability-based criterion formulated in this study attempts to follow this approach to provide a quantification of P_F due to extreme events. To include extreme events in general sense, the formulation will cover the tail properties of both high demand and low capacity.

In the following of this paper, we first introduce two different ways to define extreme events. Then we show that a quantitative link can be established between the two. And this relationship helps to provide a convenient approach to physically measure the severity of a hazard loading and numerically relate the probability of such event to the risk of structural failure. Some examples are provided to illustrate the use of the methodology in hazard load analysis and comparison.

2. RELATIONSHIP BETWEEN EXTREME EVENTS AND PROBABILITY OF FAILURE

2.1. Conceptual Understanding of Extreme Events

EEs can be viewed in many different ways. Preferably, to delineate the severity and rareness of an EE, a threshold is attached to a hazard in discussion. Conceptually, a severity-based threshold definition for load and resistance related extreme events can be described as

(1). If the hazard load (demand) produced by a hazard event onto a structure is beyond a pre-set threshold then the hazard event is referred to as an excessively high demand (HD) event. The threshold can be given by the hazard intensity, load effects or return period.

(2). If the structural system's resistance (capacity) to withstand a hazard load (demand) is lower than a pre-set threshold, then the structure is referred to as an excessively low capacity (LC) event. The threshold is measured by the resistible hazard intensity or level of load effects.

In many of natural hazard studies, extreme natural hazards are often associated with concepts such as risk, impact, vulnerability and loss. The advantage of the above definition is that it can be easily integrated with these concepts. However, as it is easily noticed that there are no definitive boundaries to determine the threshold beyond which a hazard event should be called extreme. For the purpose of this study, we propose to follow the approach of Cornell 1996 by setting the threshold for quantitative definitions of "excessively high demand (*HD*)" and "excessively low capacity (*LC*)" as the median of capacity and demand, respectively. This choice of threshold provides two benefits to be shown later: 1) it helps to tie the extreme events to a tail distribution of the event occurrence model, and 2) it provides an easy link between severity measurement of the extreme event and the overall POF.

Note that the above definition of EE is based on a single-variable (severity) threshold. It is not connected to the structural failure, which requires two-variable (capacity and demand) relationship. Following the standard definition of probability of failure, the set of structural failure (denoted as F) is considered as the collection of events such that in each case the demand exceeds the capacity.

We use the notation of pairs of demand and capacity (d, c) for the following definition. The elements in each pair represent values taken from independent distributions of D (demand) and C (capacity), respectively. The physical variables of the demand and capacity can be force-based, displacement-based or energy-based design structural design parameters from either the structural component level or system level.

A set U is defined as the complete probability event space (Eq (1)) and its probability measure is equal to unity (100%)

$$U = \{(d,c) \mid d \in D, c \in C\} \quad (1)$$

The two subclasses of extreme events HD and LC, are defined as subsets of U such that

$$HD = \{(d,c) \mid d \in D, c \in C \text{ and } d > m_C\} \quad (2)$$

$$LC = \{(d,c) \mid d \in D, c \in C \text{ and } c < m_D\} \quad (3)$$

The aggregated extreme events EE is also a subset of U and is defined as a union of HD and LC

$$EE = HD \cup LC = \{(d,c) \mid d \in D, c \in C \text{ and}, c < m_D \text{ or } d > m_C\} \quad (4)$$

where { } represents a set, and m_X is the median of random variable X. HD and LC are not mutually exclusive and not independent. δ and ε represent the probability of HD and LC as calculated by Eqs (5), (6)

$$\delta = P[HD] = 1 - F_D(m_C) \quad (5)$$

$$\varepsilon = P[LC] = F_C(m_D) \quad (6)$$

where the notation $P[\ldots]$ denotes "the probability that …" and $F_X(x)$ represents the cumulative density function (CDF) of X.

Figure 1 shows the reference markers for m_C, m_D, δ and ε..

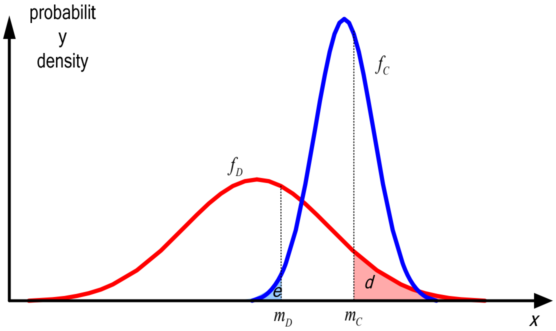


Figure 1. m_C, m_D, δ and ε..

2.2. Extreme Events Related Probability of Failure

The above defined sets of HD, LC and EE only identify the tails of demand and capacity distributions that are associated with the corresponding severity-based EE subsets. As pointed out above, the sets so defined are not directly connected to the POF. In order to evaluate the risk associated with these EE tails, we first define a failure set F as

$$F = \{(d,c) \mid d \in D, c \in C \text{ and } d > c\} \quad (7)$$

and

$$P[F] = P_F \quad (8)$$

where P_F is the POF as mentioned before. In the following, we will refine the HL, LC and EE with respect to the failure set F.

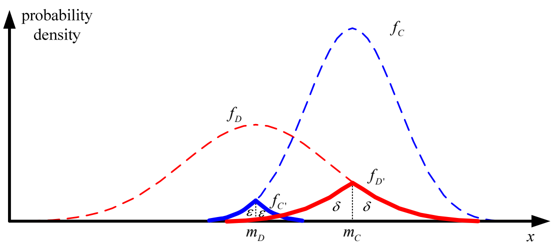


Figure 2. Tail probabilities δ and ε and tail-based PDF $f_{D'}$ and $f_{C'}$

Define a tail-based probability density function (PDF) of D can be constructed by folding (or mirroring) the upper tail part of demand PDF with respect to the axis of m_C as shown in Figure 2 where $f_X(x)$ represents probabilistic density function (PDF) of X. And we refer to this PDF as $f_{D'}$ and it is easy to show that $f_{D'}$ is located on upper part of f_D so that the integral of $f_{D'}$ is smaller than for the f_D. A failure set associated with regular capacity C and tail-based demand D' is defined as F_HD and this set is a subset of F because D' is subset of D. By analogy, tail-based PDF of C can be constructed as $f_{C'}$ and a subset of F associated with the regular demand D and tail-based capacity PDF C' is defined as F_LC. And F_EE is defined as the union of F_HD and F_LC. These definitions can be expressed by set relations given below

$$F\_HD = \{(d,c) \mid d \in D', c \in C \text{ and } d > c\} \tag{9}$$

$$F\_LC = \{(d,c) \mid d \in D, c' \in C' \text{ and } d > c\} \tag{10}$$

$$F\_EE = F\_HD \cup F\_LC \subset F \tag{11}$$

where F_HD and F_LC are subsets of F_EE. Since F_HD and F_LC are not mutually exclusive, the probability of F_EE is not a simple summation of probabilities of F_HD and F_LC. Nevertheless, based on the subset relationship, probabilities of these subsets satisfy the following condition

$$P[F\_HD] < P_F \tag{12}$$

$$P[F\_LC] < P_F \tag{13}$$

$$P[F\_EE] = P[F\_HD \cup F\_LC] < P_F \tag{14}$$

In the following subsections, we show that the probabilities of the subsets F_HD and F_LC are equal to δ and ε, the probabilities of HD and LC. Once these simple formulas are established, they can be used to assess the influence of EE related risk in P_F.

2.3. Formulation for POF Due to High Demand Extreme Events: F_HD

We first work on the relationship between $P[F\_HD]$, δ and P_F for normal (symmetric) distribution of capacity, and then prove that this relationship is also validity for lognormal capacity model.

2.3.1 Formulation between δ and P_F: symmetrical (normal) PDF of C

It is proved that when C follows the symmetrical (normal), δ is less than or equal to P_F.

P_F is given as

$$P_F = \int_{-\infty}^{+\infty} f_C(x)\{1 - F_D(x)\}dx = 1 - \int_{-\infty}^{+\infty} f_C(x) F_D(x)dx \tag{15}$$

$P[F\_HD]$ can be evaluated as (16) considering D' where the maximum value of $F_{D'}$ is 2δ instead of 1.0.

$$P[F\_HD] = \int_{-\infty}^{+\infty} f_C(x)\{2\delta - F_{D'}(x)\}dx = 2\delta - \int_{-\infty}^{+\infty} f_C(x) F_{D'}(x)dx \tag{16}$$

As shown in Figure 2, the abscissa of x is converted by setting the new origin at m_C, and assign ξ and η as starting from new origin toward the right hand direction and left hand direction, respectively. Correspondingly, PDF of C and CDF of D' have the relation of (17) in which + and – signs behind D' and C indicate the direction of ξ and η, respectively.

$$f_{C+}(\alpha) = f_C(m_C + \alpha) \qquad f_{C-}(\alpha) = f_C(m_C - \alpha) \tag{17}$$

$$F_{D'+}(\alpha) = F_{D'}(m_C + \alpha) \qquad F_{D'-}(\alpha) = F_{D'}(m_C - \alpha) \qquad F_{D'+}(\alpha) = 2\delta - F_{D'-}(\alpha) \tag{18}$$

The second term in (16) can be separated as

$$\int_{-\infty}^{+\infty} f_C(x) F_{D'}(x)\,dx = \int_{-\infty}^{m_C} f_C(x) F_{D'}(x)\,dx + \int_{m_C}^{+\infty} f_C(x) F_{D'}(x)\,dx \qquad (19)$$

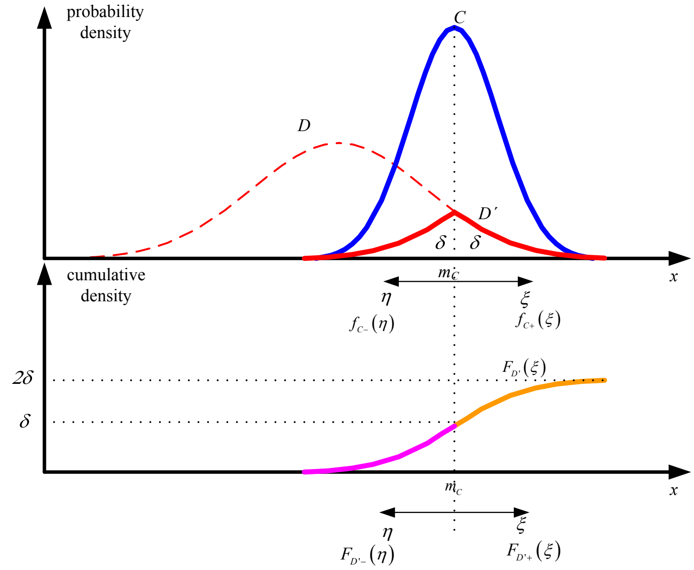


Figure 2. PDF of D' and CDF of D'

By using (17) and (18), transformations (20) and (21) can be applied to (19).

$$\eta + x = m_C,\ x = m_C - \eta,\ dx = -d\eta,\ x = m_C \to \eta = 0,\ x = -\infty \to \eta = +\infty \qquad (20)$$

$$x - m_C = \xi, x = m_C + \xi, dx = d\xi, x = +\infty \to \xi = +\infty, x = m_C \to \xi = 0 \qquad (21)$$

Then (19) becomes

$$\begin{aligned}\int_{-\infty}^{m_C} f_C(x) F_{D'}(x)\,dx + \int_{m_C}^{+\infty} f_C(x) F_{D'}(x)\,dx &= \int_0^{+\infty} f_{C-}(\eta) F_{D'-}(\eta)\,d\eta + \int_0^{+\infty} f_{C+}(\xi) F_{D'+}(\xi)\,d\xi \\ &= \delta + \int_0^{\infty} F_{D'+}(\xi)\{f_{C+}(\xi) - f_{C-}(\xi)\}\,d\xi\end{aligned} \qquad (22)$$

Substituting (22) into (19) and then (19) into (16) provide

$$P[F\_HD] = \delta + \int_0^{\infty} F_{D'+}(\xi)\{f_{C-}(\xi) - f_{C+}(\xi)\}\,d\xi \qquad (23)$$

If the PDF of C has symmetric shape, then mean, mode and median coincide. Accordingly, the relation of $f_{C-}(\xi)= f_{C+}(\xi)$ is satisfied and it makes the second term as zero in (23) on any value of ξ and then $\delta \le P_F$ is satisfied based on (12). Therefore,

$$P[F\_HD] = \delta \le P_F \qquad (24)$$

2.3.2 Formulation between δ and P_F: lognormal PDF of C

Lognormal distribution is closely related to normal distribution. Accordingly, even if lognormal distribution itself is not a symmetrical distribution, by using similar approach for symmetrical distribution, it can be shown that $P[F\_HD]=\delta \le P_F$ is valid.

When X follows lognormal distribution, the median of random variable X is given as

$$m_X = e^{\mu_{\ln X}} \qquad (25)$$

where $\mu_{\ln X}$ is the mean of ln$X$.

Given that D and C have positive values, P_F can be represented using logarithm as

$$P_F = P[D > C] = P[\ln D > \ln C] = \int_{-\infty}^{+\infty} f_{\ln C}(x)\{1 - F_{\ln D}(x)\}\,dx = 1 - \int_{-\infty}^{+\infty} f_{\ln C}(x) F_{\ln D}(x)\,dx \qquad (26)$$

Also, δ can be expressed using logarithm

$$\delta = P[D > m_C] = P[\ln D > \ln m_C] = P[\ln D > \ln(e^{\mu_{\ln C}})] = P[\ln D > \mu_{\ln C}] \quad (27)$$

Since ln$C$ is symmetrical distribution (normal distribution), $m_{\ln C}$ and $\mu_{\ln C}$ coincide.

$$\delta = P[\ln D > m_{\ln C}] \quad (28)$$

Hence, ln$C$ and ln$D$ can replace C and D in (23), respectively.

$$P[F\_HD] = \delta + \int_0^{\infty} F_{\ln D'+}(\xi)\{f_{\ln C-}(\xi) - f_{\ln C+}(\xi)\} d\xi \quad (29)$$

ln$C$ is symmetrical about the axis of $m_{\ln C}$, so the second term in (29) can be canceled out.

$$P[F\_HD] = \delta \quad (30)$$

Therefore, $\delta \le P_F$ is satisfied for lognormal C.

2.4 Formulation for Failure related to Low Capacity: F_LC

Similar to the formulation shown in the above sections for P[F_HD], δ and P_F, we can also formulate the same relations for F_LC, ε and P_F for the low capacity EEs. By analogy, when demand D follows symmetrical (normal) or lognormal distributions, the relationship P[F_LC]=$\varepsilon \le P_F$ will hold.

2.5 Formulation for Failure related to Union of High Demand and Low Capacity: F_EE

We have shown in the above that $P[F\_HD]$ and $P[F\_LC]$ are equal to δ and ε respectively. However, in most cases, since F_HD and F_LC are neither independent nor mutually exclusive, $P[F\_EE]$ is not a simple summation of δ and ε. The probability of failure p[F_EE] due to combination of high demand and low capacity events is evaluated by the following equation

$$\begin{aligned} P[F\_EE] &= P[F\_HD] + P[F\_LC] - P[F\_HD \cap F\_LC] \\ &= \delta + \varepsilon - P[F\_HD \cap F\_LC] \end{aligned} \quad (31)$$

When F_HD and F_LC are not independent event subsets $P[F\_HD \cap F\_LC]$ is not equal to $\delta\varepsilon$. Although we do not have a formula to calculate this probability, an upper bound of P[F_HD∩F_LC] can be developed. F_HD∩F_LC is the failure event set under the two tail-based EE. Therefore, P[F_HD∩F_LC] can be considered as

$$P[F\_HD \cap F\_LC] = \int_{-\infty}^{+\infty} f_{C'}(x)\left[2\delta - F_{D'}(x)\right] dx = 4\delta\varepsilon - \int_{-\infty}^{+\infty} f_{C'}(x) F_{D'}(x) dx \quad (32)$$

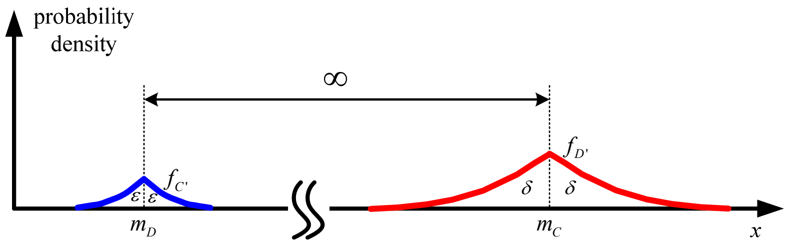


Figure 3. Infinitely Separated D' and C'

Despite that $P[F\_HD \cap F\_LC]$ cannot be evaluated in closed form, an upper bound can be developed so long as $f_{D'}(x)$ and $f_{C'}(x)$ are relatively far apart as shown in Figure 3 and this large separation condition represents the lower bound because the second term in (32) can be treated as below.

$$\int_{-\infty}^{+\infty} f_{C'}(x) F_{D'}(x) dx \ge 0 \quad (33)$$

Consequently $P[F\_HD \cap F\_LC]$ is given as

$$P[F\_HD \cap F\_LC] \le 4\delta\varepsilon \quad (34)$$

And $P[F\_HD \cup F\_LC]$ can be expressed as

$$P[F\_EE] = P[F\_HD \bigcup F\_LC] \geq \delta + \varepsilon - 4\delta\varepsilon \tag{35}$$

Therefore, (35) and (14) give

$$\delta + \varepsilon - 4\delta\varepsilon \leq P_F \tag{36}$$

If $\delta < ¼$ and $\varepsilon < ¼$ are satisfied, $\delta + \varepsilon - 4\delta\varepsilon$ is closer to P_F than δ or ε alone. Since $\delta + \varepsilon - 4\delta\varepsilon$ is a lower bound of $P[F\_EE]$, we denote this value as P_{EE}.

$$P_{EE} = \delta + \varepsilon - 4\delta\varepsilon \tag{37}$$

In this section, P_{HD} (=δ), P_{LC} (=ε) and P_{EE} are developed to show the influence of extreme event to P_F and these tail probabilities will be compared with P_F under different statistical models and parameters in following examples.

3. EXAMPLES

In this section, the aforementioned relationships are examined through three case study examples. First, the relationship between P_{HD} and P_F is numerically simulated using extreme value type-II (for largest value) model as the demand distribution model and lognormal model as the capacity distribution model. We check in this simulated case how the statistical parameters affect the P_{HD}/P_F ratio. Second, as an approximated combination of P_{HD} and P_{LC}, the portion of P_{EE} in P_F is simulated. Owing to requirement in the relationship related to *EE*, demand model is also assumed to follow lognormal distribution for evaluating *LC* events. How much P_{EE} fills P_F is inspected with respect to the statistical parameters. Third case study is for a design problem, a single-column bridge bent subjected to earthquake ground motion is examined. Considering the randomness of factors constructing demand value, Monte Carlo Simulation is carried out to generate the demand model.

3.1. Case Study 1: Quantification of P_{HD} in P_F

Given sets of COV of *C* (coefficient of variation: σ_C/μ_C), central safety factor (μ_C/μ_D) and COV_D are changed to see how P_{HD}/P_F is varied with corresponding reliability index β which can be obtained from $\Phi^{-1}(1-P_F)$. ($\Phi(\ )$: standard normal cumulative distribution function)

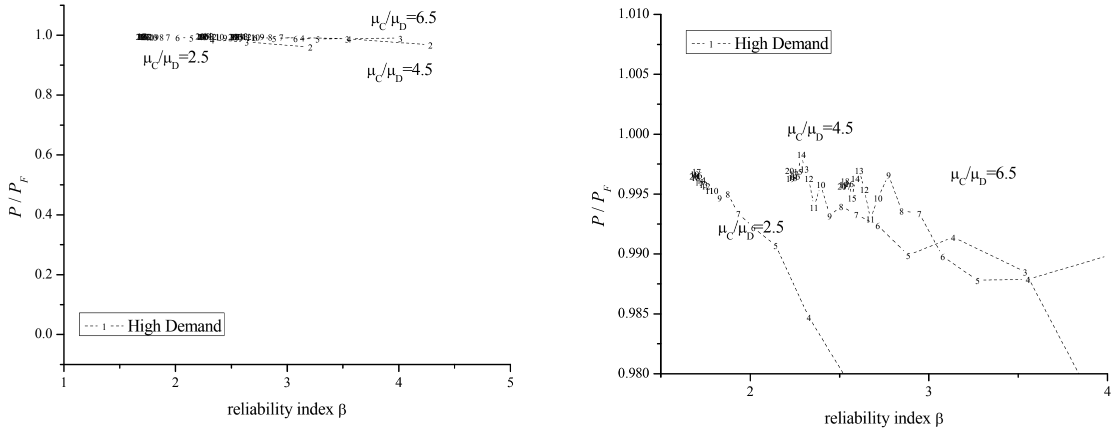
- Demand: extreme value type-II (EVT2); Capacity: Lognormal; P_F is evaluated using Simulations.
- μ_C/μ_D = 2.5, 4.5 and 6.5 ; COV_C = 0.04, 0.12, and 0.20 ; COV_D = 0.1 through 2.0: based typical statistics for load (gravity and environmental) and resistance (RC and steel) provided by Ellingwood el al. (1980) and Nowak (1999).

Table 1. Portion of P_{HD} in P_F under different statistical parameters (Case Study 1)

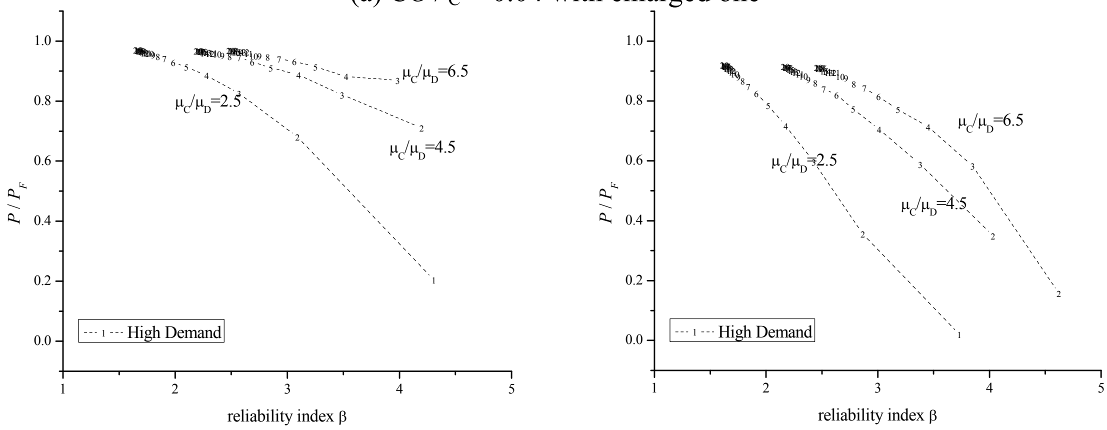
COV_D	$COV_C = 0.04$						$COV_C = 0.12$						$COV_C = 0.20$					
	$\mu_C/\mu_D = 2.5$		$\mu_C/\mu_D = 4.5$		$\mu_C/\mu_D = 6.5$		$\mu_C/\mu_D = 2.5$		$\mu_C/\mu_D = 4.5$		$\mu_C/\mu_D = 6.5$		$\mu_C/\mu_D = 2.5$		$\mu_C/\mu_D = 4.5$		$\mu_C/\mu_D = 6.5$	
	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$	β	$\frac{P_{HD}}{P_F}$
0.2	3.21	0.960	4.28	0.967			3.09	0.681	4.20	0.711			2.87	0.356	4.03	0.350	4.62	0.158
0.3	2.64	0.977	3.54	0.989	4.01	0.990	2.57	0.825	3.49	0.821	3.98	0.870	2.43	0.597	3.38	0.589	3.85	0.583
0.4	2.33	0.985	3.14	0.991	3.55	0.988	2.28	0.885	3.10	0.888	3.52	0.882	2.18	0.717	3.01	0.705	3.45	0.714
0.5	2.14	0.991	2.88	0.990	3.27	0.988	2.10	0.914	2.85	0.911	3.25	0.915	2.02	0.785	2.78	0.774	3.18	0.772
0.6	2.02	0.992	2.71	0.992	3.08	0.990	1.98	0.930	2.69	0.931	3.06	0.933	1.91	0.826	2.63	0.821	3.01	0.815
0.7	1.93	0.993	2.59	0.993	2.95	0.993	1.90	0.943	2.57	0.945	2.93	0.940	1.84	0.849	2.51	0.841	2.88	0.843
0.8	1.88	0.995	2.51	0.994	2.85	0.994	1.84	0.949	2.49	0.949	2.83	0.948	1.79	0.867	2.44	0.861	2.79	0.858
0.9	1.83	0.995	2.44	0.993	2.77	0.997	1.80	0.955	2.42	0.953	2.75	0.950	1.75	0.881	2.38	0.873	2.72	0.878
1.0	1.80	0.995	2.39	0.996	2.72	0.995	1.77	0.959	2.37	0.958	2.70	0.951	1.72	0.890	2.33	0.883	2.66	0.882
1.1	1.77	0.995	2.36	0.994	2.67	0.993	1.75	0.960	2.34	0.961	2.66	0.960	1.70	0.897	2.30	0.892	2.62	0.896
1.2	1.75	0.996	2.33	0.996	2.64	0.995	1.73	0.961	2.31	0.959	2.62	0.964	1.69	0.903	2.27	0.895	2.59	0.896
1.3	1.74	0.996	2.30	0.997	2.61	0.997	1.71	0.964	2.29	0.962	2.59	0.963	1.67	0.907	2.25	0.898	2.56	0.896
1.4	1.72	0.996	2.29	0.998	2.59	0.996	1.70	0.966	2.26	0.961	2.57	0.964	1.66	0.909	2.23	0.902	2.54	0.899
1.5	1.71	0.996	2.27	0.997	2.57	0.995	1.69	0.968	2.25	0.964	2.55	0.964	1.65	0.910	2.22	0.905	2.52	0.902
1.6	1.71	0.997	2.25	0.997	2.55	0.996	1.69	0.968	2.24	0.966	2.54	0.964	1.65	0.917	2.20	0.909	2.51	0.909
1.7	1.70	0.997	2.24	0.997	2.54	0.996	1.68	0.969	2.23	0.965	2.52	0.967	1.64	0.915	2.19	0.912	2.49	0.911
1.8	1.69	0.996	2.24	0.996	2.53	0.996	1.67	0.970	2.22	0.968	2.51	0.969	1.63	0.917	2.18	0.912	2.48	0.911
1.9	1.69	0.997	2.23	0.996	2.52	0.996	1.67	0.969	2.21	0.968	2.50	0.967	1.63	0.919	2.18	0.913	2.47	0.908
2.0	1.69	0.997	2.22	0.997	2.51	0.996	1.67	0.970	2.20	0.968	2.50	0.969	1.63	0.921	2.17	0.916	2.47	0.913

Followings are observed in the simulation results (Figure 4 and Table 1)

1) When COV_C is very low (more deterministic on capacity), P_{HD} is almost equivalent to P_F shown as more than 95% in Fig. 4(a).
2) If COV_D exceeds 1.0, P_{HD}/P_F begins to converge to a certain value (0.8~1.0 depending of COV_C) regardless of μ_C/μ_D.
3) As COV_C increases, P_{HD}/P_F is decreased as expected on same COV_D.



(a) $COV_C = 0.04$ with enlarged one



(b) $COV_C = 0.12$ (c) $COV_C = 0.20$

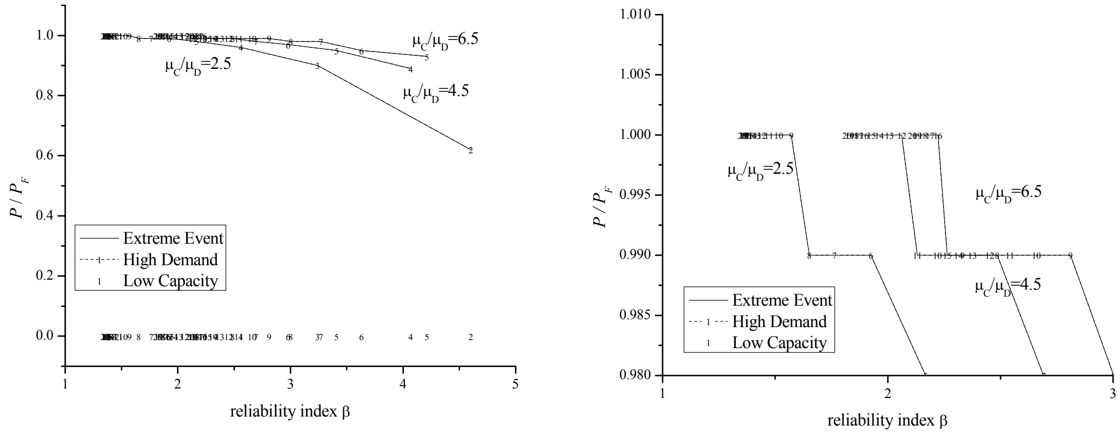
Figure 4. Portion of P_{HD} in P_F (demand: extreme value type-II & capacity: lognormal)
*note: numbers on line represent COV_D as tenths: e.g. 3➔$COV_D = 0.3$; 11➔$COV_D = 1.1$

3.2. Case Study 2: Quantification P_{EE} in P_F

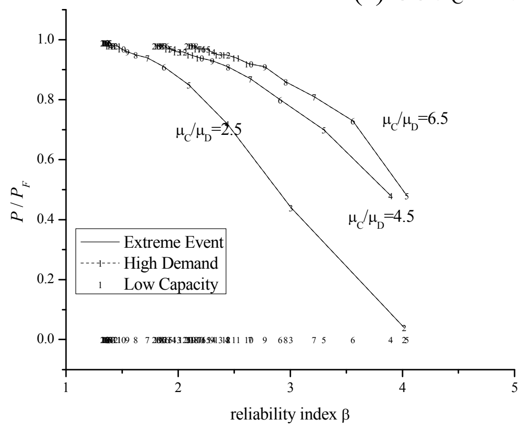
Under the same parametric conditions as the previous example except using lognormal model as the demand distribution model, in this second simulated case study, P_{HD}/P_F, P_{LC}/P_F and P_{EE}/P_F are examined where P_F is evaluated for the case such that both D and C follow lognormal distributions.

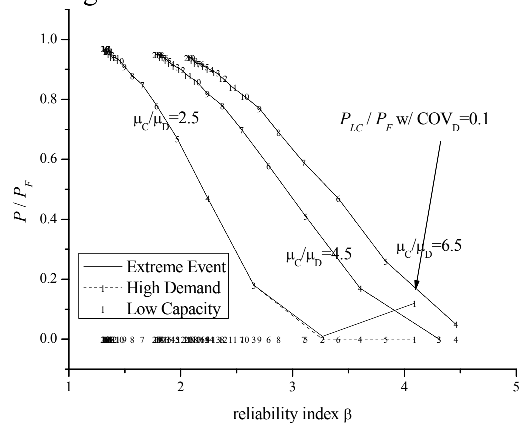
Following observations are made here. (results provided as Figure 5 and Table 2)

1) Mostly, P_{HD}'s portion coincides with P_{EE}'s. When $COV_C = 0.20$, P_{LC} seems to have little portion in P_F. P_{LC} has notable quantity only if $\mu_C/\mu_D = 2.5$ as indicated with the arrow in Figure 5(c). This is partly due to the shape of Capacity's lognormal distribution because the left hand side (lower) tail of lognormal PDF diminishes faster than right hand side (upper) tail. So lognormal distribution for capacity may not be appropriate to capture Low Capacity case.
2) Since EVT2 has fatter upper tail than lognormal, P_{HD}/P_F of EVT2 demand is larger than that of lognormal demand model when μ_C/μ_D, COV_C and COV_D are same. Analogously, in lognormal demand, reliability index β has larger value and P_{HD}/P_F converges slower than in EVT2. Also converged level (P_{HD}/P_F) is affected by μ_C/μ_D as COV_C increases unlikely to EVT2. This is due to 'tail sensitivity' as indicated by Melchers (1999).



(a) COV_C = 0.04 with enlarged one





(b) COV_C = 0.12 (c) COV_C = 0.20

Figure 5. Portion of P_{HD}, P_{LC} and P_{EE} in P_F (demand: lognormal & capacity: lognormal)
*note: numbers on line represent COV_D as tenths: e.g. 3➔COV_D = 0.3; 11➔COV_D = 1.1

For some general comparisons of the relevant parameters involved in the the above two case studies, it is observed that the ratio P_{HL}/P_F or P_{EE}/P_F will increase or decrease following the changes of β, COV_D, COV_C and parameter changes.

1) Under fixed FS and COV_C, as COV_D becomes larger, correspondingly β is decreased. In this case, P_{EE}/P_F is increased. This means that, given designed capacity, if uncertainty of demand becomes larger, then influence of extreme event is dominant in P_F.
2) Just under fixed COV_C, as COV_D becomes increasing, but β is intended to be the same level, then FS should be raised. In this case, then P_{EE}/P_F is slightly increased. This case depicts that, when the demand's uncertainty is increased, the reliability level is intended to be kept then influence of extreme event in P_F still increased but slightly.
3) Given fixed COV_D, if COV_C is increased and β is kept as same, then also FS should be increased. Then P_{EE}/P_F is decreased. Under same load condition, uncertainty of capacity becomes larger but engineering want to keep the same level of reliability, then FS should be increased. In this case, the influence of *EE* to P_F is less significant.
4) Under fixed FS and COV_D, if COV_C is decreased then β is correspondingly increased. In this case, P_{EE}/P_F is increased. Once uncertainty about capacity is reduced, e.g. by improved quality control of material property, the reliability is increased. This means that as overall reliability becomes less affected by non-extreme event, the portion of extreme event occupies more in overall P_F.
5) In general cases, if β is large enough, namely over 3.5 which corresponds to large FS. In this case, the influence of extreme event becomes insignificant. In other side, the reliability level is low that means low FS in general. So the influence of extreme event becomes dominant. So engineers should pay attention to how much the tail properties contribute to overall P_F.

6) These observations are based on the lognormal capacity model. And mostly the probability of high demand composes the combined extreme event dominantly. Conventionally, capacity is modeled to follow lognormal distribution e.g. in fragility curves and the calibration of LRFD factors for AASHTO (Nowak, 1999). Even though, that model may not enough reflect lower level of capacity. That is because lognormal PDF has longer tail on the right hand side. If normal distribution is used, lower capacity's influence to P_F can show larger.

Table 2. Portion of P_{HD}, P_{LC} and P_{EE} in P_F under different statistical parameters (Case Study 2)

COV_D	$COV_C = 0.04$												$COV_C = 0.12$					
	$\mu_C/\mu_D = 2.5$				$\mu_C/\mu_D = 4.5$				$\mu_C/\mu_D = 6.5$				$\mu_C/\mu_D = 2.5$				$\mu_C/\mu_D = 4.5$	
	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F
0.2	4.60	0.619	0	0.619									4.02	0.037	0	0.037		
0.3	3.24	0.900	0	0.900									3.01	0.443	0	0.443		
0.4	2.56	0.964	0	0.964	4.07	0.890	0.000	0.890					2.44	0.724	0	0.724	3.89	0.482
0.5	2.17	0.980	0	0.980	3.41	0.946	0.000	0.946	4.21	0.929	0.000	0.929	2.10	0.847	0	0.847	3.30	0.696
0.6	1.92	0.988	0	0.988	2.98	0.971	0.000	0.971	3.63	0.953	0.000	0.953	1.87	0.905	0	0.905	2.91	0.803
0.7	1.76	0.993	0	0.993	2.69	0.985	0.000	0.985	3.27	0.977	0.000	0.977	1.72	0.936	0	0.936	2.64	0.872
0.8	1.65	0.995	0	0.995	2.49	0.989	0.000	0.989	3.00	0.983	0.000	0.983	1.62	0.954	0	0.954	2.44	0.907
0.9	1.57	0.996	0	0.996	2.33	0.992	0.000	0.992	2.81	0.994	0.000	0.994	1.55	0.963	0	0.963	2.30	0.933
1.0	1.51	0.996	0	0.996	2.22	0.994	0.000	0.994	2.66	0.989	0.000	0.989	1.49	0.971	0	0.971	2.19	0.944
1.1	1.47	0.997	0	0.997	2.13	0.994	0.000	0.994	2.54	0.994	0.000	0.994	1.45	0.975	0	0.975	2.11	0.954
1.2	1.44	0.998	0	0.998	2.06	0.996	0.000	0.996	2.45	0.993	0.000	0.993	1.42	0.980	0	0.980	2.04	0.962
1.3	1.42	0.998	0	0.998	2.01	0.996	0.000	0.996	2.37	0.994	0.000	0.994	1.40	0.981	0	0.981	1.99	0.965
1.4	1.40	0.997	0	0.997	1.96	0.996	0.000	0.996	2.31	0.994	0.000	0.994	1.38	0.983	0	0.983	1.95	0.970
1.5	1.38	0.998	0	0.998	1.93	0.996	0.000	0.996	2.26	0.995	0.000	0.995	1.37	0.985	0	0.985	1.91	0.973
1.6	1.38	0.999	0	0.999	1.90	0.997	0.000	0.997	2.22	0.996	0.000	0.996	1.36	0.986	0	0.986	1.88	0.977
1.7	1.37	0.999	0	0.999	1.87	0.998	0.000	0.998	2.19	0.997	0.000	0.997	1.36	0.987	0	0.987	1.86	0.978
1.8	1.36	0.999	0	0.999	1.85	0.997	0.000	0.997	2.16	0.998	0.000	0.998	1.35	0.988	0	0.988	1.84	0.979
1.9	1.36	0.998	0	0.998	1.83	0.998	0.000	0.998	2.13	0.997	0.000	0.997	1.35	0.987	0	0.987	1.82	0.982
2.0	1.35	0.998	0	0.998	1.82	0.998	0.000	0.998	2.11	0.998	0.000	0.998	1.34	0.990	0	0.990	1.81	0.982

COV_D	$COV_C = 0.12$						$COV_C = 0.20$											
	$\mu_C/\mu_D = 4.5$		$\mu_C/\mu_D = 6.5$				$\mu_C/\mu_D = 2.5$				$\mu_C/\mu_D = 4.5$				$\mu_C/\mu_D = 6.5$			
	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F	β	P_{HD}/P_F	P_{LC}/P_F	P_{EE}/P_F
0.2							3.27	0.002	0.004	0.006								
0.3							2.65	0.176	0.000	0.177	4.31	0.000	0.000	0.000				
0.4	0.000	0.482					2.24	0.470	0.000	0.470	3.60	0.167	0.000	0.167	4.46	0.049	0.000	0.049
0.5	0.000	0.696	4.04	0.478	0.000	0.478	1.97	0.670	0.000	0.670	3.12	0.409	0.000	0.409	3.83	0.264	0.000	0.264
0.6	0.000	0.803	3.56	0.733	0.000	0.733	1.78	0.779	0.000	0.779	2.79	0.582	0.000	0.582	3.41	0.471	0.000	0.471
0.7	0.000	0.872	3.21	0.808	0.000	0.808	1.66	0.846	0.000	0.846	2.54	0.699	0.000	0.699	3.10	0.594	0.000	0.594
0.8	0.000	0.907	2.96	0.858	0.000	0.858	1.57	0.884	0.000	0.884	2.37	0.777	0.000	0.777	2.88	0.695	0.000	0.695
0.9	0.000	0.933	2.77	0.905	0.000	0.905	1.50	0.907	0.000	0.907	2.24	0.825	0.000	0.825	2.70	0.771	0.000	0.771
1.0	0.000	0.944	2.63	0.922	0.000	0.922	1.45	0.926	0.000	0.926	2.14	0.859	0.000	0.859	2.57	0.813	0.000	0.813
1.1	0.000	0.954	2.52	0.936	0.000	0.936	1.42	0.938	0.000	0.938	2.06	0.883	0.000	0.883	2.47	0.841	0.000	0.841
1.2	0.000	0.962	2.43	0.946	0.000	0.946	1.39	0.945	0.000	0.945	2.00	0.903	0.000	0.903	2.38	0.868	0.000	0.868
1.3	0.000	0.965	2.36	0.955	0.000	0.955	1.37	0.951	0.000	0.951	1.95	0.912	0.000	0.912	2.32	0.887	0.000	0.887
1.4	0.000	0.970	2.30	0.961	0.000	0.961	1.36	0.957	0.000	0.957	1.91	0.924	0.000	0.924	2.26	0.898	0.000	0.898
1.5	0.000	0.973	2.25	0.967	0.000	0.967	1.35	0.961	0.000	0.961	1.88	0.933	0.000	0.933	2.21	0.911	0.000	0.911
1.6	0.000	0.977	2.21	0.967	0.000	0.967	1.34	0.964	0.000	0.964	1.85	0.938	0.000	0.938	2.17	0.921	0.000	0.921
1.7	0.000	0.978	2.17	0.973	0.000	0.973	1.33	0.966	0.000	0.966	1.83	0.945	0.000	0.945	2.14	0.924	0.000	0.924
1.8	0.000	0.979	2.14	0.975	0.000	0.975	1.33	0.968	0.000	0.968	1.81	0.946	0.000	0.946	2.11	0.932	0.000	0.932
1.9	0.000	0.982	2.12	0.977	0.000	0.977	1.33	0.970	0.000	0.970	1.80	0.950	0.000	0.950	2.09	0.937	0.000	0.937
2.0	1.69	0.997	2.22	0.997	2.51	0.996	1.32	0.971	0.000	0.971	1.78	0.954	0.000	0.954	2.07	0.939	0.000	0.939

In summary, the above change patterns are shown in Table 3:

Table 3. Relationship among design parameters.

Observation	COV_D	COV_C	β	P_F	FS	P_{EE}/P_F
1	increased	fixed	decreased	increased	Fixed	increased
2	increased	fixed	kept	kept	increased	slightly increased
3	fixed	increased	kept	kept	increased	decreased
4	fixed	decreased	increased	decreased	Fixed	increased

Note: shaded cells contain the changing parameters, un-shaded cells are the influenced outcomes.

3.3. Case Study 3: Bridge Bent subjected to Earthquake

In this example, the seismic reliability of a single-RC column bridge bent (Figure 6) is examined. Flexural failure of column is assumed as a critical failure mode in the single-column bent design, and designed flexural strength in the example (NCHRP 489: Ghosn et al. 2003) is determined following NEHRP design spectra of four sites with different seismic hazards in the U.S.

- Demand: simulated model; Capacity: lognormal flexural resistance (fixed COV = 0.17).
- The bridge is classified as an essential structure, so Response Modification Factor (R_m)= 2.0 and design earthquake = 2500-year return period ground motion are chosen.
- Parameters of composed demand and capacity model are provided in Table 4. Other statistical parameters used to evaluate flexural demand and capacity can be found in NCHRP 489. All probabilistic parameters are modeled to follow lognormal in this study.

Table 4. Statistical parameter of flexural demand and capacity for bridge bent

Site	Demand (simulated)		Capacity (lognormal)		μ_C/μ_D
	Mean [kips]	COV	Mean [kips]	COV	
San Francisco	9881	1.01	30465	0.17	3.08
Seattle	6045	1.21	17029	0.17	2.82
New York	1097	3.41	4566	0.17	4.16
Memphis	3311	2.44	13174	0.17	3.98

Following observations are made in this case study. (Figure 7)

1) Due to large COV_D of flexural demand, P_{HD}/P_F's have the values near 1.0. This COV comes from the large uncertain property of earthquake hazard intensity (PGA). Especially, since COV of conventional RC column's strength has been recognized as 0.13~0.17 (Ghosn et al. 2003), P_{HD}/P_F is expected to be greater than 0.9.
2) Reflecting COV_D and μ_C/μ_D for each site's hazard demand model, SF and Seattle's demand models seem close to EVT2 and NYC and Memphis's close to lognormal.

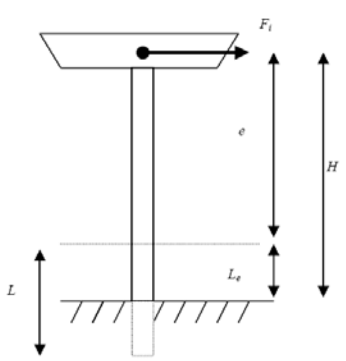


Figure 6. Bridge bent in example (Ghosn et al. 2003)

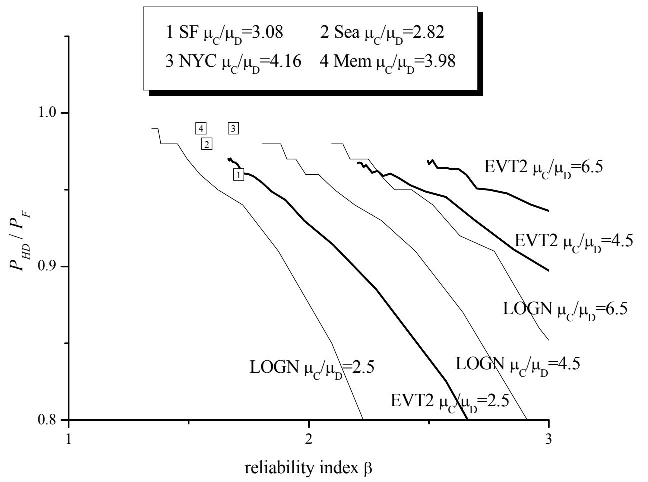


Figure 7. P_{HD}/P_F of bridge bent subjected to earthquake for COV_C=0.17 overlapped with Fig. 4(b) and 5(b)

4. CONCLUSIONS

In this paper, we examined the composition of POF and tried to quantify the portion of POF due to EE (high demand and low capacity. Defining EE by severity-based threshold or as subsets of failure events is provided and quantitative relationship between the two is developed. Through case study examples it is shown that a dominant portion of failure probability is attributed to EEs with large uncertainty in the demand model. The formulated criterion has three meanings: 1) this helps to represent the tail property of demand and capacity in POF decomposition; 2) it is directly linked severity-based EE concept to POF based EE concept; 3) it offers an easy measure of EE by simply looking at the two tail probabilities.

5. ACKNOWLEDGEMENT

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