DYNAMIC STRESS-STRAIN RELATIONSHIP OF NANJING FLAKE-SHAPED SAND UNDER CYCLIC LOADING

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ABSTRACT :

Comparing with the common quartz sand soils composed of the circular-shaped particle, the flake-shaped sand soils have remarkable differences in physical mechanics properties. With the dynamic pore-water pressure rising, the stiffness degradation of saturated sand soils is caused under cyclic loading. And liquidation of saturated sand soils is triggered when the dynamic pore-water pressure accumulates to some level, so that soil structure is broken down. Considering different static deviator stress levels, axial cyclic stress ratio levels and the number of cycles, a series of dynamic stress-strain relationship tests of Nanjing flake-shaped sand were performed by using the WFI cyclic triaxial apparatus made in England. It is observed that initial static deviator stress levels, axial cyclic stress ratio levels and the number of cycles have marked effect on stress-strain relationship of Nanjing flake-shaped sand. By considering the properties of secant shear modulus in each unload-reload loop of dynamic stress-strain relationship, an empirical equation on degradation of secant shear modulus is established based on series of test results. The static deviator stress level has remarkable effect on the degradation of shear modulus. With the numbers of cycles increasing, the stress-strain hysterisis loops separate each other gradually and the stress-strain hysterisis loops are leaning gradually towards the coordinate axis of strain. In addition, based on the empirical equation of shear modulus degradation, the dynamic stress-strain relationship of Najing flake-shaped sand was described by using modified Masing's rule.

KEYWORDS: cyclic loading; flake-shaped sand; dynamic stress-strain relationship; degradation of dynamic shear modulus

1. INTRODUCTION

Structure of soil refers to characters and arrangement modes of soil particles and its void, also interaction among soil particles. The main influencing factors on soil structure are mineral compositions, shapes, sizes and characteristics of particles, space arrangement of particles, status of porous; contact between particles, connection features and so on. The flake-shaped sand soils, containing quartz, chlorite and muscovite minerals, are widely distributed throughout the middle and lower reaches of the Yangtze River. Comparing with the standard quartz sand soils composed of circular-shaped particles, the flake-shaped sand soils have some differences in physical mechanics properties. The flake-shaped particle components give rise to the anisotropic property in physical mechanics properties of flake-shaped sand soils, and its strength in horizontal direction is smaller than that of vertical direction. For example, flake-shaped sand soil is widely distributed in Jiangsu section of the Beijing-Shanghai high-speed railway line. Academician Zhou Jing (1999) first paid attention to the mechanical properties of the flake-shaped sand soil, and named it as Nanjing sand. Although many research works about structural clay have been done, little literatures about structural sand have been reported, especially research on dynamic characters under cyclic loadings.

The relationship between dynamic stress and strain under cyclic loading exhibits nonlinear, hysteretic nature and softening characters. There are two kind models used to describe dynamic constitutive relation of soils under cyclic loading. They are the viscoelasticity model and the elastoplastic model. The viscoelasticity models include Hardin-Drnevich model, Ramberg-Osgood model, hyperbolic model and other combination curve model. As for elastoplastic models, there are multiple yield surface model, boundary surface model and elastoplastic model based on the generalized plasticity theory. But most of them can not exhibit the important character of strain softening. A. Puzrin（1995）considered that the main reason of deviation using Masing’s rules to describe dynamic stress-strain relationship of soft clay lies in cyclic softening character of this kind soil. M. Vucetic (1990) used Masing’s rules to describe initial loading curve and built backbone curve with M. Idriss’s model, then described further loading and unloading curve. However, M. Idriss’s model only exhibit variation law of backbone curve’s stiffness and can not exhibit softening rule of soil in each cycle. So, the results using this method coincided with actual measurement only in unloading and further loading curve’s initial point. Mohajeri (2003) modified Masing formula and described dynamic constitutive relation of sand soil material based on modified Masing formula, but this method can not exhibit softening rule in each cycle, too. Wang Jun (2007) improved the method proposed by Mohajeri and built the relation among stress-strain and cycle number. And dynamic stress-strain relation of Xiaoshan soft soil under cycled loadings can be described well with this model.

This paper researched the effects of static deviator stress and dynamic stress on the dynamic properties of Nanjing flake-shaped sand under cycled loading and built an empirical formula reflecting degradation of dynamic shear modulus by analyzing change law of dynamic shear modulus during unloading and re-loading curves. Based on this experiential formula, Masing’s rule was modified to describe dynamic stress-strain relationship of Nanjing flake-shaped.

2. SOIL SAMPLES AND TEST METHODS

The static triaxial compression tests and cyclic triaxial tests were carried out using the WFI cyclic triaxial system. The soil samples, taken from the Xu fu xiang station site of Nanjing Metro, are flaked-shaped fine sand, which is gray with densities from medium compactness to compactness. Its morphogenesis belongs to the Yangtze River floodplain and ancient river accumulation. And liquation phenomenon may occur under dynamic loading. The sand soils contain some quartz particles, small number of chlorite, muscovite, clay sliver and weathered heavy minerals. The proportion of quartz particles in sand is from 50% to 60%. There are some differences between Nanjing flake-shaped sand and the circular particle standard quartz sand in particle composition and gradation. Compared with Fujian standard sand, Nanjing sand is much looser, its void ratio and density are bigger and the average particle size is smaller. The soil samples had following mechanical indexes: the average nature density $\rho = 18.5kN/m^3$, the maximum void ratio $e_{\max} = 1.14$, the minimum void ratio $e_{\min} = 0.62$, relative density $D_r = 0.5$, uniformity coefficient Cu=2.31 and curvature coefficient Cc=1.07. The height and diameter of the soil sample is 8.0cm and 3.91cm, respectively. The particle size distribution curve of Nanjing flake-shaped fine sand is shown in Figure1.

The cyclic triaxial tests of Nanjing flake-shaped fine sand were completed with WFI cyclic system. Firstly, soil samples were consolidated under confining pressure ($\sigma_c = 100kPa$), and drain valves were closed after completion of consolidation. Then, vertical static deviator stress was applied. After static deviator stress reached predetermined value, dynamic stress was applied and loading frequency was 1Hz. The cyclic triaxial shear tests under different static deviator stress were shown in table 1.

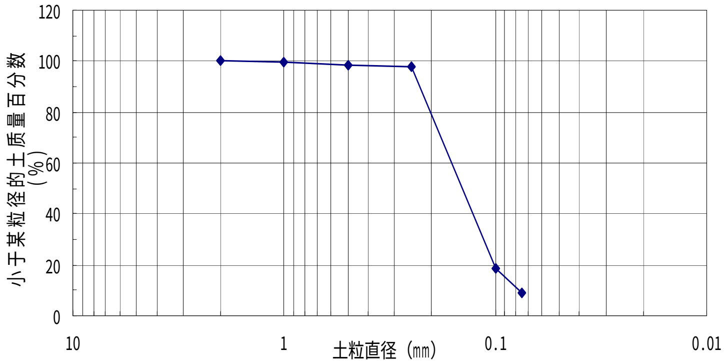


Figure1 Particle size distribution curve of Nanjing flake-shaped fine sand

Table 1. Details of undrained cyclic triaxial tests under different static deviator stress

Number	static deviator stress q_s (kPa)	static deviator stress ratio q_s / p_c'	Dynamic deviator stress q_{cyc} (kPa)	Dynamic deviator stress ratio q_{cyc} / p_c'
0-1			18	0.18
0-2	0	0.0	35	0.35
0-3			55	0.55
25-1			18	0.17
25-2	25	0.23	35	0.32
25-3			50	0.46
50-1			35	0.30
50-2	50	0.43	42	0.36
50-3			50	0.43

Note: p_c' is initial average effective stress

3. ANALYSIS OF EXPERIMENTAL RESULTS

3.1. Degradation of dynamic shear modulus and the empirical formula

Under cyclic loading, the relationship curves of deviator stress q and axial strain were series of closed hysterisis loops, as shown in Figure 2. Each hysterisis loop was made up of unloading and further loading curves. So, secant modulus of unloading and re-loading curves was defined in Figure 2, where $q = \sigma_1 - \sigma_3$, and ε_a is axial strain. $\varepsilon_{a\max}$ and $\varepsilon_{a\min}$ are the maximum and minimum values of axial strain in unloading curves respectively. $q_{\max}$ and $q_{\min}$ are the values of deviator stress corresponding to maximum and minimum axial strain in unloading curves respectively. The relationship between dynamic elastic modulus and shear modulus was shown in Eqn. 3.1.

$$G = \frac{E_d}{2(1+\nu)} = \frac{q}{2\varepsilon_a(1+\nu)} \tag{3.1}$$

For saturated and undrained shear tests, set $\nu = 0.5$, then

$$G = \frac{q}{3\varepsilon_a} = \frac{E_d}{3} \tag{3.2}$$

Secant shear modulus of unloading curves

$$G_{\sec} = \frac{q_{\max} - q}{3(\varepsilon_{a\max} - \varepsilon_a)} = \frac{q_{\max} - q}{3\varepsilon_s} \tag{3.3}$$

Secant shear modulus of re-loading curves

$$G_{\sec}^{'} = \frac{q - q_{\min}}{3(\varepsilon_a - \varepsilon_{a\min})} = \frac{q - q_{\min}}{3\varepsilon_s} \tag{3.4}$$

Where ν is Poisson ratio of soil and ε_s is cyclic strain amplitude.

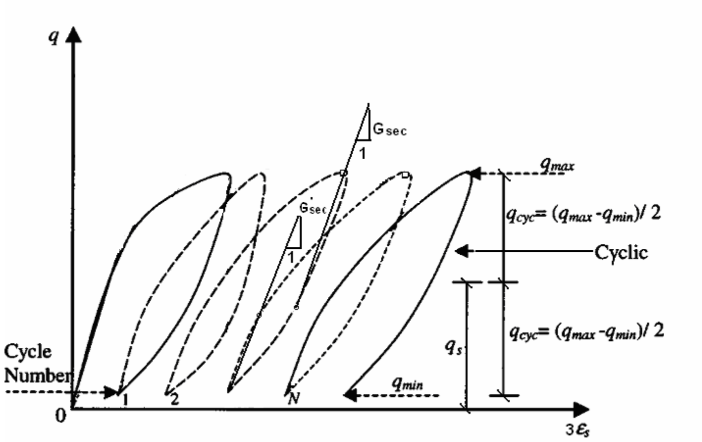
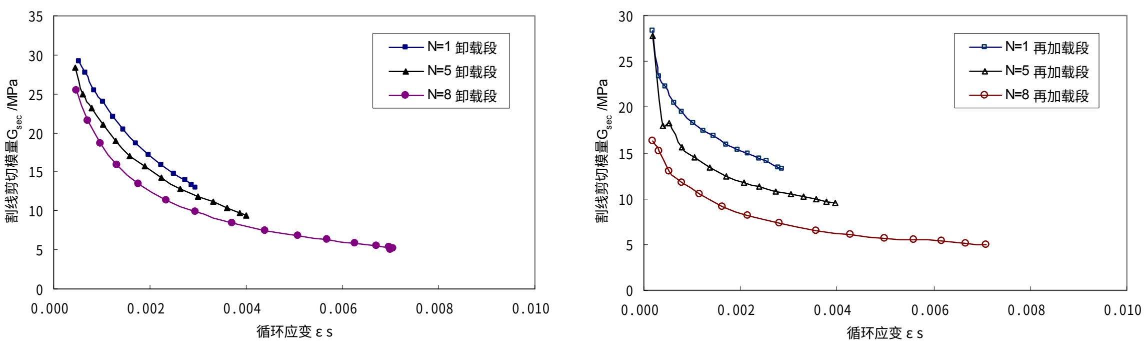


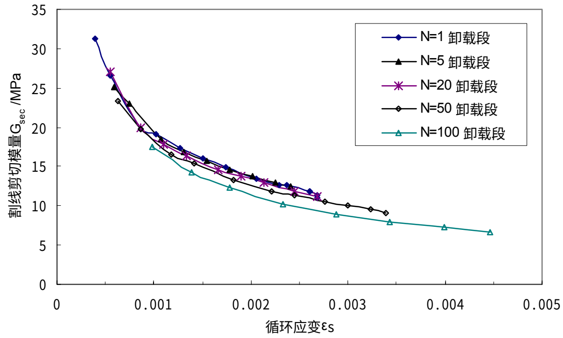
Figure 2 Definition of unloading-reloading secant shear modulus



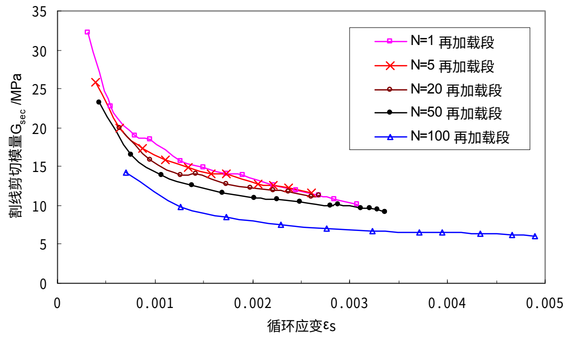
(a) Unloading stage (b) Reloading stage

Figure 3 The degradation curves of unloading and reloading secant modulus on flake-shaped Nanjing fine sand from cyclic triaxial tests with static deviator stress qs=0

Figure 3 to Figure 5 are degradation curves of dynamic shear modulus of Nanjing flake-shaped sand under cyclic loading when static deviator stress is 0, 25 and 50kPa respectively. The results show that Nanjing flake-shaped sand occurred degradation of dynamic shear modulus under cyclic loading and dynamic shear modulus reduces with cyclic number increasing. As for each cycle, secant shear modulus of unloading and reloading curve quickly reduced as cyclic strain increases. When the cyclic number is large, the attenuation rate of dynamic shear modulus is smaller with strain increasing and finally become moderate gradually. Also, secant shear modulus both unloading and re-loading meets exponential function relation. Shumbhu and WangJun get similar conclusion and so this paper only studied the degradation of dynamic shear modulus of NanJing flake-shaped sand during unloading.

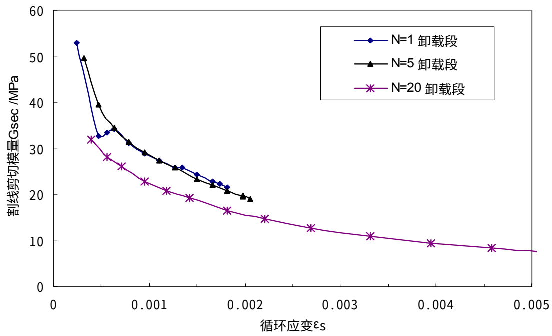


(a) Unloading stage

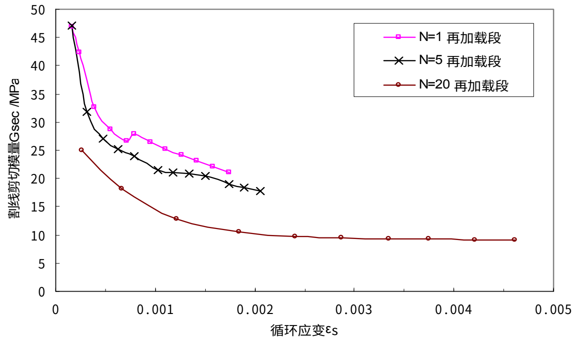


(b) Reloading stage

Fig.4 The degradation curves of unloading and reloading secant modulus on flake-shaped Nanjing fine sand from cyclic triaxial tests with static deviator stress qs=25kPa



(a) Unloading stage



(b) Reloading stage

Fig.5 The degradation curves of unloading and reloading secant modulus on flake-shaped Nanjing fine sand from cyclic triaxial tests with static deviator stress qs=50kPa

Dimensionless modulus ratio G/Gmax and cyclic strain curve was shown in Figure 6. It is clearly that the relation between dynamic shear modulus ration and strain can be described by an exponential function, as following equation 3.5.

$$G_{sec}/G_{max} = \alpha \varepsilon_s^{\beta} \tag{3.5}$$

Where, α and β are experience parameters

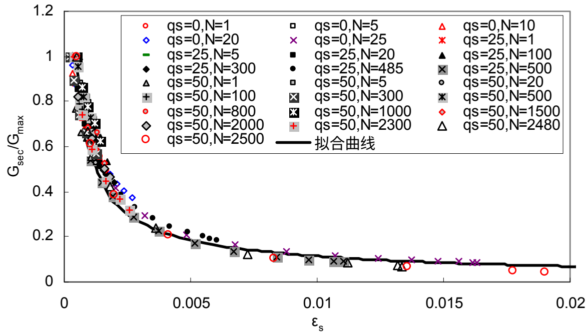
Meanwhile, the regressive analysis shows that the parameter value of function of $G_{sec}/G_{max} \sim \varepsilon_s$ is similar under different stress level. And it shows that cyclic stress level has little effect on dynamic shear modulus G_{sec}/G_{max}. An empirical formula of degradation of dynamic shear modulus on flake-shaped Nanjing fine sand are given by average all parameters, as the following equation.

$$G_{sec}/G_{max} = 0.0054\varepsilon_s^{-0.71} \tag{3.6}$$

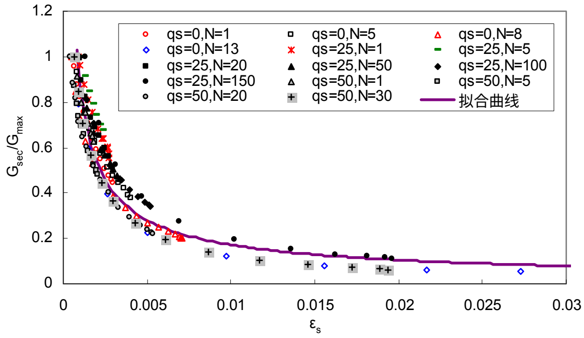
Figure 7 is the relationship curve between maximum dynamic shear modulus G_{max} of the unloading curve and the cyclic number N. Similarly, an exponent function can be used to describe the relation between $G_{\max}$ and N as the following equation 6. Under semilog coordination, the curve between $G_{\max}$ and $\ln N$ has a linear relationship. As static deviator stress increase, maximum dynamic shear modulus increase and curves of $G_{\max}$ keep parallel under different deviator stress, but have a decreasing trend with the increase of the cyclic number N. So, it shows that static deviator stress is a parameter which affect on maximum dynamic shear modulus.

$$G_{\max} = \alpha_1 N^{\beta_1} \tag{3.7}$$

α_1 and β_1 are parameters related to q_s and N is the cyclic number



(a) $q_{cyc} = 35kPa$



(b) $q_{cyc} = 50kPa$

Figure 6 The degradation curves of unloading secant modulus on flake-shaped Nanjing fine sand from cyclic triaxial tests

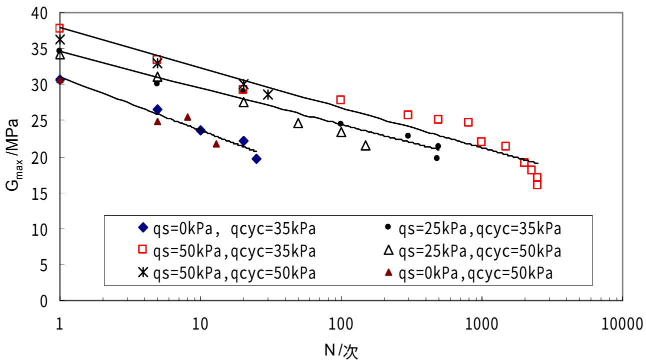


Figure 7 Variation of $G_{\max}$ with increasing number of cycles

Figure 8 is the relationship curves of q_s with α_1 and β_1 under different static deviator stress conditions. The relationship between q_s and α_1, β_1 is in closely approximation 1o linearity, and can be described with Eqn. 3.8.

$$\alpha_1 = A + Bq_s \tag{3.8a}$$
$$\beta_1 = C + Dq_s \tag{3.8b}$$

Where A, B, C and D were fitting parameters. By the regression analysis, their values were $A = 29.902$, $B = 0.1552$, $C = -0.0885$ and $D = 0.0002$ respectively. The maximum dynamic shear modulus can be formulated as the following empirical formula.

$$G_{\max} = (A + Bq_s)N^{(C+Dq_s)} \tag{3.9}$$

Finally, dynamic shear modulus was calculated by using the following empirical formula.

$$G_{\sec} = (0.1615 + 0.00084q_s)N^{(0.0002q_s - 0.0885)}\varepsilon_s^{-0.71} \tag{3.10}$$

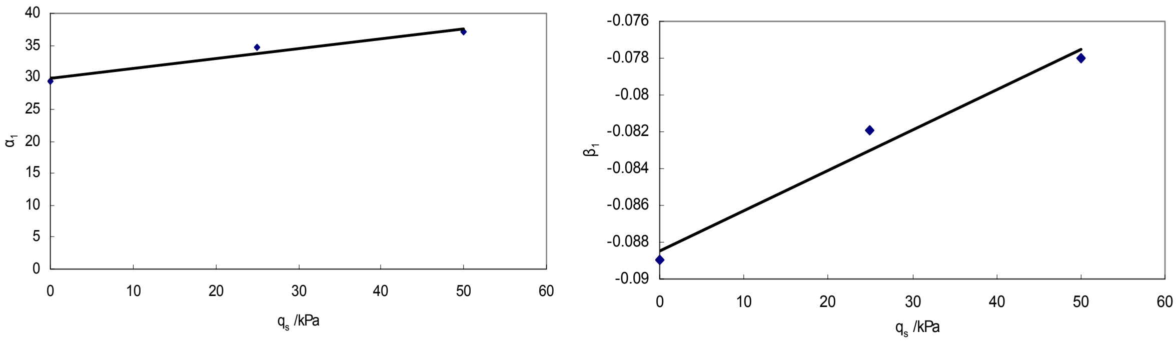


Figure8 Relationship between α_1, β_1 and q_s

4. DYNAMIC STRESS-STRAIN RELATIONSHIP OF NANJING FLAKE-SHAPED FINE SAND CONSIDERING SOFTENING EFFECT

M. Mohajeri (2003) modified Masing's equation and proposed a hyperbolic model which can describe dynamic stress-strain relationship of silty clay and sand. But this model did not consider the change law of secant line shear modulus in each cyclic. So, using method proposed by Mohajeri and considering change law of secant shear modulus by Eqn.3.10, a hyperbolic model was used to describe dynamic stress-strain relationship of flake-shaped Nanjing fine sand.

The method to build the dynamic constitutive equation based on modified Mohajeri's hyperbolic model is introduced as follows.

(1) First loading stage, $0 \le \varepsilon < \varepsilon_{a\max}$

$$q = \frac{3G_0\varepsilon}{1 + \dfrac{3G_0\varepsilon}{R}} \tag{3.11}$$

(2) Unloading stage, $\varepsilon_{a\min} \le \varepsilon < \varepsilon_{a\max}$

$$q = q_{\max} + \frac{3G_0 n_1(\varepsilon - \varepsilon_{a\max})}{1 - \dfrac{3n_1G_0}{R}(\varepsilon - \varepsilon_{a\max})} \tag{3.12}$$

(3) Re-loading stage, $\varepsilon_{a\min} \le \varepsilon < \varepsilon'_{a\max}$

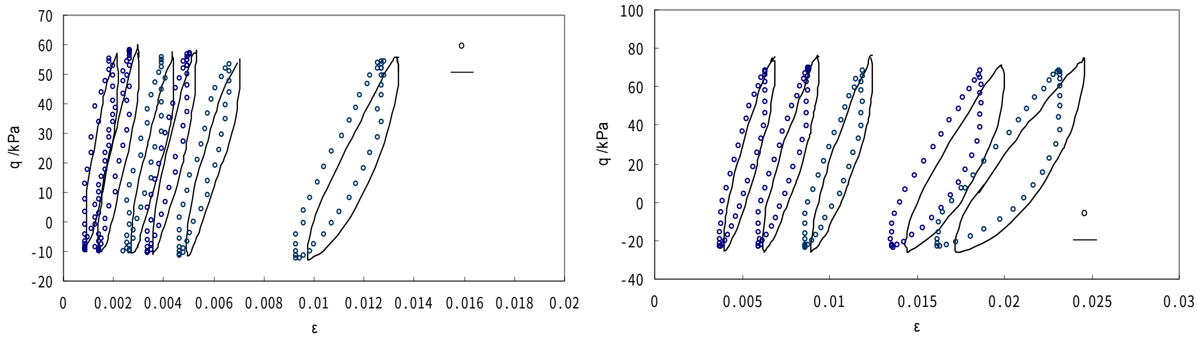
$$q = q_{\min} + \frac{3G_0 n_2(\varepsilon - \varepsilon_{a\min})}{1 - \dfrac{3n_2G_0}{R}(\varepsilon - \varepsilon_{a\min})} \tag{3.13}$$

where, G_0 is shear modulus under small strain from free-vibration column test, $G_0 = 58MPa$ in this paper. R is peak strength of soil. $\varepsilon'_{a\max}$ is maximum strain value of re-loading stage. Parameters n_1 and n_2 were calculated from Eqn. 3.14 and Eqn. 3.15, respectively.

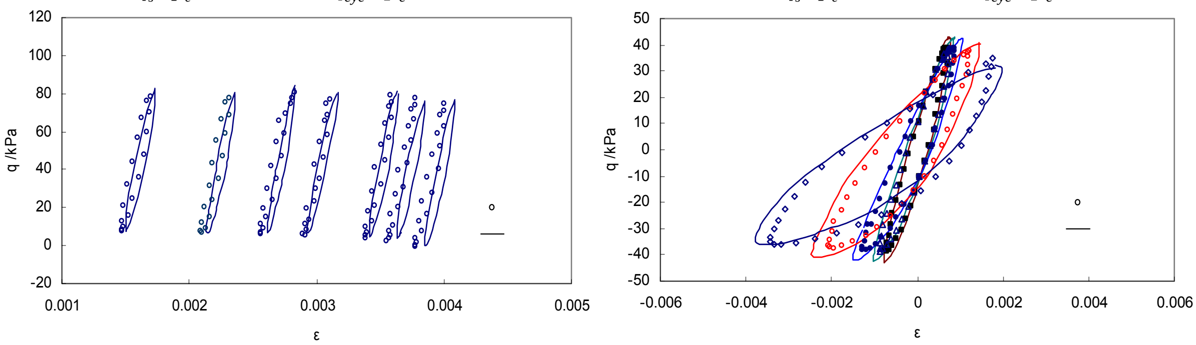
$$n_1 = \frac{R(q-q_{\max})}{3G_0(\varepsilon-\varepsilon_{a\max})[R+(q-q_{\max})]} = \frac{RG_{\sec}}{G_0(R+q-q_{\max})} = \frac{RG_{\sec}}{G_0(R+3G_{\sec}\varepsilon_s)} \tag{3.14}$$

$$n_2 = \frac{R(q-q_{\max})}{3G_0(\varepsilon-\varepsilon_{a\min})[R-(q-q_{\min})]} = \frac{RG_{\sec}^{'}}{G_0[R-(q-q_{\min})]} = \frac{RG_{\sec}}{G_0(R-3G_{\sec}^{'}\varepsilon_s)} \tag{3.15}$$

Figure 9 is the hysterics curve of relationship between dynamic stress and strain about Nanjing flake-shaped fine sand based on test results. By the comparison between test results and calculation results with hyperbolic model considering softening effect, the calculating results agree well with experimental ones when the cycle number is small. But when the cycle number become large, both of them had some difference and strains of calculation result developed more quick than those tests. Deflection of the hysterics curve shows that softening phenomenon of Nanjing flake-shaped fine sand occurred under cyclic loading and this hyperbolic model with considering softening effect is effective and feasible. Meanwhile, both test and calculate results show that strain and plastic strain increased gradually with the increasing of the cycle number and the hysterics curve gradually separated and the area of hysterics loop increased but the shape was similar. The hysterics loop did not separate because plastic strain of soil did not accumulate for no static deviator stress situation. However, the dynamic constitutive relationship characters are similar as the static deviator stress condition. It is worthy to say that this paper only gave the results before the failure of the test and it is difficult to simulate large deformation after liquefaction.



(a) $q_s/p_c^{'}=0.23$ and $q_{cyc}/p_c^{'}$=0.32 (b) $q_s/p_c^{'}=0.23$ and $q_{cyc}/p_c^{'}$=0.46



(c) $q_s/p_c^{'}=0.43$ and $q_{cyc}/p_c^{'}$=0.30 (d) $q_s/p_c^{'}=0$ and $q_{cyc}/p_c^{'}$=0.35

Figure9 Dynamic stress-strain relationship of flake-shaped Nanjing fine sand

4. CONCLUSIONS

This paper analyzed softening character of dynamic shear modulus of NanJing flake-shaped fine sand under cyclic loading by undrained triaxial tests. The change law of dynamic shear modulus was analyzed during unloading and re-loading curves. And based on modified Masing's rule, considering modulus softening effects, a new stress-strain relationship model was built to describe dynamic stress-strain relationship of Nanjing

N=5 20 100 180 300 450 试验 计算

flake-shaped fine sand. Some conclusions are as follows:

(1) The strain softening occurred when Nanjing flake-shaped sand subjected to cyclic loading and dynamic shear modulus reduced with increase of cyclic number. As for each cyclic, secant shear modulus of unloading and reloading curves quickly reduced as cyclic strain increased. When the cyclic number is large, the attenuation rate of dynamic shear modulus is smaller with strain increasing and finally tends to be stable. Secant shear modulus of unloading and re-loading curve meets exponential function relation.

(2) The calculating results agree well with experimental ones when the cycle number is small. But when the cycle number became large, both of them had some difference and strains of calculation result developed more quick than those tests.

(3) Strain and plastic strain increased gradually with the increasing of the cycle number and hysterics curve separated gradually. And the area of the hysterics loop increased but the shape was similar. The hysterics loop did not separate because plastic strain of soil did not accumulate for no static deviator stress situation. However, the dynamic constitutive relationship characters are similar as the static deviator stress condition.

(4) Softening phenomenon of Nanjing flake-shaped fine sand occurred under cyclic loading and this hyperbolic model with considering softening effect is effective and feasible.

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