VARIATION OF DYNAMIC PROPERTIES WITH DISPLACEMENT IN A 3-STORY REINFORCED CONCRETE FLAT PLATE STRUCTURE

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ABSTRACT :

A full-scale three-story reinforced concrete flat-plate building was subjected to a series of quasi-static load cycles. At various stages of the quasi-static test dynamic tests in the forms of free and forced vibrations, ambient vibrations and modal impact-hammer tests, were performed to collect low-amplitude dynamic response data from the building. This paper presents results from the structure after three different drift levels during the quasi-static test. These drift levels are: as-built/zero drift, 1.5% overall drift and 3.0% overall drift (damaged condition). Data on natural frequencies and damping ratios are provided and discussed. It is shown that at low displacement levels the structure exhibits non-linear elastic behavior regardless of its past displacement level. Consequently, natural frequency is a displacement-amplitude-dependent parameter in the range of low displacement amplitudes. It is recommended that comparison of natural frequency before and after an event be made using results from tests with same displacement amplitude. The equivalent viscous damping ratio was observed to remain nearly constant in the range of low displacements while being sensitive to past maximum displacement level. However, once the structure reached damaged condition, damping ratio became sensitive to the displacement level of the dynamic tests as well. More data are needed to conclude whether a robust health monitoring technique based on damping measured at small amplitudes could be suggested for reinforced concrete structures.

KEYWORDS: Dynamic properties, damping, small amplitudes, health monitoring

1. INTRODUCTION

Vibration tests provide information about the dynamic properties of structures. To list the most important, natural frequencies (modal frequencies), mode shapes, shear-wave velocity and damping in a structure could be obtained using vibration tests. Examples of such observations can be seen in the literature [for example, Trifunac 1972; Gulkan, et al. 1974; Kaminoso, et al. 1985, and Snieder 2006]. Modal frequencies and mode shapes are the typical choice of properties used in investigating the integrity of a structure, i.e., in so-called structural health monitoring/structural damage detection studies [Doebling et al. 1996].
Ambient vibration is the easiest type of data to acquire from an existing structure. An effective damage identification technique should be able to extract the information from such small amplitude vibrations. This paper presents estimates of natural frequencies and damping ratios using measurements obtained from a full-scale 3-story reinforced concrete flat plate building at small amplitude vibrations.

1.1. Description of the Building

A full-scale two-spans-by-one-span three-story flat-plate structure is the specimen used in this research (see Figure 1). The building was built and tested at the Robert L. and Terry L. Bowen Laboratory for Large Scale Civil Engineering Research at Purdue University [Fick, 2008]. The 30-ft tall building has six columns spaced at 20 ft in each direction. The 7-in thick concrete slabs cantilever 5 ft around the perimeter of the structure. The slab reinforcement was designed and detailed [Fick, 2008] to meet 2002 ACI code requirements [ACI Committee, 2002]. A superimposed gravity load was added to the structure to represent an approximate load of an occupied building. This load was applied by filling 55-gallon drums with water and placing them uniformly on the slab.

Figure 1 General view of the test specimen (without the superimposed gravity load)

1.2. Load Cycles

Lateral loads were applied to the test specimen along its longer plan axis through steel load frames along the edge of the slab at each story. Six hydraulic actuators (two at each level) were connected to the strong wall to provide axial load to the steel load frames. Figure 2 shows the load-displacement relationship in the building represented by the total base shear coefficient versus roof drift [Fick, 2008]. Failure produced by punching shear of the column into the slab at the third level and at a roof drift of about 3% is illustrated in Figure 3.

1.3. Dynamic Tests

Ambient vibration, free vibration, forced vibration, and impact-hammer tests were performed in the building at low-amplitude vibration levels. During the tests, the maximum drift was limited to the overall drift at which initial cracking would be produced in the columns. Accordingly, the structure was considered to behave elastically during the dynamic tests. The initial cracking drift level can be computed by considering flexural and axial deformation on a

laterally distorted column with restrained rotation at its ends. Given the columns (10ftX18inX18in) and their axial load demand (0.05*f'c*Ag), the lateral displacement at which each column would start exhibiting cracks is 0.10in (0.03% drift). The tests on the pristine condition were restricted to one-fifth of that limit.
Ambient vibration records were collected at the beginning and the end of each set of dynamic tests. Free vibration tests were carried out by pull/release of the building at each floor. Forced vibration tests were done by using a linear actuator located at the top of the structure. Impact-hammer tests were carried out by driving the force on a given floor and measuring the response on all three floors.
Dynamic tests were conducted after three different drift levels in the structure: (a) as-built structure (named here as pristine condition), (b) after the structure experienced $\pm 1.5\%$ roof drift, and (c) after the structure experienced $\pm 3.0\%$ roof drift (named here as damaged condition). These last two displacement levels correspond to cycles 3 and 4 from Figure 2 respectively. To avoid interrupting the quasi-static tests, no dynamic tests were made after cycles 1 and 2.

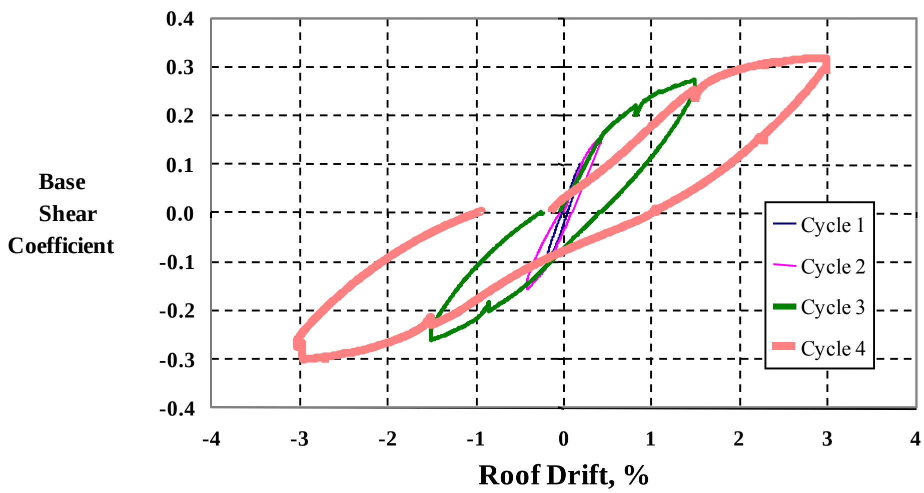

Figure 2 Quasi-static loading cycles [Fick, 2008]

Figure 3 Punching shear failure of slab-column connection

2. MATHEMATICAL MODEL AND EXPERIMENTAL RESULTS

Comparison of results from a finite element model and actual results on the pristine condition was made. Use of the mathematical model helped in distinguishing which of the natural frequencies measured (i.e. peaks in the Fourier Spectra) corresponded to those for the response in longitudinal direction.
A three-dimensional model was developed in SAP2000NL structural analysis software [Computers and Structures, 1998], by using shell elements for the floor systems (8,820 elements) and flexural elements for the

columns (180 elements). Modulus of elasticity of concrete was taken as $E = 3,850ksi$ [ACI Committee, 2002], according to the cylinder compressive strength of $f'_c = 4,000psi$ measured on the day quasi-static test started. Figure 4 illustrates the first three mode shapes and natural frequencies in the longitudinal direction from the SAP2000NL model.
Actual natural frequencies of the building computed from free vibration test can be seen in Figure 5 (for the pristine condition). A Fourier spectrum of the response when the building is pulled and released at second floor shows three natural frequencies in the longitudinal direction. Maximum difference between actual values and those estimated by the finite element model is less than 15%.

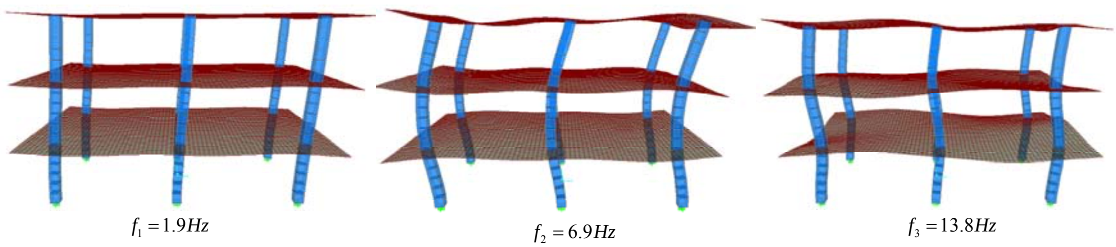

Figure 4 Mode shapes and natural frequencies in the longitudinal direction, from SAP2000NL model

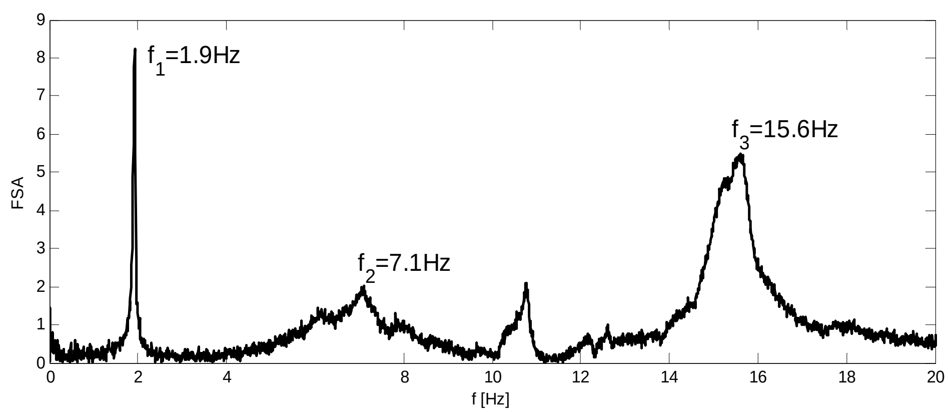

Figure 5 Fourier spectrum of free vibration response at second floor

3. NONLINEAR ELASTIC BEHAVIOR

A nonlinear elastic behavior in the structure was identified at each of the drift levels considered in this work. Instantaneous frequency computed via Hilbert transform [Bendat and Piersol, 1986] was used to obtain time-frequency distribution of the building response and to investigate the nonlinearity in its stiffness. According to the Hilbert transform, the analytical signal $x(t)$ is expressed as a function of instantaneous amplitude *A(t)* and phase angle $\theta(t)$, as $x(t) = s(t) + i\tilde{s}(t) = A(t)e^{i\theta(t)}$,; where $s(t)$ is the original time-history record and $\tilde{s}(t)$ is its Hilbert transform. Instantaneous frequency is obtained as time derivative of the phase angle.
Figure 6 shows the instantaneous frequency computed for the roof displacement obtained through forced vibration of the structure in pristine condition. During the first 10 seconds of the response the system is in steady-state, then the actuator is stopped and 10 more seconds were recorded in free vibration. Two different tests named as *test1* and *test2* with displacement-amplitude of 0.003in and 0.010in respectively, are shown. It can be seen that instantaneous frequency computed during free vibration part is larger for the smaller displacement-amplitude level (*test1*), as typical for a nonlinear-softening system. However, it must be said that this is a weak stiffness-nonlinearity.

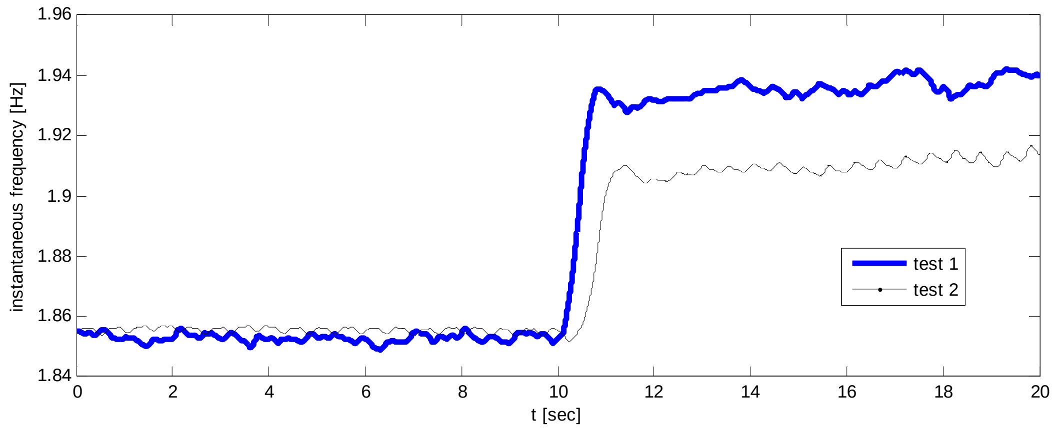

Figure 6 Time-frequency distribution during forced vibration test

Evidence of stronger stiffness-nonlinearity behavior in the structure is illustrated in Figure 7. Free vibration response of the roof after the structure has experienced $\pm 3.0\%$ roof drift (damaged condition) is shown. The *actual* displacement response of the building is simulated using two models: by assuming linear behavior (Figure 7a) and nonlinear elastic behavior (Figure 7b) for the stiffness of the structure. In order to simulate the response, solution to the dynamic equation $m\ddot{x} + c\dot{x} + kx = 0$ is obtained for the initial conditions of the experiment. For the linear stiffness model, a stiffness value equal to the stiffness measured in the test at the instant of building release is used. For nonlinear elastic stiffness model, the stiffness is computed from the actual response at each time instant through the instantaneous frequency $f(t)$ as $k = 4\pi^2 f(t)^2 m$, where m is the mass of the system. It is evident that the nonlinear elastic assumption matches better the actual response than the linear assumption.

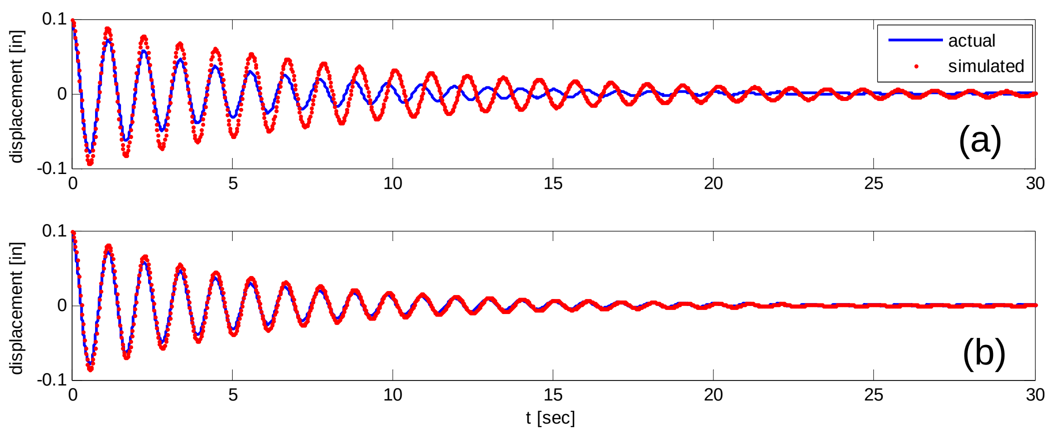

Figure 7 Actual and simulated free vibration response assuming (a) linear elastic stiffness behavior, (b) nonlinear elastic stiffness behavior

4. DAMPING MEASUREMENTS IN THE STRUCTURE

Two different methods were used to determine the viscous damping ratio ξ in the structure: *logarithmic decrement* from free vibration response and *curve-fit* in the frequency response function (FRF) from forced vibration tests. The damping ratio for the free vibration decays is defined by the amplitude of the response at cycles i and $i+n$ as $\xi = \frac{1}{2\pi n} ln(\frac{u_i}{u_{i+n}})$. An FRF is defined by the ratio between the response amplitude and the force magnitude driven at a given frequency. When the output measured is displacement, the magnitude of the analytical FRF for a SDOF is given by

$$FRF(\Omega) = \frac{1/k}{\sqrt{\left[1-(\Omega/\omega)^2\right]^2 + \left[2\xi\Omega/\omega\right]^2}} \qquad (4.1)$$

where k, ω, and ξ are stiffness, natural frequency, and damping ratio, respectively. Estimation of damping ratio from forced vibration results has been made by fitting the analytical FRF to the experimental results. Such procedure is referred here as curve-fit method to estimate damping. Values of damping ratio obtained like this are in excellent agreement with those estimated by half-power method from the same FRF.

Logarithmic decrement and *curve-fit* in the FRF are both defined for linear systems. Any deviation of the actual response from a linear system would produce inconsistency in the damping measurements. In order to determine damping consistently from forced vibrations, FRFs were obtained for a constant displacement response. Thus, stroke of the actuator was modified on each frequency step to reach a preset target displacement response in the building. This procedure leaded to a 'linearization' of the nonlinear elastic system by oscillating within a constant displacement-amplitude. Such linearization cannot be done in the free vibration response since there is no control over the vibration-amplitude as the oscillation decays (as seen in Figure 7).

Figure 8 shows a comparison of the damping values estimated at the pristine condition and after $\pm 1.5\%$ roof drift. Figure 8 (a) and (b) correspond to logarithmic decrement and curve-fit methods on the pristine condition respectively. Figure 8 (c) and (d) correspond to logarithmic decrement and curve-fit methods after $\pm 1.5\%$ roof drift respectively.

The envelope of free vibration response for a linear system, i.e. $e^{-\xi\omega_n t}$, is drawn in Figure 8 (a) and (c) to illustrate how much the actual responses deviate from those for linear systems. It is observed stronger nonlinearity in the free vibration response of the structure after $\pm 1.5\%$ roof drift. Figure 8 (b) and (d) show both a good fit of the analytical FRF on the experimental data which proves the efficacy of the linearization procedure. Values of damping ratio estimated by each method seem to be more consistent on the pristine condition than after $\pm 1.5\%$ roof drift. This situation could be due to the weaker nonlinearity observed in the pristine condition for which the damping estimated by *logarithmic decrement* is closer to that of a linear system. On the other hand, due to stronger nonlinearity in the response after the structure has experienced $\pm 1.5\%$ roof drift, damping ratio estimated by *logarithmic decrement* (Figure 8 (c)) does not compare very well with that of the 'linear' system obtained through "constant peak-displacement" forced vibration FRF (Figure 8 (d)).

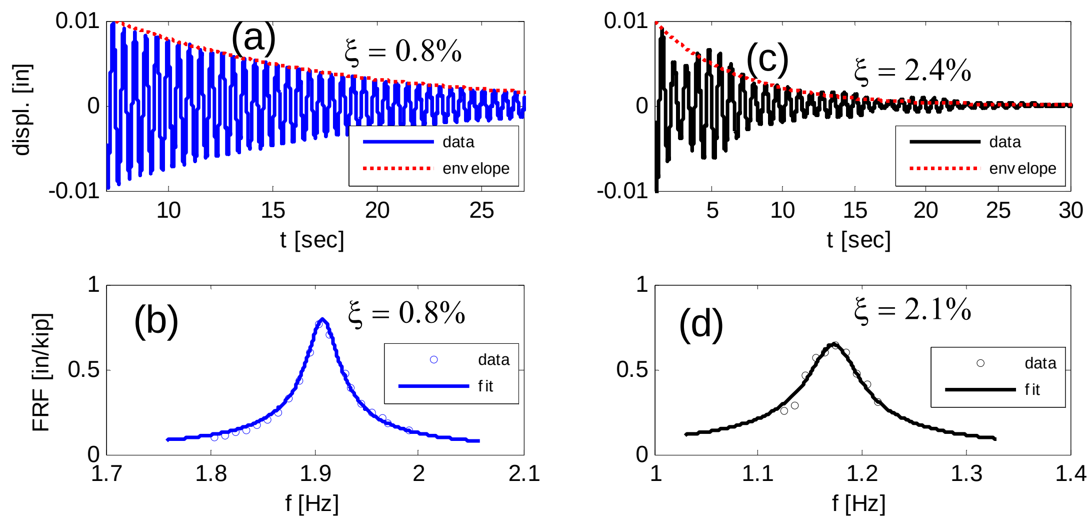

Figure 8 Damping comparison. Measurements from free and forced vibrations respectively: (a) and (b) at pristine condition; (c) and (d) after $\pm \mathbf{1.5}\%$ roof drift

Table 4.1 shows a complete list of damping estimations at different displacement amplitudes after the structure experienced $\pm 1.5\%$ roof drift. Values from logarithmic decrement and curve-fit methods are illustrated. Values of damping ratios computed by curve-fit method might suggest existence of a low-amplitude plateau for damping in reinforced concrete members [Jeary, 1996].

Table 4.1 Damping Measurements after $\pm 1.5\%$ roof drift

	ξ [%]	
Δ_roof [in]	log. dec	curve-fit
0.01	2.4	2.1
0.02	2.4	2.0
0.04	2.4	2.1
0.10	2.9	2.0

Estimation of damping from curve-fit method on constant-peak-response FRF is more consistent with the formulation of a linear system (equation 4.1). For this reason the values of damping ratios obtained using curve-fit method will be used hereafter.

5. VARIATION OF NATURAL FREQUENCY AND DAMPING

A summary of fundamental frequencies and damping ratios of the building for its fundamental translational mode along its longitudinal axis and measured in the range of small amplitudes can be seen in Table 5.1. Natural frequency is observed to change as much as 45% in the range of low amplitudes for a given drift level, indicating the extreme dependency of natural frequency to the dynamic test displacement amplitude. It is seen that when the dynamic tests are done before and after an event for the same low-amplitude displacement, the natural frequency is sensitive to the large overall drift level. If testing at identical dynamic displacement level approach is not followed, erroneous conclusions could be made. For example, comparison of fundamental frequency measured at 0.10-in drift after experiencing 1.5% overall drift and measured at 0.01-in after experiencing 3.0% overall drift (i.e. damaged condition) would lead to indifference at best.

Damping ratio was observed to be nearly constant within the range of low displacement amplitudes in the structure after it has experienced 1.5% roof drift. However no such observation could be made after 3.0% roof drift in the building. It should be noted that 3.0% drift level corresponded to a shear failure of the structure which might produce important variations in the damping mechanism of the structure.

Table 5.1 Summary of fundamental frequency and damping ratio estimates for fundamental mode in the longitudinal direction

	Δ_roof [in]				
	< 0.01(ambient)	**0.01**	**0.02**	**0.04**	**0.10**
Pristine Condition					
f [Hz]	2.00	1.91			
ξ [%]	-	0.8			
1.5% Overall drift					
f [Hz]	1.48	1.17	1.15	1.06	1.02
ξ [%]	-	2.1	2.0	2.1	2.0
3.0% Overall drift (damaged)					
f [Hz]	1.08	0.96	0.92	0.89	0.84
ξ [%]	-	2.2	2.6	2.5	2.5

6. CONCLUSIONS

A full-scale 3-story reinforced concrete building has been subjected to a series of quasi-static loading cycles from its pristine condition to the point where a punching shear failure happened in one of the slab-column connections (damaged condition). Dynamic tests in the range of small displacement amplitudes were performed after three different drift levels. Nonlinear elastic behavior of the structure in the range of low amplitudes was evidenced by fluctuation of the natural frequency during the free vibration tests and when running forced vibration tests at different displacement amplitudes.

Natural frequency and damping ratio were seen to be sensitive parameters to damage in the structure. However, it was shown that to avoid misleading estimates through comparison of natural frequency before and after an event, the data need to be collected via dynamic tests that use identical target displacement amplitude. Such an approach would be more reliable for health monitoring purposes given the nonlinear elastic behavior of the structure even at low displacement amplitude levels. Damping ratio in the range of small amplitudes might be a constant parameter for a given past maximum drift-level condition. However further data are needed to investigate if such constant-amplitude damping plateau exists for reinforced concrete structures, and if and how other details of past loading history might affect the damping estimates. Such data would also help conclude if damping ratio could be used as a robust measure to estimate past maximum drift levels in a building.

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