DAMAGE IDENTIFICATION FOR STRUCTURES SUBJECTED TO SEVERE EARTHQUAKES USING WAVELET TRANSFORM

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ABSTRACT :
The availability of methods for damage identification of civil infrastructures under sever earthquake is crucial for safety assessment and repair decision of structures post earthquake. Two novel damage identification methods based on wavelet transform are proposed in this paper. In the first method, the structural seismic responses are dealt with by the continuous wavelet transform directly. The time-varying frequency of structure is identified, and then the occurrence time and degree of structural damage can be observed. However, this method can not work well for multi-degree of freedom system because of the complicated relationship between damage and structural frequencies. Then the second method is presented subsequently to identify nonlinear hysteresis curves of structures during strong earthquake. By this method, not only the location but the degree of structural damage can be identified. This method alternately uses the extended Kalman filter (EKF) and wavelet (W) multiresolution analysis. Then it is entitled as the EKF-W method. In each time step, the full structural states are first estimated by using the EKF based on limited observations; then the tangent stiffness as well as hysteresis curves of structures is identified by using the wavelet multiresolution analysis based on the estimated full states. Simulation results from two structures are presented to illustrate the power and efficiency of the proposed two methods.

KEYWORDS:
nonlinear, damage identification, severe earthquake, wavelet transform, extended Kalman filter

1. INTRODUCTION

It is very important to assess damage of civil infrastructures after earthquake accurately and as soon as possible. Because the assessment provides reference for reinforcement and repair of these structures. Until now, damage assessment of civil structures post earthquake is mainly conducted through visual inspection and some other nondestructive detect instruments. These methods can not provide overall assessment on structural damage and can not detect damage inside structural elements. So a better strategy is to identify structural damage by analyzing structural seismic responses measured by sensors on structures. However, damage identification based on structural seismic responses suffers from two problems: (1) it is impractical to measure the structural responses on every DDOF for real buildings; and (2) structural characteristics are time variant and nonlinear. Therefore, some traditional damage identification methods for linear system and those methods need structural measurements on total DDOFs can work well no longer.

Wavelet transform has the ability to provide information simultaneously in time and frequency with adaptive windows. Then it offers promising tools for identifying structural time varying characteristics under intensive earthquake. Damage identification methods based on wavelet transform can be divided into three categories: (1) detecting damage occurrence by the change of wavelet transform coefficients [1-3]; (2) identifying time varying modal parameters by extracting ridge of wavelet scalograms [4-5]; and (3) identifying structural nonlinear hysteresis curves by using wavelet multiresolution analysis [6]. However, the modal parameters identification for hysteretic nonlinear system under earthquake excitation is seldom discussed. Moreover, structural seismic responses on total DDOFs are needed for hysteresis curves identification in literature [6], which is impractical.

In this paper, two novel damage identification methods based on wavelet transform are proposed. In the first method, the structural seismic responses are dealt with by the continuous wavelet transform. The time-varying frequency of structure is identified, and then the occurrence time and degree of structural damage can be observed. However, this method can not work well for multi-degree of freedom system because of the complicated relationship between damage and structural frequencies. Then the second method is presented subsequently to identify nonlinear hysteresis curves of structures during strong earthquake. By this method, not only the location but the degree of structural damage can be identified. This method alternately uses the extended Kalman filter (EKF) and wavelet (W) multiresolution analysis. Then it is entitled as the EKF-W method. In each time step, the full structural states are first estimated by using the EKF based on limited observations; then the tangent stiffness as well as hysteresis curves of structures is identified by using the wavelet multiresolution analysis based on the estimated full states. Simulation results from two structures are presented to illustrate the power and efficiency of the proposed two methods.

2. METHOD ONE: TIME VARYING FREQUENCY IDENTIFICATION BASED ON WAVELET TRANSFORM

It is generally recognized that the strength and stiffness of a civil structure will decrease under an intensive earthquake and that the force-displacement relationship of structural members will exhibit hysteretic nonlinear characteristics. In such cases, the frequencies of structures are time variant and the variation of frequency is associated with the level of nonlinearity in the system. So the nonlinearity and the damage of a system can be characterized by its time varying frequency. As a powerful time-frequency analysis tool, the wavelet transform has been extensively employed by many researchers to capture frequency variation with time of nonlinear systems. However, the study of time-frequency responses of hysteretic nonlinear systems is insufficient and its accurate physical meaning is not yet clear. In the method one, the structural seismic response is dealt with by continuous wavelet transform first. Then the wavelet transform coefficients are integrated along each vibration period to obtain the periodic energy spectrum. The ridge of the energy spectrum is extracted and the average frequency in each vibration period is identified. Also, the physical meaning of the time-varying frequency of hysteretic nonlinear system is discussed.

The wavelet transform of a signal $x(t)$ is presented as an example of a time-scale decomposition obtained by dilating and translating a chosen analytical function (wavelet) along the time axis. The continuous wavelet transform is defined as follows:

$$W_\psi(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt \tag{2.1}$$

where b is the parameter localizing the wavelet function in the time domain, a is the dilation parameter defining the analytical window stretching and ψ^* is the complex conjugate of the basic wavelet function. Therefore, b represents a time parameter and a is related to frequency. The Gabor wavelet function is used in this paper to obtain the structural time-frequency response. The Gabor wavelet function is defined as

$$\psi = \frac{1}{(\sigma^2\pi)^{1/4}} e^{-t^2/(2\sigma^2)} \cdot e^{i\eta t} \tag{2.2}$$

where parameter σ and the initial scale define the time and frequency spread of the Gabor wavelet function, and η is the parameter of frequency modulation. For discussing the physical meaning of time varying frequency of hysteretic system, the wavelet coefficient $W_\psi(a,b)$ is integrated along each vibration period. Since the dilation parameter a and translation parameter b have clear relationship with frequency ω and time t of system, the wavelet coefficient can also be expressed as $W_\psi(\omega,t)$. Then the energy $E_i(\omega)$ at each vibration period can be written as

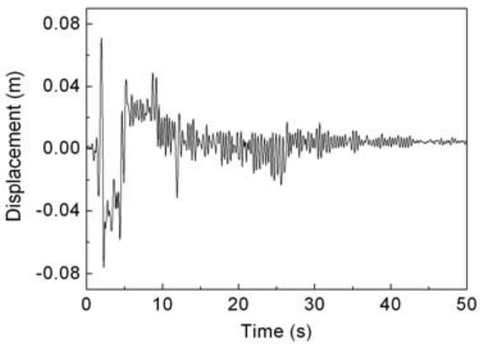
$$E_i(\omega)=\int_{t_i}^{t_i+T}W_\psi(\omega,t)dt \tag{2.3}$$

where t_i is the start time of the ith vibration period, T is the vibration period. Then the periodic energy spectrum, i.e. the variation of periodic energy $E_i(\omega)$ with time, can be plot. With the same meaning of the ridge of the scalogram of wavelet transform, the ridge of the periodic energy spectrum also describes the frequency variation of structures. Detailed explanations of the various methods for ridge extraction can be found in [4-5] and will not be included here because of space limitations.

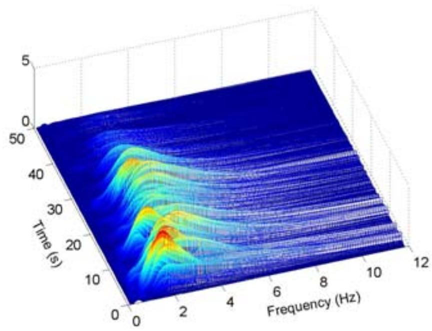
To verify this method, a one-story structure with bilinear hysteretic model is considered here. The characteristic parameters of this model are list in Table 1, in which f_y and x_y denote the yield force and displacement; k_0 and k_1 denote the initial and post yield stiffness.

Table 1.1 Characteristic parameters and their quantities of the structures

(i) One-story structure					
Story	m (kg)	f_y (kN)	x_y (m)	k_0 (kN/m)	k_1 (kN/m)
1	990147	3.44×10^3	0.012	2.87×10^5	2.12×10^4
(ii) Three-story structure (the weak story is at the first floor)					
Stroy	m (kg)	f_y (kN)	x_y (m)	k_0 (kN/m)	k_1 (kN/m)
1	990147	3.44×10^3	0.012	2.87×10^5	2.12×10^4
2	646512	6.30×10^4	0.172	3.65×10^5	1.76×10^4
3	646512	6.30×10^4	0.172	3.65×10^5	1.76×10^4

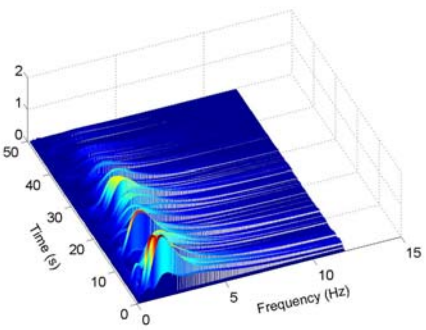


(a) displacement response

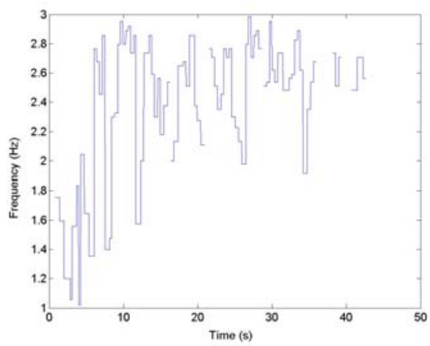


(b) wavelet transform scalogram

Figure 1 Displacement and its wavelet scalogram of the structure



(a) periodic energy spectrum



(b) time varying frequency

Figure 2 Periodic energy spectrum and time varying frequency of the structure

N-S component of the EL Centro earthquake with the peak acceleration normalized to 8m/s^2 is adopted as input excitation. The seismic responses of the structure are simulated by the Newmark numerical integration method. The displacement responses are plot in Figure 1. The displacement is then dealt with by the continuous wavelet transform and the scalogram is also shown in Figure 1. The periodic energy spectrum of displacement response is depicted in Figure 2. The time varying frequency of the structure obtained from the ridge of the periodic energy spectrum is also shown in Figure 2. It can be noted that the frequency is proportional to displacement amplitude. Then the variation of frequency reflects damage level of the structure.

3. METHOD 2: NONLINEAR SYSTEM IDENTIFICATION BY USING KALMAN FILTER AND WAVE TRANSFORM

Method one can not work well for multi-degree of freedom structures because of the complicated relationship between damage and structural frequencies. Then the method two is proposed here for damage identification of multi-degree of freedom structures. The extended Kalman filter and wavelet multiresolution analysis are simultaneously used in this method to identify hysteresis curves of structures, which reflect damage location and degree of structures.

3.1. Problem Formulations and Solution Procedure

Consider a shear-type multi-degree-of-freedom nonlinear system subjected to earthquake excitation. The governing equation can be written in matrix form as

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}(t)\mathbf{X}(t)=-\mathbf{M1}\ddot{x}_g(t) \tag{3.1}$$

where $\ddot{\mathbf{X}}(t)$, $\dot{\mathbf{X}}(t)$, $\mathbf{X}(t)=$ acceleration, velocity, and displacement vectors of the system. Suppose that only structural responses at limited DDOFs can be measured. $\mathbf{1}=$ an unity vector; $\ddot{x}_g(t)=$ ground acceleration. $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}(t)=$ mass, damping and stiffness matrices respectively and their value at the initial state can be obtained according to the dimension and material property of structural elements. Suppose that $\mathbf{M}$, $\mathbf{C}$ keep constant during earthquake in this study, while $\mathbf{K}(t)$ is time-variant after damage occurs and need to be identified. If we assume that the stiffness matrix keeps constant in each time step and equal to the tangent stiffness matrix at the beginning of this time step, then Eq. (3.1) is easily included in the incremental representation as follows:

$$\mathbf{M}\Delta\ddot{\mathbf{X}}(t)+\mathbf{C}\Delta\dot{\mathbf{X}}(t)+\mathbf{S}(t)\Delta\mathbf{X}(t)=-\mathbf{M1}\Delta\ddot{x}_g(t) \tag{3.2}$$

where $\mathbf{S}(t)=$ tangent stiffness matrix at the beginning of each time step. For an L-story shear-type frame structure, Eq. (3.2) can be expanded as:

$$\begin{bmatrix} m_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & m_2 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & m_{L\text{-}1} & 0 \\ 0 & 0 & 0 & \dots & 0 & m_L \end{bmatrix} \begin{bmatrix} \Delta\ddot{x}_1(t) \\ \Delta\ddot{x}_2(t) \\ \dots \\ \Delta\ddot{x}_{L-1}(t) \\ \Delta\ddot{x}_L(t) \end{bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 & \dots & 0 & 0 & 0 \\ -c_2 & c_2+c_3 & -c_3 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -c_{L\text{-}1} & c_{L\text{-}1}+c_L & -c_L \\ 0 & 0 & 0 & \dots & 0 & -c_L & c_L \end{bmatrix} \begin{bmatrix} \Delta\dot{x}_1(t) \\ \Delta\dot{x}_2(t) \\ \dots \\ \Delta\dot{x}_{L-1}(t) \\ \Delta\dot{x}_L(t) \end{bmatrix}$$
$$+\begin{bmatrix} s_1(t)+s_2(t) & -s_2(t) & 0 & \dots & 0 & 0 & 0 \\ -s_2(t) & s_2(t)+s_3(t) & -s_3(t) & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -s_{L\text{-}1}(t) & s_{L\text{-}1}(t)+s_L(t) & -s_L(t) \\ 0 & 0 & 0 & \dots & 0 & -s_L(t) & s_L(t) \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \dots \\ \Delta x_{L-1}(t) \\ \Delta x_L(t) \end{bmatrix} = \begin{bmatrix} -m_1\Delta\ddot{x}_g(t) \\ -m_2\Delta\ddot{x}_g(t) \\ \dots \\ -m_{L\text{-}1}\Delta\ddot{x}_g(t) \\ -m_L\Delta\ddot{x}_g(t) \end{bmatrix} \tag{3.3}$$

where $L=$ total number of stories; m_i, c_i and $s_i(t)$ ($i=1, 2, \dots, L$) = mass, damping and tangent stiffness coefficients at the ith story; $\Delta x_i(t)$, $\Delta\dot{x}_i(t)$ and $\Delta\ddot{x}_i(t)$ =incremental displacement, velocity and acceleration at the ith story relative to the ground.

As mentioned above, the whole identification in each time step in the present method can be divided into two steps: (1) estimating structural responses at all DDOFs using the EKF based on a given tangent stiffness matrix; and (2) identifying the tangent stiffness matrix as well as hysteresis curves at each story using wavelet multiresolution analysis based on the total structural responses. It should be noted in this method that the structural mass, damping and stiffness matrices, together with the structural responses, at initial state are required to be known as the condition to initiate the identification.

Step I: Estimation of structural responses at all DDOFs using the EKF.

For estimating the structural responses at all DDOFs using EKF, Eq. (3.1) need to be expressed in a state-space representation as

$$\dot{\mathbf{Z}}(t)=\begin{bmatrix}\dot{\mathbf{X}}(t)\\ \ddot{\mathbf{X}}(t)\end{bmatrix}=\begin{bmatrix}\dot{\mathbf{X}}(t)\\ -\mathbf{M}^{-1}(\mathbf{K}(t)\mathbf{X}(t)+\mathbf{C}\dot{\mathbf{X}}(t))-\mathbf{1}\ddot{x}_g(t)\end{bmatrix} \tag{3.4}$$

where $\mathbf{Z}(t)=\left[\mathbf{X}(t)\ \ \dot{\mathbf{X}}(t)\right]^T$, is the state vector at time t. Since the stiffness matrix $\mathbf{K}(t)$ is time-variant and nonlinear, in general Eq. (3.4) can be expressed as

$$\dot{\mathbf{Z}}(t)=g[\mathbf{Z}(t),t] \tag{3.5}$$

The observation equation of the nonlinear system is

$$\mathbf{Y}(t)=\mathbf{H}\mathbf{Z}(t)+\mathbf{V}(t) \tag{3.6}$$

where $\mathbf{Y}(t)=u\times 1$ observation vector at time t that may include displacements, velocities and accelerations; $u=$ total number of observations; $\mathbf{H}=u\times 2L$ matrix associated with observations and $u\le 2L$; $\mathbf{V}(t)=u\times 1$ measurement noise vector and is assumed to be independent, white, and with normal probability distributions.

$$p(\mathbf{V})\sim N(0,\mathbf{R}) \tag{3.7}$$

where $\mathbf{R}$ is the covariance matrix of $\mathbf{V}(t)$.

The initial structural state vector $\hat{\mathbf{Z}}(t_1)$ and its estimation error covariance matrix $\hat{\mathbf{P}}(t_1)$ at time t_1 are assumed to be known according to the initial conditions. The initial tangent stiffness matrix $\mathbf{S}(t_1)$ can be obtained according to material properties and geometry of members. The full state at time t_2 can be estimated based on the initial tangent stiffness matrix $\mathbf{S}(t_1)$, and the initial state $\hat{\mathbf{Z}}(t_1)$ and $\hat{\mathbf{P}}(t_1)$. Then the state vector $\hat{\mathbf{Z}}(t_{r+1})$ and error covariance matrix $\hat{\mathbf{P}}(t_{r+1})$ at time t_{r+1} ($r=1\sim(\lambda-1)$; $\lambda=$number of sampling data) can be estimated according to the $\hat{\mathbf{Z}}(t_r)$, $\hat{\mathbf{P}}(t_r)$ and the tangent stiffness matrix at time t_r. The sub-steps in the state estimation using EKF are as follows:

(i) Start with filtered state $\hat{\mathbf{Z}}(t_r)$ and its error covariance matrix $\hat{\mathbf{P}}(t_r)$, which is obtained based on the initial conditions or equal to their estimated value at previous time step.

(ii) Evaluate the predicted state $\tilde{\mathbf{Z}}(t_{r+1})$ and its error covariance matrix $\tilde{\mathbf{P}}(t_{r+1})$ by

$$\tilde{\mathbf{Z}}(t_{r+1})=\hat{\mathbf{Z}}(t_r)+\int_{t_r}^{t_{r+1}} g\left(\hat{\mathbf{Z}}(t_r),t\right)dt \tag{3.8}$$

$$\tilde{\mathbf{P}}(t_{r+1})=\Phi(t_{r+1})\hat{\mathbf{P}}(t_r)\Phi(t_{r+1})^T \tag{3.9}$$

where $\Phi(t_{r+1})=$nonsingular state transition matrix of the system; for small time interval Δt, it can be approximately obtained as

$$\mathbf{\Phi}(t_{r+1})\approx\mathbf{I}+\Delta t\left[\frac{\partial g_i\left[\hat{\mathbf{Z}}(t)\right]}{\partial\hat{\mathbf{Z}}_j}\right]_{t=t_r} \tag{3.10}$$

where

$$\left[\frac{\partial g_i\left[\hat{\mathbf{Z}}(t)\right]}{\partial \hat{\mathbf{Z}}_j}\right]_{t=t_r} = \begin{bmatrix} \mathbf{0} & \mathrm{I} \\ -\mathrm{M}^{-1}\mathbf{S}(t_r) & -\mathrm{M}^{-1}\mathrm{C} \end{bmatrix} \tag{3.11}$$

Where $\mathbf{S}(t_r)=$ tangent stiffness matrix at time t_r consisting of tangent stiffness $s_i(t_r)$ at each story. It is either equal to the initial stiffness matrix or the tangent stiffness matrix identified at time step t_r .

(iii) Estimate the Kalman gain matrix $\boldsymbol{\kappa}(t_{r+1})$

$$\boldsymbol{\kappa}(t_{r+1}) = \widetilde{\mathbf{P}}(t_{r+1})\mathbf{H}^T\left[\mathbf{H}\widetilde{\mathbf{P}}(t_{r+1})\mathbf{H}^T + \mathbf{R}(t_{r+1})\right]^{-1} \tag{3.12}$$

(iv) Estimate the filtered state $\hat{\mathbf{Z}}(t_{r+1})$ and its error covariance matrix $\hat{\mathbf{P}}(t_{r+1})$ by

$$\hat{\mathbf{Z}}(t_{r+1}) = \widetilde{\mathbf{Z}}(t_{r+1}) + \boldsymbol{\kappa}(t_{r+1})\left[\mathbf{Y}(t_{r+1}) - \mathbf{H}\widetilde{\mathbf{Z}}(t_{r+1})\right] \tag{3.13}$$

$$\hat{\mathbf{P}}(t_{r+1}) = \left(\mathbf{I} - \boldsymbol{\kappa}(t_{r+1})\mathbf{H}\right)\widetilde{\mathbf{P}}(t_{r+1}) \tag{3.14}$$

The full states are then estimated with observations at limited DDOFs, $\mathbf{Y}(t_{r+1})$.

Step II: Identification of tangent stiffness and hysteresis curves of each story according to the said estimated full states up to time t_{r+1} using the wavelet multiresolution technique.

Matrix equation (3.3) can be rewritten as follows:

$$s_i(t)\left(\Delta x_i(t) - \Delta x_{i-1}(t)\right) = -\sum_{q=i}^{L} m_q\left(\Delta\ddot{x}_q(t) + \Delta\ddot{x}_g(t)\right) - c_i\left(\Delta\dot{x}_i(t) - \Delta\dot{x}_{i-1}(t)\right) \quad (2 \le i \le L)$$

$$s_1(t)\Delta x_1(t) = -\sum_{q=1}^{L} m_q\left(\Delta\ddot{x}_q(t) + \Delta\ddot{x}_g(t)\right) - c_1\Delta\dot{x}_1(t) \tag{3.15}$$

where $\Delta x_i(t) = x_i(t) - x_i(t-\Delta t)$ denotes incremental displacement relative to the ground at the ith story; $\Delta x_i(t) - \Delta x_{i-1}(t)$, $\Delta\dot{x}_i(t) - \Delta\dot{x}_{i-1}(t)$ and $\Delta\ddot{x}_i(t) - \Delta\ddot{x}_i(t)$ are incremental interstroy drifts, velocities and accelerations, respectively; and $\Delta\ddot{x}_g(t) =$ incremental ground acceleration. $s_i(t)$ denotes tangent stiffness of each story. For identifying tangent stiffness $s_i(t)$ in Eq. (3.15), it is approximated using scaling function in subspace v_j as follows:

$$s_i(t) = \sum_{n=0}^{l} p_{i,j,n}\phi_i\left(2^{-j}t - n\right) \tag{3.16}$$

where $n =$ translation parameter of scaling function $\phi_i(t)$; $p_{i,j,n}$ = integral coefficients of scaling functions. Substituting Eq. (3.16) into Eq. (3.15), then the incremental governing equation of the i th story becomes

$$\sum_{n=0}^{l} p_{i,j,n}\phi_i\left(2^{-j}t - n\right)\left(\Delta x_i(t) - \Delta x_{i-1}(t)\right) = -\sum_{q=i}^{L} m_q\left(\Delta\ddot{x}_q(t) + \Delta\ddot{x}_g(t)\right) - c_i\left(\Delta\dot{x}_i(t) - \Delta\dot{x}_{i-1}(t)\right) \; (2 \le i \le L)$$

$$\sum_{n=0}^{l} p_{1,j,n}\phi_1\left(2^{-j}t - n\right)\Delta x_1(t) = -\sum_{q=1}^{L} m_q\left(\Delta\ddot{x}_q(t) + \Delta\ddot{x}_g(t)\right) - c_1\Delta\dot{x}_1(t) \tag{3.17}$$

By setting $\Delta\varepsilon_i(t) = \Delta x_i(t) - \Delta x_{i-1}(t)$, and $\Delta\dot{\varepsilon}_i(t) = \Delta\dot{x}_i(t) - \Delta\dot{x}_{i-1}(t)$ for notational convenience, Eq.(3.17) can be rewritten as follow:

$$\sum_{n=0}^{l} p_{i,j,n}\phi_i\left(2^{-j}t - n\right)\Delta\varepsilon_i(t) = -\sum_{q=i}^{L} m_q\left(\Delta\ddot{x}_q(t) + \Delta\ddot{x}_g(t)\right) - c_i\Delta\dot{\varepsilon}_i(t) \tag{3.18}$$

Substitution of structural response obtained by observation and estimation at $t = t_w \sim t_{r+1}$ into Eq. (3.18) yields

$$\mathbf{A}_i\Theta_i = \mathbf{B}_i \tag{3.19}$$

where

$$\mathbf{A}_{i,(r-w+2)\times(l+1)} = \begin{bmatrix} \phi_i\left(2^{-j}t_w\right)\Delta\varepsilon_i\left(t_w\right) & \cdots & \phi_i\left(2^{-j}t_w - l\right)\Delta\varepsilon_i\left(t_w\right) \\ \vdots & \ddots & \vdots \\ \phi_i\left(2^{-j}t_{r+1}\right)\Delta\varepsilon_i\left(t_{r+1}\right) & \cdots & \phi_i\left(2^{-j}t_{r+1} - l\right)\Delta\varepsilon_i\left(t_{r+1}\right) \end{bmatrix}$$

$$\mathbf{\Theta}_{i,(l+1)\times 1} = \begin{bmatrix} p_{i,j,0} \\ \vdots \\ p_{i,j,l} \end{bmatrix} \qquad \mathbf{B}_{i,(r-w+2)\times 1} = \begin{bmatrix} -\sum_{q=i}^{L} m_q(\Delta\ddot{x}_q\left(t_w\right) + \Delta\ddot{x}_g(t_w)) - c_i\Delta\dot{\varepsilon}_i(t_w) \\ \vdots \\ -\sum_{q=i}^{L} m_q(\Delta\ddot{x}_q\left(t_{r+1}\right) + \Delta\ddot{x}_g(t_{r+1})) - c_i\Delta\dot{\varepsilon}_i(t_{r+1}) \end{bmatrix}$$

$\mathbf{A}_i$ and $\mathbf{B}_i$ are prior-known matrices composed of observations, estimated states and scaling functions. The unknown parameters, vector $\mathbf{\Theta}_i$, are determined by means of the least-squares method

$$\mathbf{\Theta}_i = \left(\mathbf{A}_i^T\mathbf{A}_i\right)^{-1}\mathbf{A}_i^T\mathbf{B}_i \tag{3.20}$$

where the superscript T is transpose of a matrix. Once $\mathbf{\Theta}_i$ is computed, the tangent stiffness $s_i(t)$ ($t = t_w \sim t_{r+1}$) is approximated by substituting $p_{i,j,n}$ into Eq. (3.16). The identified tangent stiffness $s_i(t_{r+1})$ at time t_{r+1} will be used to estimate structural full states at time t_{r+2}.

After the stiffness matrix is identified, taking the increment $r = r+1$ and returning back to Eq. (3.8), the same procedure is conducted until $r = \lambda$. Finally, the structural responses at each DDOF, together with the time-varying tangent stiffness are obtained by alternately using the EKF technique and wavelet multiresolution analysis in each time step (abbreviated to EKF-W method). The incremental nonlinear restoring force in each story is also obtained by multiplying the tangent stiffness by incremental interstory drift and the nonlinear restoring force is the sum of all incremental nonlinear restoring force before t_r. Then the hysteresis curves of each story can be easily obtained.

3.2. Numerical Examples

The EKF-W method is tested through simulation on a base-excited shear-type three-story frame with bilinear hysteretic model at each story. The characteristic parameters of this structure are also list in Table 1. The damping ratio of the structure is 0.05. Responses of the bilinear system subjected to white noise (the peak accelerations is 14.44 m/s^2) are simulated by the Newmark numerical integration method. After that, responses at some stories are selected as observations. The total structural seismic responses and hysteresis curves at each story are then identified by the EKF-W method. To investigate impact of observation and damage location on precision of identification, total two weak story cases (the weak story is at the first and third story respectively) and twelve schemes of sensor placement are considered. However, only the case with the weak story at the first story and observation at the bottom two stories is shown here because of the space limitation.

In this identification, the mass and damping coefficients are assumed to be constant during excitation and prior-known; the initial stiffness is also assumed to be known beforehand. In each time step, the Daubechies 4 wavelet is adopted here and the quantity of scaling function is 1/8 of the sampling interval to approximate tangent stiffness of each story. The matrix equation $\mathbf{A}_i^{\mathbf{T}}\mathbf{A}_i\mathbf{\Theta}_i = \mathbf{A}_i^{\mathbf{T}}\mathbf{B}_i$ is solved by the Choleski decomposition for obtaining the $\mathbf{\Theta}_i$. Figure 3 shows the identified hysteresis curves at each story. Also, the simulated hysteresis curves are depicted in this Figure as the real hysteresis curves. It can be noted that the EKF-W method can identify structural hysteresis curves accurately. Then the location and level of structural damage due to earthquake can be observed clearly through the identified hysteresis curves. It also should be noted that in the EKF-W method, the sensors should be placed at or near the weak story to get better identification results.

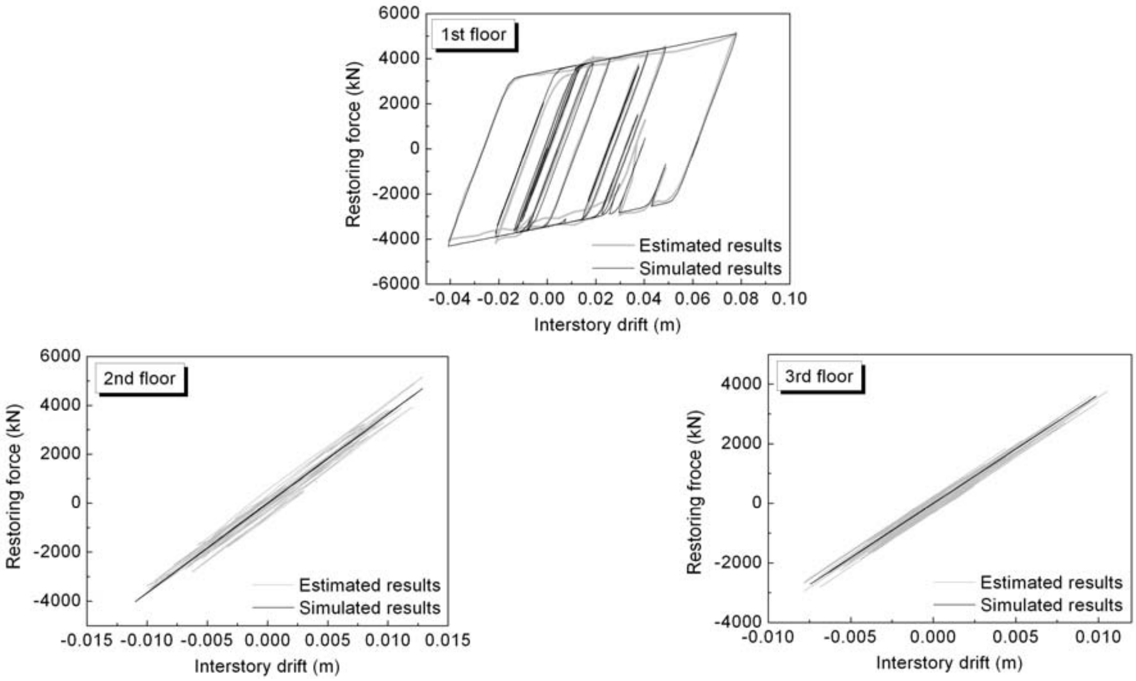


Figure 3 The identified and simulated hysteresis curves at each story

4. CONCLUSIONS

Two damage identification methods based on wavelet transform are proposed in this paper. In the method one, structural time varying frequency can be identified by extracting the ridge of periodic energy spectrum, which respects damage level of the structure. However, this method can not work well for multi-degree of freedom structure because of the completed relationship between the frequency and structural damage. This problem is solved by the method two, in which the hysteresis curves at each story are identified by using wavelet transform and Kalman filter. Then the location and level of structural damage can be observed clearly through the identified hysteresis curves. The efficiency of these two methods is verified by two simulation examples.

ACKNOWLEDGE

This study is financially supported by NSFC with Grant No. 50538020, the National Distinguished Youth Funds with Grant No. 50525823 and the Ministry of Science and Technology with Grant No. 2007CB714204, 2007CB714205, 2006BAJ03B05-B06 and 2006BAJ13B03.

REFERENCES

[1] Kim, H., and Melhem, H. (2004). Damage detection of structures by wavelet analysis. *Engineering Structures* **26**, 347-362.
[2] Hou, Z., Noori, M. and Amand, R. St. (2000). Wavelet-based approach for structural damage detection. *Journal of Engineering Mechanics* **126:7**, 677-683.
[3] Hera, A. and Hou, Z. (2004). Application of wavelet approach for ASCE structural health monitoring benchmark studies. *Journal of Engineering Mechanics* **130:1**, 96-104.
[4] Lardies, J. and Gouttebroze, S. (2002). Identification of modal parameters using the wavelet transform. *International Journal of Mechanical Sciences* **44**, 2263-2283.
[5] Slavic, J., Simonovski, I. and Boltezar, M. (2003). Damping identification using a continuous wavelet transform: application to real data. *Journal of Sound and Vibration* **262**, 291-307.
[6] Kitada, Y. (1998). Identification of nonlinear structural dynamic systems using wavelets. *Journal of Engineering Mechanics*, *ASCE* **124**, 1059-1066.