# APPLICATIONS OF THE HILBERT-HUANG TRANSFORM TO MICROTREMOR DATA ANALYSIS ENHANCEMENT

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## ABSTRACT :

In this paper we discuss the use of the Hilbert-Huang transform (HHT) to enhance the time-frequency analysis of microtremor measurements. HHT is a powerful algorithm that combines the process of empirical mode decomposition (EMD) and the Hilbert transform to compose the Hilbert-Huang spectrum that contains the time-frequency information of the recorded signals. HHT is essentially an empirical algorithm and does not require the signals to be linear or stationary. HHT is advantageous to analyzing microtremor data, since observed microtremors are commonly contaminated by non-stationary transient noises close to the recording instruments. This is especially true when microtremors are measured in an urban environment. In this paper, using HHT we present the enhancement of the microtremor data analysis by 1) eliminating the unwanted transient component from microtremor data with EMD analysis by using only the coherent portion of the data to carry out the widely used horizontal to vertical spectral ratio (H/V) method; and 2) enhancing the H/V analysis by using the Hilbert-Huang spectrum (HHS). Application of the proposed approach to a microtremor field dataset acquired in metropolitan Beijing area demonstrated the efficacy of this procedure. This project is supported by Ministry of Science and Technology of China with Project No. 2006DFA21650 and the Institute of Earthquake Science (Project No. 0207690229).

**KEYWORDS:** Microtremor, Hilbert-Huang transform, empirical mode decomposition, horizontal to vertical spectral ratio

## 1. INTRODUCTION

Using microtremors to assess subsurface conditions have gained great momentum in the last decade, in quite a few sub-disciplines in earth science and geotechnical engineering such as earthquake engineering (e.g., Konno and Ohmachi, 1998), and petroleum reservoir characterization and monitoring (e.g., Dangel et al, 2003; Draganov et al, 2007; Lambert et al, 2007).

Originally proposed by scientists and engineers in Japan (e.g., Okada, 2003) and represented by Nakamura's (1989) widely cited work, microtremor were analyzed with the horizontal to vertical spectral ratio (H/V) method to extract useful information concerning earthquake engineering and geotechnical engineering. There would not be much objection from the geotechnical community if one declares that the H/V method is the most widely used method to study microtremors. In conjunction with borehole measurements at control points, the H/V method has been widely employed by the earthquake engineering community to map the thickness of near-surface sedimentary layers (e.g., Ibs-von Seht and Wohlenberg, 1999; Bodin and Horton, 1999; Parolai et al, 2002; Chen et al, 2008). Though the rigorous derivation based on the first principles of physics for the H/V method is still lacking, engineering practices in many parts of the world demonstrated its robustness and practicality (Atakan et al, 2004).

Nevertheless, the fast Fourier transform (FFT) remains the major workhorse for fundamental time-frequency analysis of microtremor data. In this paper we introduce the Hilbert-Huang transform (HHT), as a powerful time-frequency analysis tool with an empirical nature (Huang et al, 1998), to enhance the microtremor data analysis, specifically the H/V ratio analysis. Examples on using HHT to field microtremor data collected in Beijing during the summer 2007 (Chen et al, 2008) demonstrated the efficacy of this approach.

## 2. THE HILBERT-HUANG TRANSFORM

In contrast to the traditional Fourier transform (FT) which assumes any time-domain signal is the superposition of a family of simple harmonic functions with different magnitudes, the time-frequency analysis method termed Hilbert-Huang transform consists of 2 major steps: 1) the use of the empirical mode decomposition (EMD) and 2) the Hilbert spectral analysis method (Huang et al., 1998). The first step of HHT is EMD with which any complicated data set can be decomposed into a finite and often small number of intrinsic mode functions (IMFs). An IMF is defined as any function having the same number of zero crossings and extrema and also having symmetric envelopes defined by the local maxima and minima, respectively. The IMFs also admit well-behaved Hilbert transforms. This decomposition method is adaptive and, therefore, highly efficient. Since the decomposition is based on the local characteristic timescale of the data and computes instantaneous frequency through the Hilbert transform, it can reveal the intrawave frequency modulations as functions of time and thus give sharp identifications of imbedded structures. The final presentation of the results is an amplitude–frequency–time distribution, designated as the Hilbert-Huang spectrum (HHS). Because of these advantages, it is equally applicable to nonlinear and non-stationary processes and gives a physically meaningful interpretation of the data. Since it was initially proposed in the study of fluid mechanics (Huang et al., 1998), HHT has found immediate applications in many scientific and engineering disciplines (e.g., Adam, 2006; Salisbury and Sun, 2007).

According to Huang et al. (1998), any time sequence X(t) can be expressed as the sum of a series of intrinsic mode functions (IMFs) $X_j(t)$, i.e.,

$$X(t) = \sum_{j}^{n} X_j(t) \tag{2.1}$$

The mathematical process to get the IMFs is called empirical mode decomposition, which was described elsewhere (Huang, et al., 1998). After getting the IMFs by EMD, an IMF can be expressed as

$$X_j(t) = a_j(t)\exp(i\int \omega_j(t)dt) \tag{2.2}$$

where $a_j(t)$ is the instantaneous amplitude, and $\omega_j(t)$ is the instantaneous frequency of the *j*-th IMF. These parameters can be obtained through the Hilbert transform of the IMF

$$Y_j(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X_j(t)}{t-\tau} d\tau \tag{2.3}$$

where *P* is the Cauchy principal value of the integral. The instantaneous amplitude $a_j(t)$ is defined as

$$a_j(t) = \sqrt{X_j(t)^2 + Y_j(t)^2} \tag{2.4}$$

And the instantaneous phase is

$$\theta_j(t) = \tan^{-1}\frac{Y_j(t)}{X_j(t)} \tag{2.5}$$

Finally, the instantaneous frequency $\omega_j(t)$ is

$$\omega_j(t) = \frac{d\theta_j(t)}{dt} \tag{2.6}$$

Theoretically, the original time sequence X(t) should be recovered as the real part of the IMF summation. Meanwhile, as the function of frequency ω and time t, it can also be viewed as an instantaneous spectrum, i.e., the Hilbert spectrum:

$$\begin{aligned} h(\omega,t) &= \mathrm{Re}[\sum_j^n X_j(t)] \\ &= \mathrm{Re}[\sum_j^n a_j(t)\exp(i\int \omega_j(t)dt)] \\ &= \sum_j^n a_j(t)\int \cos\omega_j(t)dt \end{aligned} \tag{2.7}$$

## 3. APPLICATION OF HHT TO FIELD MICROTREMOR DATA

Applications of HHT to enhance the microtremor data analysis are carried out in 2 aspects: 1) the elimination of the local transient by using the EMD analysis; and 2) the development of a new approach via the Hilbert-Huang spectrum to determine the H/V ratio that has more intrinsic meaning with a narrower uncertainty range. We illustrate our approach through an application using the microtremor data collected from the Beijing area (Chen et al 2008). As an example, Figure 1 shows the application of EMD to a piece of 1-minute long, 3-component microtremor data. The top row represents the original microtremor record in EW, NS and vertical directions, with the rows representing the IMFs in sequence from 1 to 6 for the 3 components. In this particular example, IMF1 and IMF2 are the predominant contributors in the EW and NS components, and IMF1 is the predominant contributor in the vertical component. In general, IMF1 always possesses the highest frequency content and is most likely associated with transient noise close to the instrument.

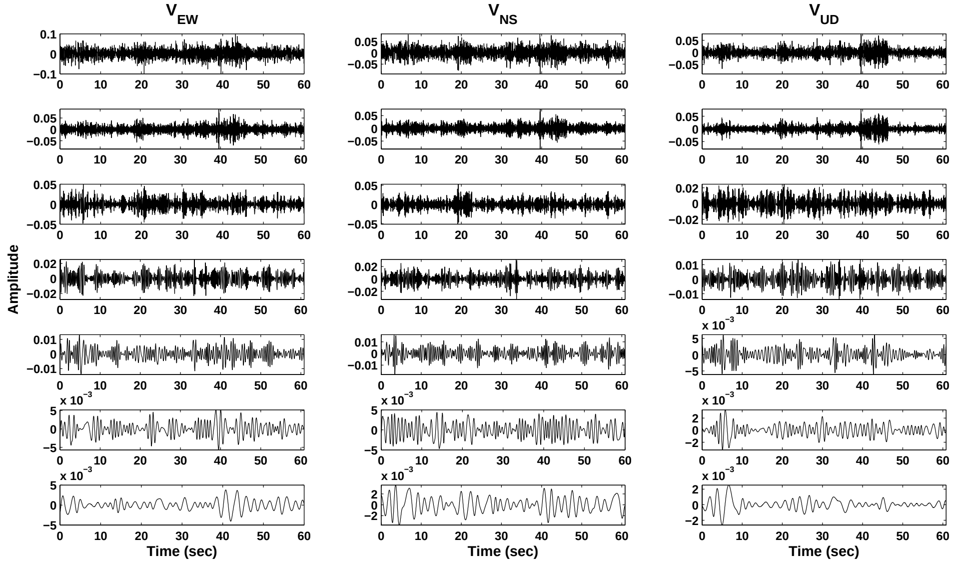


Figure 1 An example of the 3-component microtremor records (top row) and their IMFs from the 1st to the 6th mode (from second to the top to bottom rows) after the empirical mode decomposition

Taking the vertical component we can further the analysis by using the Fourier transform to discuss the microtremors' behavior in frequency domain. The left column of Figure 2 shows the characteristics of the vertical component microtremor record shown in Figure 1, its IMF1 and the remnant in time domain; and the right column shows the their corresponding behavior in frequency domain. Clearly, IMF1 is the dominant portion in this piece of microtremor data in vertical direction, as shown in the left column. Meanwhile, the frequency analysis in the right column shows that IMF1's domination occurs mostly in the high frequency band above 10 Hz. Thus we conclude that IMF1 originates locally.

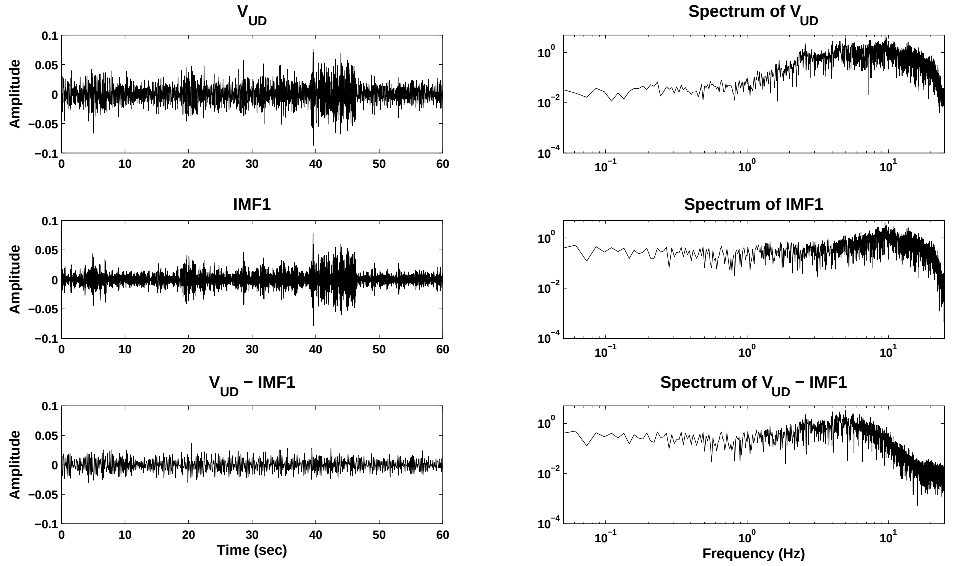
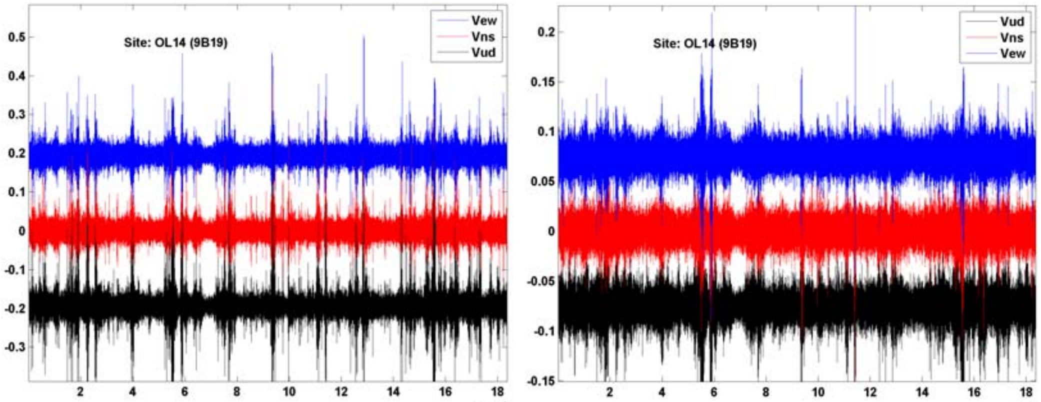


Figure 2 An example of the time domain record for the original vertical component, IMF1, and the remnants (left column) and their corresponding amplitude spectra in frequency domain (right column)

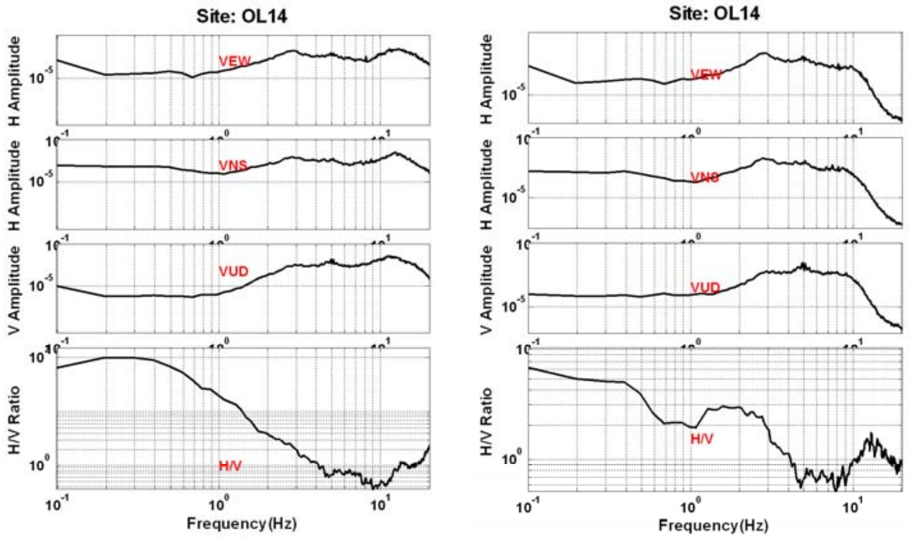
The behavior of the sample microtremor data has its generality. The high frequency constituents in the microtremor records are mostly local and caused by transient noise and may not contain much useful geological or engineering information. Theoretical analysis (Arai and Tokimatsu, 2004) also indicated that noises within one wavelength to the site maybe useless.

Now let us focus on microtremor analysis by applying the EMD and HHS techniques to the original data collected at the site of OL14 (ϕ=40.006°, λ=116.450°, and h=40.5 m), a site near the Olympic Green and north of the City center as the example to show the HHT approach. The OL14 data set were collected over one hour. As can be seen, the original data at OL14 were heavily contaminated by local transients, as shown in Figure 3a. With the use of EMD and after eliminating IMF1, the power of local transient is substantially suppressed, as shown in Figure 3b.



(a) (b)

Figure 3 (a) The original 3-component microtremor data recorded at site OL14; and (b) same data with IMF1 removed using the EMD analysis.



(a) (b)

Figure 4 (a) The Fourier spectra and H/V results from the original 3-component microtremor data recorded at site OL14; and (b) same results with IMF1 removed using the EMD analysis.

The effect of eliminating local transients by using EMD can be further articulated by demonstrating the effect on the H/V analysis as shown in Figure 4. Using only the original data, no clear resonance peaks can be identified in the FFT-derived H/V curve (Figure 4a). However, by simply eliminating IMF1, a resonance peak appears in the range of 1.2-2.2 Hz, though the peak is quite wide implying low resolution in determining the resonance frequency (Figure 4b). This fact leads to the discussion of the second aspect of analysis enhancement with HHT.

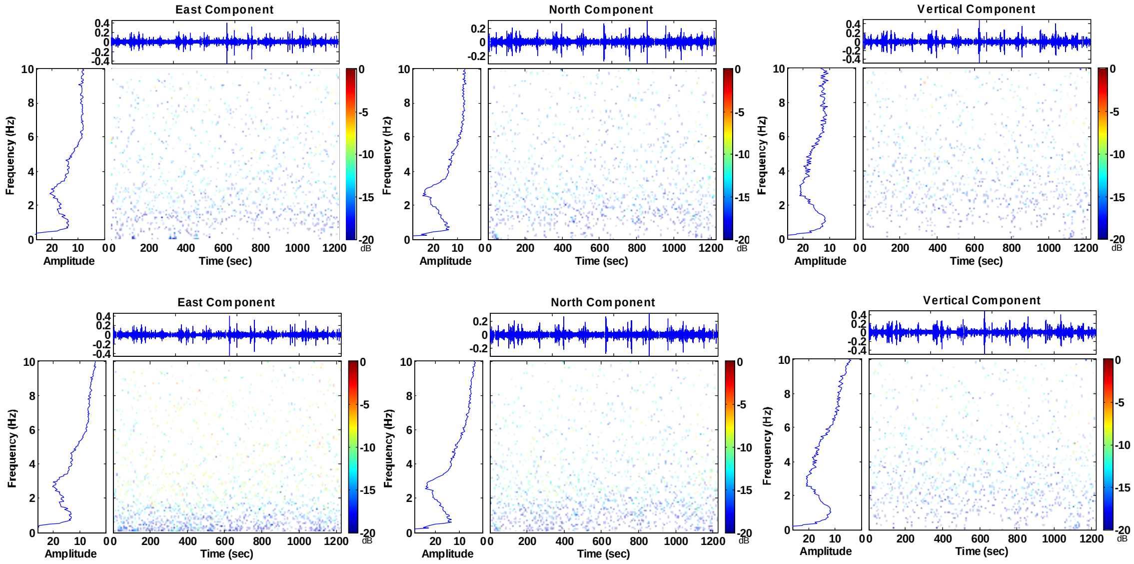
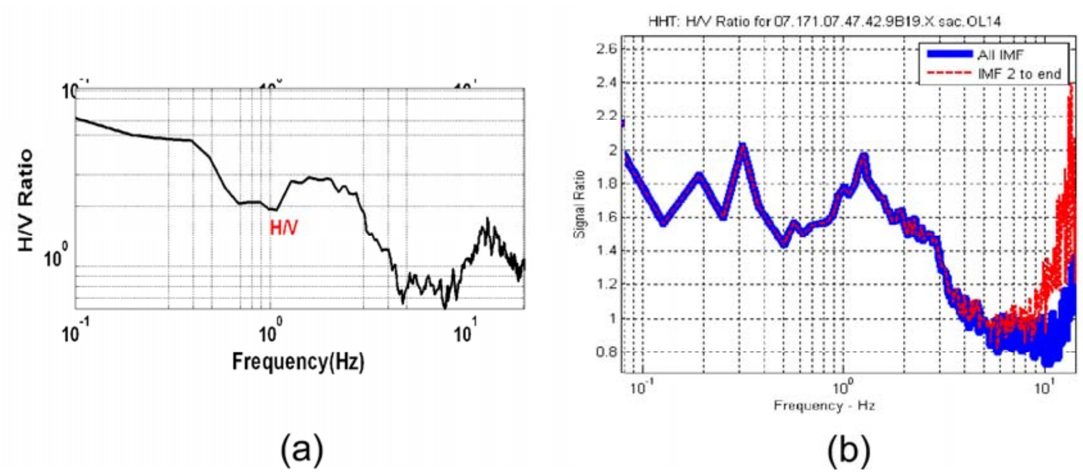


Figure 5 HHS of the original 3-component data at OL14 (top row, left to right are for the EW, NS, and Vertical components, respectively), and the HHS of the 3-component data after eliminating IMF1 (bottom row, left to right are for the EW, NS, and Vertical components, respectively)



(a)

(b)

Figure 6 (a) The H/V result from the FFT algorithm with the elimination of IMF1; and (b) the H/V result from the HHS approach with (the solid blue curve) and without (the broken red curve) the elimination of IMF1

Figure 5 shows the HHS spectra for the 3-component data from site OL14. In each panel, the top profile is the time domain record; the large rectangular area shows the amplitude distribution of the time-domain record of the top in the time-frequency domain with color code. Clearly, the amplitude of the microtremor record is random without much coherence. The vertical profile on the left shows amplitude spectrum which is the summation of the amplitudes in the time-frequency domain with respect to the time axis and can be interpreted similarly as the Fourier spectrum. Again, the HHS analysis shows that eliminating IMF1 results in the suppression of the high frequency constituents of the microtremor data.

Getting the H/V ratio from the amplitude spectra in Figure 5 results in Figure 6b. Compared Figure 6b with the FFT-derived H/V results as shown again as Figure 6a, it becomes clear that the peak resonance frequency in Figure 6b appears to be sharper, implies higher resolution in resonance frequency determination. Also, as shown in Figure 6b, elimination of IMF1 makes not much difference in determining the resonance frequency peaks at about 1.2 Hz; the effect is only in the high frequency portion above 10 Hz, in agreement with the analysis associated with Figure 2.

## 4. RESULT DISCUSSION

From the comparison of the H/V results shown in Figure 6, we can see that the HHT-derived H/V ratio has sharper peaks than the FFT-derived ratio. Consequently it implies a higher resolution in determining the fundamental resonance frequency, which is critical to infer sediment thickness with a better constraint (Parolai et al, 2002). As a matter of fact, a borehole drilled as water well in this area ($\phi$=40.0167°, $\lambda$=116.35°), 8 km away from site OL14, revealed the thickness of the Quaternary overburden to be 92 m (Liu et al, 1989). By plugging in the HHT-derived resonance frequency peak of 1.2 Hz, as shown in Figure 6b, into the resonance frequency-sediment thickness relationship derived in Cologne area (Parolai, et al 2002):

$$h = 108\, f_r^{-1.551} \tag{4.1}$$

for which we found also suitable for Beijing area (Chen et al, 2008; Wang et al, 2008), a sediment thickness of 82 m can be calculated. In general this is consistent with the ‘ground truth’ shown in the borehole ~10 km away from site OL14.

## 5. CONCLUSIONS

In conclusion, we have implemented the EMD decomposition with HHS analysis to enhance microtremor data analysis. We have demonstrated that by using EMD we can effectively eliminate the local transient noise effect and improve the stationarity of the intrinsic signal from the microtremor data. Moreover, by using the HHS spectra, rather than FFT spectra, we can gain higher resolution in the H/V analysis. This procedure will be beneficial in further reducing the uncertainties of sediment thickness estimation, which is one of the major objectives of microtremors studies for earthquake engineering.

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