

# Inelastic earthquake spectra ----- damage spectra

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# **ABSTRACT :**

In this paper, firstly, a procedure of analysis of inelastic response spectra based on Park and Ang damage model, called  $R_D$  spectra, is proposed. Secondly, considering ultimate limit state of structures and due to programming DBDS, a set of  $R_D$  spectra are obtained.  $R_D$  spectra consider both the effects of maximum inelastic displacement and the effects of cumulative hysteretic energy, fairly according with the actual inelastic behavior due to ground motions. Thirdly, corresponding to lots of time history analysis of SDOF oscillators and regress of the datum, an expression about  $R_D$  spectra is presented, which can be used in seismic evaluation of structures in the future. Finally, the proposed  $R_D$  spectra are compared with  $R_\mu$  spectra in qualitative, and the comparison shows the importance of considering the duration of ground motions.

**KEYWORDS**: damage index, inelastic response spectra, cumulative energy, ductility factor, duration.

# **1. INTRODUTION**

As known to all, elastic spectra theory is very grown-up and widely used in the world as one of methods for seismic design. The newly executing Seismic Design Code<sup>[i]</sup> in China, expresses "three levels and two stages" for the requests of seismic fortification. During the second and third fortification intensity level, seismic fortification criterion allows the structure to exhibit the inelastic deformation to dissipate energy due to ground motions. At present, many countries have been taking their efforts to the new seismic design theory—performance-based seismic design (PBSD), which have been used to some actual engineering projects in the U.S., Japan and China, etc. As proved, this theory will be the new direction of seismic design theory in future. Pushover analysis method (POA) is very popular in PBSD, which is often used, combining with capacity spectra method (ATC40 or FEMA273) which needs inelastic response spectra, to figure out the demands of structures. Therefore, it makes great sense to research inelastic response spectra of structures subjected to ground motions.

In the beginning of 1950s, research about inelastic response spectra was firstly evolved. From then on, many relative results had been obtained. Some famous research results are the achievements of Housner<sup>[ii,iii]</sup>, Newmark & Veletos<sup>[iv]</sup>, Newmark & Hall<sup>[v,vi]</sup>, Fajfar & Bertero<sup>[vii-ix]</sup>, etc. From 1965, the similar research was presented by many researchers in China, such as Qianxin Wang<sup>[x]</sup>, Chen Dan<sup>[xi]</sup>, Chengji Wei<sup>[xii]</sup>, Minxiang Chen<sup>[xiii]</sup> and so on. They obtained all kinds of inelastic response spectra, respectively. Recently, Mingkui Xiao<sup>[xiv,xv]</sup>, Xilin Lu<sup>[xvi]</sup> and Changhai Zhai<sup>[xvii,xviii]</sup> also achieved corresponding simplified formulas due to lots of time history analysis (THA) of SDOF oscillators and regress of their datum, which can be offered to the application in seismic estimation of structures. But it is deserved to mention that these results as mentioned above have some deficiencies of their own. In other words, those either can not reflect the effects of the maximum inelastic deformation or effects of cumulative damage. Nowadays, with the further research of seismic design theory, people have achieved a common sense about the destroy criterion of structures. In 1985, Park & Ang<sup>[xi,xxi]</sup> provided a damage model considering both two factors, which is widely used in the world until now, for its according with the double parameters destroy criterion. Therefore, in this paper, a procedure of analysis of inelastic response spectra based on Park & Ang damage model, called  $R_D$  spectra, is presented.  $R_D$  spectra make contribution to handle the weaknesses of previous kinds of inelastic response spectra.



# 2. INELASTIC EARTHQUAKE SPECTRA-RD SPEATRA

## 2.1 Damage Model and Description of RD Spectra

In 1985, Park & Ang provided a double parameters damage model based on many experiments of structural components, which is comparatively in accord with the destroy behavior of structures and so widely used in the world until now. The model is as follows :

$$D = \frac{x_m}{x_{cu}} + \beta \frac{E_h}{F_v x_{cu}}$$
(2.1)

where  $x_{cu}$  = ultimate displacement capacity of the components under monotonic loading;  $F_y$  = yield strength of the component;  $x_m$  = actual peak response displacement;  $E_h$  = plastic strain energy dissipated by the component, called hysteretic energy and  $\beta$  is a constant representing the rate of cumulative damage through hysteretic energy due to cyclic loading.  $\beta$  may be associated with the strength degradation in the hysteretic behavior. For poor detailed systems, it should take high value, otherwise takes low value. Its variation is from 0.025 to 0.25 normally. From which, Park *et al.*<sup>[xix]</sup> suggested a rather small value of 0.05 for reinforced concrete structures. Since cumulative damage is dependent on the  $\beta$  value, a small  $\beta$  value leads to a low damage index. Therefore, RC structures that possess high  $\beta$  value are subjected to high damage due to a high proportion of cumulative damage. The value of  $\beta$  =0.15 is chosen as a mean value to represent typical reinforcement details<sup>[xxi-xxiii]</sup>.

Equation (6) can be rewritten as

$$D = \frac{\mu_m}{\mu_{cu}} + \beta \frac{E_h}{F_v x_v \mu_{cu}}$$
(2.2)

where  $x_y$  = yield displacement;  $\mu_m$  and  $\mu_{cu}$  corresponds to the actual maximum displacement ductility and the monotonic displacement ductility, respectively.

As for the elastic-perfectly plastic (EPP) SDOF system (figure 1), introduce a  $R_D$  factor based on the damage index as mentioned above, which is defined as

$$R_{D} = \frac{F(\mu = 1)}{F(D = D_{i}, \mu = \mu_{cu})} = \frac{F_{e}}{F_{v}}$$
(2.3)

where  $F(\mu=1)$  =minimum strength value to keep elastic state due to the ground motion, i.e. the maximum elastic strength value  $F_e$ ;  $F(D=D_i,\mu=\mu_{cu})$  = minimum strength value, under prescribed ultimate ductility  $\mu_{cu}$ , to control the damage index *D* to a specified target damage  $D_i$  due to the same ground motion.

In terms of  $R_D$  factor definition of equation (9),  $F_y$  and  $x_y$  can be expressed as follows:





$$F_e = mS_a \tag{2.4}$$

$$F_{y} = \frac{F_{e}}{R_{p}} = \frac{mS_{a}}{R_{p}}$$
(2.5)

$$x_{y} = \frac{F_{y}}{k} = \frac{S_{a}}{R_{D}} \cdot (\frac{T}{2\pi})^{2}$$
(2.6)

where  $S_a$  is the value from the elastic response spectra for the ground motion,  $\ddot{x}_g$ ; *m* is the mass of system; *k* is the initial stiffness and *T* is initial elastic period.

After substituting (2.5) and (2.6) into (2.2), the relationship of  $R_D - \mu_{cu} - T - D_i$  is established as

$$D_{i} = \frac{\mu_{m}}{\mu_{cu}} + \beta \left[ \frac{E_{h}}{m(\frac{S_{a}T}{2\pi R_{D}})^{2} \mu_{cu}} \right]$$
(2.7)

Therefore, given  $\beta$ ,  $\mu_{cu}$  and specified *T*,  $R_D$  value can be figured out under all levels of  $D_i$  values by THA method. Because the relation accords with *T*, it can be expressed in the form of spectra. Especially, for considering the ultimate limit state of structures, set  $D_i = 1.0$ , which is substituted to (2.7), it is finally built up the relation of  $R_D$  - $\mu_{cu}$  –*T*, called  $R_D$  spectra, in order to be used in the seismic evaluation of structures in PBSD. The procedure will be presented in details in the following.

## 2.2 Earthquake Damage Levels and Damage Index Range

In general, the structural damage states due to earthquake can be classified into five levels: negligible damage, minor damage, moderate damage, severe damage and collapse. Table 1 gives the relation between damage levels and damage index range based on Park & Ang damage model<sup>[xxiv]</sup>.

damage levels	damage levels description			
negligible damage	Some of beams or columns have minor non-continuous cracks; some of walls have minor cracks, only needs slightly repair for occupation.	0~0.20		
minor damage	Some of beams or columns have minor continuous cracks; the cover of some joints is gone; most of walls have continuous cracks; easy to recover	0.20 ~ 0.40		
moderate damage	Cracks arise around both ends of columns; concrete crushes and steel bars reveal; severe cracks in joints; beams broken; walls have severe cracks or cracks expanding; hard to repair.	0.40 ~ 0.60		
severe damage	Ends of columns crushed; steel bars yielding; beams and planks broken; joints crushed and bars revealed; walls collapsed	0.60 ~ 0.9		
collapse	Main components broken, collapse or wholly collapse; all function missing	>0.9		

# 2.3 Selections of Structural Dynamic Parameters

In order to establish  $R_D$  spectra of SDOF systems through THA method, consider the parameters of systems as follows: (1) totally computed 60 SDOF oscillators with the initial period *T* from 0.05 to 3.0s; the viscous damping ratio  $\xi$  is assumed to be 5% in all case for simplicity; (2) EPP system is simply to use and also can well describe the inelastic behavior of system, which will be discussed in details in the following; (3) take a series of



 $\mu_{cu}$  as 4,6,8 and 10 and set  $\beta$ =0.15; (4) considering ultimate limit state of structure, set  $D_i$ =1.0, in order to form the relation of  $R_D - \mu_{cu} - T$  directly.

# 2.4 Selection, Classification and Magnitude Adjustment of Input Ground Motions

Firstly, a total of more than 1000 horizontal earthquake acceleration time histories recorded from 18 different earthquakes are found in this study, which have been obtained from PEER Strong Motion Database, as shown in table 3. Due to the influence of site conditions, the USGS site classification criterion is adopted herein. The method is based on the average shear wave velocity and specifies four sites classes, namely A—(hard) rock site, B—stiff soil site, C—normal soil site and D—soft soil site. Table 3 shows the quantities of newly classified input ground motions

Secondly, the discussion in this study is mainly concerned about the district of 8 intensity of seismic fortification in China. According to the executing Chinese Seismic Code, it should be adjusted the magnitude of any recorder to 400gal. It is worthwhile to point out that some research shows the magnitude of accelerations may not affect the previous  $R_{\mu}$  spectra <sup>[xvi]</sup>. Trial analysis indicates that the same conclusion is obtained for the  $R_D$  spectra.

Finally, due to consideration the effect of earthquake duration, it is important to choose a definition of the duration since there is not a uniform reference. In this study, effective energy duration is adopted herein, which was widely used, namely time between 5% and 95% Arias intensity is served as acceleration time history<sup>[xxv]</sup>.

Table 2 Input ground motion recorders						
Earthquake name	Date	Magnitude	Recorders ' number			
Chi-Chi	09/20/1999	7.6	146			
Coalinga	05/02/1983	6.5	114			
Coyote Lake	08/06/1979	5.6	19			
Duzce	11/12/1999	7.3	43			
Imperial Valley	10/15/1979	6.9	39			
Kobe	01/16/1995	7.2	21			
Kocaeli	08/17/1999	7.8	40			
Landers	06/28/1992	7.4	79			
Livermore Valley	01/24/1980	5.8	30			
Loma Prieta	10/18/1989	7.1	86			
Morgan Hill	04/24/1984	6.1	52			
Northridge	01/17/1994	6.7	148			
Palm Springs	07/08/1986	6	60			
Parkfield	06/28/1966	5.9	21			
San Fernando	02/09/1971	6.6	56			
Whittier Narrows	10/01/1987	5.7	106			
Cape Mendocino	04/25/1992	7.1	11			

#### Table 3 USGS site classification criterion

Site groups	Site description	Shear wave velocity $V_s(m/s)$	Recorders ' number
А	Normal (hard) rock	$V_{s} > 750$	102
В	Stiff soil	$750 \ge V_s > 350$	284
С	Normal soil	$350 \ge V_s \ge 180$	351
D	Soft soil	$180 > V_s$	76



## 2.5 Analysis Method and Process of DBDS Program

First of all, it is necessary to introduce the program DBDS, where the key problems are how to deal with the inflexion points of EPP system and how to solve the energy accurately. As to prove whether program is accurate enough or not, the EPP model and energy curves are shown in figure2 and figure 3, respectively, due to a detailed SDOF oscillator. Trial analysis indicates that, for the hysteretic parameters varying in a reasonable range as expected in an adequate ductile design, the results of  $R_D$  factor are not sensitive to the selection of the hysteretic parameters. For this reason, a typical pattern hysteretic model—EPP model, as mentioned before, is employed, shown in figure 2, which shows that the problems of inflexion points are well treated.



Fig.3 shows kinds of energy curves, where  $E_I$  is the total input energy;  $E_k$  is kinetic energy;  $E_d$  is the energy dissipated by damping;  $E_y$  is the total deformation energy, which includes elastic deformation energy  $E_s$  and inelastic hysteretic energy  $E_h$  and  $E_a$  equals to the add of  $E_k$ ,  $E_d$  and  $E_y$ . As for SDOF system, the equation of energy is expressed as

$$E_{I} = E_{k} + E_{d} + E_{v} = E_{k} + E_{d} + E_{s} + E_{h}$$
(2.8)

As shown in fig. 3,  $E_I$  is nearly equal to  $E_a$ , which indicates that precision of solution for energy is accurate enough. Because plastic energy is unrecoverable, in general, it is common to recognize  $E_h$  as the destroy energy.



Fig.4 shows the flowchart for the calculation of the damage-based  $R_D$  factor. The steps of the procedure are listed as follows:

- (1) Given  $\xi$ ,  $\mu_{cu}$ ,  $\beta$  and  $D_i$ ;
- (2) Choose the first initial period  $T_l$ ;
- (3) Input any ground motion, and then calculate  $F_e$ ;
- (4) Introduction of  $R_D$  factor, solve the  $F_y$  and  $x_y$ ;
- (5) Time history analysis for  $E_h \& \mu_m$ ;
- (6) Substituting  $E_h$  and  $\mu_m$  to equation (2.7), calculate D;
- (7) Updating  $R_D$  factor, re-obtain  $F_y$  and  $x_y$  and repeat the steps from (4) to (7) until a target damage level,  $D_i$ , is achieved, such as 0.3, 0.5, 0.7 etc. In this study set  $D_i = 1.0$  for considering ultimate limit state of structures.
- (8) Obtain the  $R_D(T_1, \mu_{cu})$  value, at a given period  $T_1$ , when  $D_i=1.0$  is satisfied;
- (9) Increment of period as  $T_l = T_l + \Delta T$ , and return to step (2);
- (10) Carry on all steps above until the period reaches the prescribed period *T*, then find  $R_D$  (*T*,  $\mu_{cu}$ ) curve, namely  $R_D$  spectra.



Fig.4 Procedure for constructing a  $R_D$  spectra

(11) Proceed to another  $\mu_{cu}$ , and repeat (2)~(10), then construct a set of  $R_D$  spectra; End

## 2.6 Basic Characters of RD Spectra

Following the above program, fig.5 plots a set of mean  $R_D$  spectra for the four site groups of A site—(hard) rock, B site—stiff soil, C site—normal soil and D site—soft soil, and a set of typical coefficient of variation (*COV*). As shown in the fig.5, some basic characters of mean  $R_D$  spectra and *COV* curves can be obtained as follows:

(1) As for a specified period T, the bigger  $\mu_{cu}$  value, the bigger  $R_D$  value; the degree of increments varies by periods.

(2) As for a specified  $\mu_{cu}$  value,  $R_D$  value varies by periods. In the range of short periods, it makes great influence on  $R_D$  value for the variation of periods. With the increasing of periods,  $R_D$  value increases fast. While, in the range of long periods, it makes little influence on  $R_D$  value for the variation of periods. With the increasing of periods,  $R_D$  value nearly keep a steady constant value  $\mu$  and obviously, correspondingly  $\mu < \mu_{cu}$ .

(3) As for each site group, given different  $\mu_{cu}$  values, *COV* curves represent the same trends, and corresponding *COV* value increases with the incremental  $\mu_{cu}$  value.

In the following, a nonlinear regression analysis will be carried out on the mean  $R_D$  spectra of all kinds of site groups.

## 2.7 Regression of Mean RD Spectra

For the purpose of establishing the function of  $R_D$  spectra, regression analysis is carried out, so as to conveniently use it in the seismic design or estimation.

Based on the observations described above (fig.5), the  $R_D$  spectra for different groups of ground motions do not vary significantly. Therefore, it is reasonable to construct the mean  $R_D$  spectra that could be use as a uniform



reference, i.e.

$$R_D = f(T, \mu_{cu}) \tag{2.9}$$

The boundary limits can be identified in a similar way of  $R_{\mu}$  spectra as follows:

(1) 
$$R_D(T = 0, \mu_{cu}) = 1$$
;  
(2)  $R_D(T > 0, \mu_{cu}) > 1$ ;

$$(3) R_D(T = \infty, \mu_{cu}) = \mu$$

To be noticed, the constant value  $\mu$  (ductility demand) in condition (3) depends on the value  $\mu_{cu}$  (ductility capacity) and with the increasing of  $\mu_{cu}$ , the  $\mu$  value increases, but obviously correspondingly  $\mu < \mu_{cu}$ .

In terms of the conditions above, a simple empirical formula as a function of T and  $\mu_{cu}$  is presented as follows:

$$R_D(\mu_{cu},T) = 1 + a(1 - e^{-bT})$$
(2.10)

and cubic polynomial expression is advised to regress the parameters of a and b as

$$a = a_1 \mu_{cu}^3 + a_2 \mu_{cu}^2 + a_3 \mu_{cu} + a_4$$
 (2.11)

$$b = b_1 \mu_{cu}^3 + b_2 \mu_{cu}^2 + b_3 \mu_{cu} + b_4$$
(2.12)

where  $a_{1}$ ,  $a_{2}$ ,  $a_{3}$ ,  $a_{4}$ ,  $b_{1}$ ,  $b_{2}$ ,  $b_{3}$  and  $b_{4}$  are the regressive parameters for a and b, respectively.

Table 4 gives the results of regressive parameters for each site group. Fig.6 plots the comparison between regressed  $R_D$  curves and actual mean  $R_D$  spectra curves.













Figure 6 Mean curves comparison with regressed curves of  $R_D$  spectra

Table 4 Parameters on regressive formulas								
Site groups	<i>a</i> <sub>1</sub>	$a_2$	<i>a</i> <sub>3</sub>	$a_4$	<b>b</b> <sub>1</sub>	$\boldsymbol{b}_2$	<b>b</b> <sub>3</sub>	$b_4$
A site (rock)	-0.00373	0.0675	-0.0286	1.989	-0.01100	0.2265	-1.5540	6.0100
B site (stiff soil)	0.00004	-0.0058	0.5043	0.5840	0.00329	-0.0615	0.2968	1.6210
C site (normal soil)	0.00696	-0.1626	1.6430	-2.408	-0.00302	0.0715	-0.5979	3.7030
D site (soft soil)	-0.00058	0.00513	0.3396	0.3860	0.01106	-0.2234	1.331	-0.3690

## **3 RD SPECTRA COMPARRISON WITH PREVIOUS Rµ SPECTRA**

Owning to different considerations of research on  $R_{\mu}$  spectra, such as the quantities of input ground motions, site groups, resilience model and damping etc, the results have shown some differences by the researchers at home and abroad (see fig.7). In this study, the research of  $R_D$  spectra, in one hand, consider the ultimate limit state of structures, actually which is also the form of collapse spectra, and in the other hand, consider the influence of the duration of ground motions, where are said to be the most difference with  $R_{\mu}$  spectra. Figure 7 plots the comparison of the results by different researchers between  $R_D$  spectra and  $R_{\mu}$  spectra, given  $\mu = 6$  and  $\mu_{cu} = 6$ , 8. As known to all, for a well designed ductile structure, it does request the ductility demand should be less than



the ductility capacity, i.e.,  $\mu < \mu_{cu}$ . Hence, on the purpose of qualitative comparison, set  $\mu_{cu}$  to be 6 and 8. In a word, it draws the same trends and gets some accordant conclusions about the previous research of  $R_{\mu}$  spectra, in spite of different considerations. But to be noticed herein, the results of this paper might be said that the smallest values compared to other results are due to two main reasons as mentioned above. One is to consider the ultimate limit state of structures and the other is to consider the influence of the duration of ground motions. Hence, it is important to care about the influence of duration of ground motions during the seismic design or estimation for the structures.

#### 4. CONCLUSIONS

From the beginning of 1950s, there are many results of research about inelastic response spectra. Especially, when PBSD was put forward, research on inelastic response spectra was attracted more attention to by the researchers at home and abroad. In this paper, some weaknesses are pointed out for the previous studies and some conclusions are drawn as follows:

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(1) A procedure of analysis of inelastic response spectra based on Park & Ang damage model ( $R_D$  spectra) is proposed.  $R_D$  spectra overcomes the shortcomings of previous research about inelastic response spectra, which both considers the effects of maximum inelastic displacement and effects of cumulative hysteretic energy, according with the actual inelastic behavior due to ground motions.

(2) As for a specified period *T*, the bigger  $\mu_{cu}$  value, the bigger  $R_D$  value; the degree of increments varies by periods. As for a specified  $\mu_{cu}$  value,  $R_D$  value varies by periods. In the range of short periods, it makes great influence on  $R_D$  value for the variation of periods. With the increasing of periods,  $R_D$  value increases fast. While, in the range of long periods, it makes little influence on  $R_D$  value for the variation of periods. With the increasing of periods,  $R_D$  value nearly keep a steady constant value  $\mu$  and obviously, correspondingly  $\mu < \mu_{cu}$ . And as for each site group, given different  $\mu_{cu}$  values, *COV* curves represent the same trends, and corresponding *COV* value increases with the incremental  $\mu_{cu}$  value.

(3) Due to lots of time history analysis of SDOF oscillators and regress of datum, the expression of  $R_D$  spectra is constructed, which can be offered to the application in seismic evaluation of structures in PBSD.

(4) The results of this paper might be said that the smallest values compared to other results are due to two main reasons: One is to consider the ultimate limit state of structure and the other is to consider the influence of the duration of ground motions, which should be attracted more attention to during the seismic design or estimation for the structures.

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