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A PROCEDURE FOR DERIVING ANALYTICAL FRAGILITY CURVES FOR MASONRY BUILDINGS

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ABSTRACT :

A new analytical procedure for the derivation of fragility curves for masonry buildings, based on nonlinear static and dynamic stochastic analyses of the whole structure, is presented. The procedure is applied to a prototype building to illustrate all the steps. All the mechanical parameters of the buildings are considered as random variables and Monte Carlo simulations are used to define the input parameters for the numerical model. Nonlinear static (pushover) analyses of the structures are carried out to identify the probability distributions of 4 selected damage states; then nonlinear dynamic time history analyses are performed to define the probability distribution of the displacement demand corresponding to different levels of ground motion. These probability distributions of capacity and demand are then convolved to obtain fragility points, which will be fitted by lognormal distributions to obtain fragility curves.

KEYWORDS: Fragility curves, masonry buildings, Monte Carlo simulation, stochastic nonlinear analyses

1. INTRODUCTION

Masonry buildings are a very diffused type of structures, that often suffers significant damage in case of earthquakes due to its characteristics of high rigidity, low tensile and shear strength, low ductility and low capacity of bearing reverse loading. Nevertheless, the efforts put in the study and improvement of this type of construction are very limited since masonry buildings tend to be regarded as out-of-date, even though they are still diffusely built even in seismic areas and, in any case, they constitute a very large proportion of the world existing building stock.
In particular, the scarcity of research efforts concentrating on the derivation of analytical fragility curves for masonry buildings has suggested the development of a new procedure. Due to the lack of reliable models for performing nonlinear analyses of masonry buildings, most of the analyses carried out for evaluating vulnerability of masonry structures and assessing the appropriate strategy for retrofitting them have been confined to elastic and/or “equivalent” linear models. This is also due to the inherent complexity and relative unfamiliarity with nonlinear dynamic analyses, which are more computationally demanding and time consuming than any other procedure.
In this study, a new advanced methodology for obtaining analytical fragility curves for classes of buildings is proposed and applied to a prototype building, in order to illustrate all its steps. Such approach is based on nonlinear static and dynamic analyses of the whole building, taking advantage of the capabilities of the software TREMURI, a frame-type macro-element global analysis program developed at the University of Genoa [Galasco *et al.*, 2006], which is able to perform nonlinear time history analyses on masonry buildings [Penna, 2002].
First of all, the assumptions adopted for identifying the four considered mechanical damage states are presented. Monte Carlo simulations are used to draw the values of each mechanical parameter from appropriately defined probability distributions. Nonlinear static (pushover) analyses are then performed on the numerical model of the building and probability distributions of the different damage states are obtained. Such distributions are then convolved with the cumulative distribution of the displacement demand imposed on the structure by different levels of ground motion, obtained from time history analyses with real accelerograms. Such convolution provides the fragility points, which will then be fitted by lognormal distributions, in order to obtain analytical fragility curves.

2. DEFINITION OF MECHANICAL DAMAGE STATES

Four mechanical damage states have been considered for the derivation of fragility curves, as shown in Figure 1, where they are identified on the global pushover curve of the structure. Two of these damage states can be identified from the response of a single masonry pier, while the other two are found from the global response of the building.

The first damage state, DS1, is identified by the attainment of the yield displacement δ_y of the bilinear approximation to the capacity curve of a single masonry pier, as indicated in Figure 1. The bilinear approximation is obtained by fixing the initial stiffness as being secant to the point corresponding to 70% of the maximum resistance and the equivalent resistance so that the area below the bilinear curve up to the ultimate limit state coincides with the area below the capacity curve up to the same point [OPCM 3274/2003]. It should be noticed that δ_y is always larger than the drift corresponding to 70% of the shear resistance and hence it is likely that some cracks have already occurred at such deformation value. The second damage state, DS2, is identified by the drift corresponding to the first shear cracking of the pier, δ_s and is shown again in Figure 1. This value can be derived from experimental test results, whenever available. In this work, values of δ_s have been obtained directly from the experimental test report, where they were explicitly indicated. Sometimes they are not explicitly reported since the first cracking is not easy to detect during experimental tests, especially if the wall surface is not painted or covered by plaster. For this reason the experimental values associated to δ_s are typically larger than those associated to δ_y .
The damage levels DS3 and DS4 have been derived from global pushover curves of the building. In particular, DS3 is assumed to correspond to the attainment of the maximum shear resistance, while DS4 corresponds to attainment of 80% of that value.

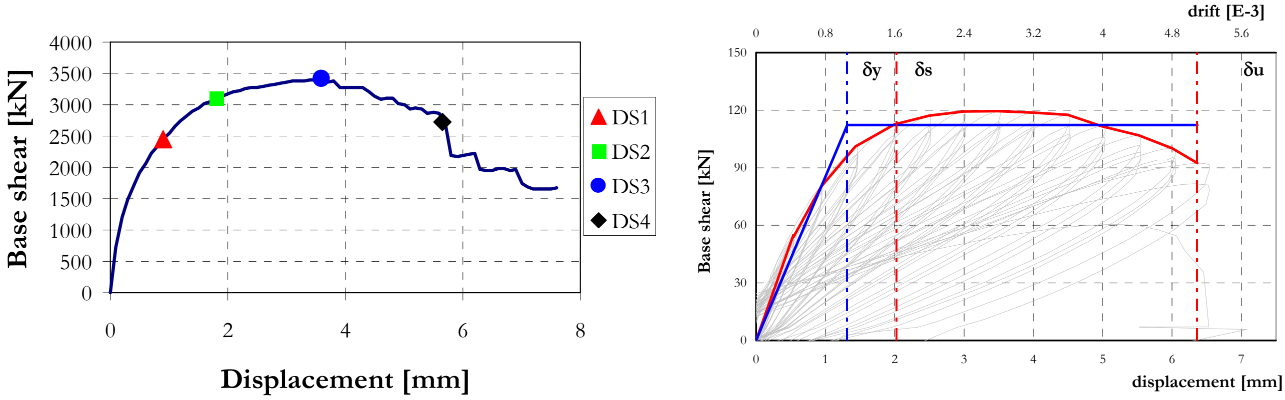


Figure 1 Left: identification of the considered damage states on a pushover curve; right: identification of the yield, cracking and ultimate drifts on the pushover curve of a single pier and its bilinear approximation.

3. STOCHASTIC MONTE CARLO SIMULATIONS

The proposed methodology for deriving analytical fragility curves requires analyses of a population of buildings, subjected to different levels of ground motion. For each building typology, a prototype is identified and the population of buildings belonging to that typology is generated through the treatment of selected structural parameters as random variables. This allows to take into account both the fact that in most real cases building properties are not well known and also the variability intrinsic in the definition of building typologies, grouping together buildings with different characteristics and mechanical properties.
For each random variable, a Gaussian probability distribution function is defined, based on realistic ranges of variation derived from experimental tests and other sources of information. Values of the parameters are then extracted from the distributions using appropriate sampling techniques. Finally, the sampled values are combined to define a series of structures with different characteristics, all nominally representing the same building.
Clearly, consideration of all possible uncertainties in the global and local structural characteristics and ground motion yields an extremely large number of permutations for the analysis. In the proposed approach, the geometry of the building is treated deterministically, whilst all the mechanical properties of masonry, necessary for the definition of the building model, have been considered as random variables.
The Monte Carlo algorithm has been used for sampling the uncertain parameters from their associated probability distributions. Monte Carlo analyses have been performed using a software called STAC, developed at the CIMNE (International Centre for Numerical Methods in Engineering) of Barcelona [Zàrate *et al.*, 2002]. This software allows to perform stochastic simulations using any type of analysis tool. The user is required to define appropriate probability distributions of the input variables of interest. The software then calls the structural program, which runs the analysis, and returns the probability distributions for the output variables. A number of Monte Carlo simulations is performed until convergence of both the input and output variables to their mean is reached and the value of standard deviation

becomes stable.

Two sets of Monte Carlo analyses have been carried out. In the first type, values of the mechanical properties have been varied within the selected ranges, and randomly assigned uniformly over the entire building model. In the second type, an additional randomness was introduced, with the creation of a library of randomly defined materials, which are then randomly assigned to the different structural elements of the model.

4. PROTOTYPE BUILDING

4.1. Building description and model

The selected building prototype is a 3-storey masonry building in Benevento (southern Italy), typical of the construction typologies of the neighbourhood called Rione Libertà, in which it is located. It has been constructed in 1952 and has plane dimensions of 17.7 x 14.3m and a height of 11.05m. The masonry bearing structure is entirely realised with tuff units, while floors are made of reinforced concrete. Reinforced concrete tie beams guarantee the connection between floors and masonry walls. A picture of the selected building and the typical plan of the building (which is practically identical at all storeys) are reported in Figure 2. The geometry of the structure has been considered deterministic, whilst, for what concerns the material characteristics, they have been assumed as random variables, since this building is a prototype considered to be representative of a class of similar buildings.

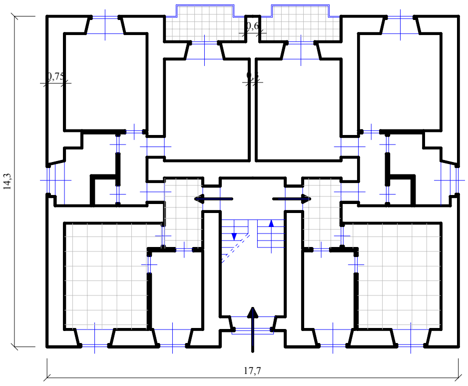


Figure 2 Prototype building located in Benevento: a picture (left) and the plan of a storey (right).

The building has been modelled using the program TREMURI, which allows to perform nonlinear seismic analyses of entire unreinforced masonry buildings. The software and the algorithms embedded in it are described in detail in several literature works [e.g. Penna, 2002; Lagomarsino and Penna, 2003; Galasco *et al.*, 2004]. The program is based on the nonlinear macro-element model proposed by Gambarotta and Lagomarsino [1996], further modified by Penna [2002], representative of a whole masonry panel (pier or spandrel beam).

3D modelling of the building is based on the identification within the construction of the seismically resistant structure, constituted by walls and floors. The walls are the bearing elements, while the floors, apart from sharing vertical loads to the walls, are considered as planar stiffening elements (orthotropic 3-4 nodes membrane elements), on which the horizontal actions distribution between the walls depends.

The local flexural behaviour of the floors and the wall out-of-plane response are not computed because they are considered negligible with respect to the global building response, which is governed by their in-plane behaviour. This is acceptable for this type of building, where stiff diaphragms and low height/thickness ratios for the walls render out-of-plane response a secondary phenomenon. Notice that a global seismic response is possible only if vertical and horizontal elements are properly connected.

A frame-type representation of the in-plane behaviour of masonry walls is adopted: each wall of the building is subdivided into piers and lintels (2 nodes macro-elements) connected by rigid areas (nodes).

4.3. Ranges of variations of the mechanical parameters

The building under study is made of tuff masonry. Among the different experimental tests available in the literature for such type of masonry, the tests carried out by Faella *et al.* [1991] have been used for identifying realistic ranges of variation of the various parameters needed for the numerical model. All the tests carried out on the walls of type T1 and T2 have been simulated with the program TREMURI, in order to identify meaningful ranges of variation for the

different model parameters. Different cyclic pushover analyses were carried out, playing with the model parameters, until a good fit of the experimental tests' results was obtained. The comparison, in terms of cyclic force-displacement curves, between experimental results and numerical results can be found in Rota [2007].
Based on the comparison with experimental results, it has been possible to calibrate only the parameters related to shear failure modes, since all experimental specimens failed in shear. In particular, the following parameters have been obtained: shear modulus G, initial shear resistance for zero compression (cohesion) f_{vo}, friction coefficient μ, nonlinear shear deformability ratio G_c, softening parameter β, ultimate shear drift δ_v. The results of the nonlinear identification can be found in Rota [2007]. The missing parameters, which cannot be directly obtained from cyclic tests, have been derived from the indications reported in the Annex 11.D of the OPCM 3274 [2003]. In addition, for what concerns the ultimate flexural drift δ_f, a value of 0.8% has been adopted as the mean value (as also suggested in EC8-3 [2005]), with an assumed coefficient of variation of 10%. The minimum and maximum values reported in Table 4.1 have been adopted. Normal distributions have been assumed for all the mechanical parameters, with a mean value corresponding to the central value of the interval and a standard deviation such that the extreme values of the interval approximately correspond to the 95% percentile. The parameters of the normal distribution of each mechanical quantity needed for the numerical analysis are also summarised in Table 4.1.

Table 4.1 Values of the parameters used in stochastic analyses

	E [MPa]	G [MPa]	f_m [MPa]	f_{vo} [MPa]	μ [-]	G_c [-]	β [-]	δ_v [%]	δ_f [%]	δ_s [‰]	δ_y [‰]
Min	1350	500	1.2	0.105	0.05	4	0.2	0.52	-	-	-
Max	1890	750	2.7	0.2	0.08	10	0.4	0.78	-	-	-
Mean	1620	625	1.95	0.1525	0.065	7	0.3	0.65	0.8	1.825	0.905
St. dev.	135	62.5	0.375	0.02375	0.0075	1.5	0.05	0.065	0.08	0.1125	0.1275

In order to define the damage states, a probability distribution of values of the drift corresponding to first cracking is also required. The interpretation of the experimental tests provides two sets of drift values: the first (δ_s) is obtained directly from the tests and is defined as corresponding to the first evidence of shear cracking; the second (δ_y) is derived from the bilinear approximation of the envelope capacity curve and represents the drift associated to the elastic limit of the equivalent bilinear curve. The values of these two parameters for the different tests are reported in Rota [2007], while the parameters of the probability distributions adopted in the damage state definition are summarised in Table 4.1.

5. NONLINEAR STATIC (PUSHOVER) ANALYSES

Pushover analyses have been carried out only in the x direction, since the building appears to be more vulnerable in x than in y; moreover the first vibration mode is along the x direction. The force distribution proportional to the first vibration mode has been selected. Damage in spandrel beams has been neglected for the individuation of the limit states of interest: this is due to the fact that the model has been calibrated on experimental tests on masonry piers, while results concerning spandrel beams were not available (as it is usually the case). In some cases, results of pushover analyses may be strongly affected by the choice of the control node; however, this is not an issue for this building, since floors are rigid and hence any node at the top storey can be assumed as the control one.
Two different sets of stochastic pushover analyses have been carried out, each one consisting of 1000 analyses. In this paper only the second set will be discussed, in which not only are the mechanical parameters random variables, but also the material associated to each macro-element is randomly selected from a predefined library of materials. In particular, 30 different materials have been defined for the 165 structural elements of the model; each material is characterised by values of the mechanical parameters randomly selected from the intervals. In each analysis, materials are redefined. The number of analyses performed was sufficient to observe convergence of each input and output variable to its mean value and stabilisation of the standard deviations.
The left part of Figure 3 shows some of the pushover curves obtained, with identification of the mean curve and the mean plus or minus one standard deviation. From each pushover curve it is possible to identify two global limit states, one corresponding to the attainment of the maximum base shear resistance, the other corresponding to the ultimate condition, identified by a value of base shear deterioration corresponding to 20% of the maximum base shear. Since a

set of 1000 analyses is available, it is possible to define a distribution of values for these two limit states. Both the mean value and the mean plus or minus one standard deviation values obtained form the analyses are identified in the left part of Figure 3 by vertical lines.
To derive a probability distribution associated to damage state DS2, it is necessary to define a relationship (in probabilistic terms) between global structural displacement and corresponding maximum element drift. The results for some of the performed analyses are reported in Figure 3 (right), where also the average relationship is highlighted. The results are not much scattered from the mean curve; the curves' dispersion is not symmetrical around the mean curve, possibly because only some curves (230) have been reported in the plot.

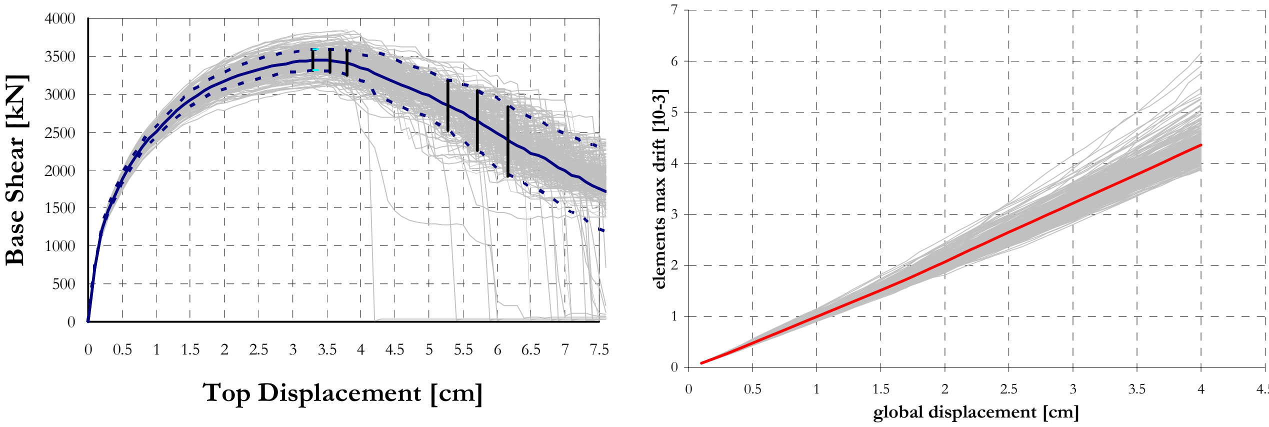


Figure 3 Left: Some of the obtained pushover curves, with identification of the mean and mean plus or minus one standard deviation curves and of global damage states (vertical lines). Right: relationship between global structural displacement and maximum element drift. The thick line represents the average curve.

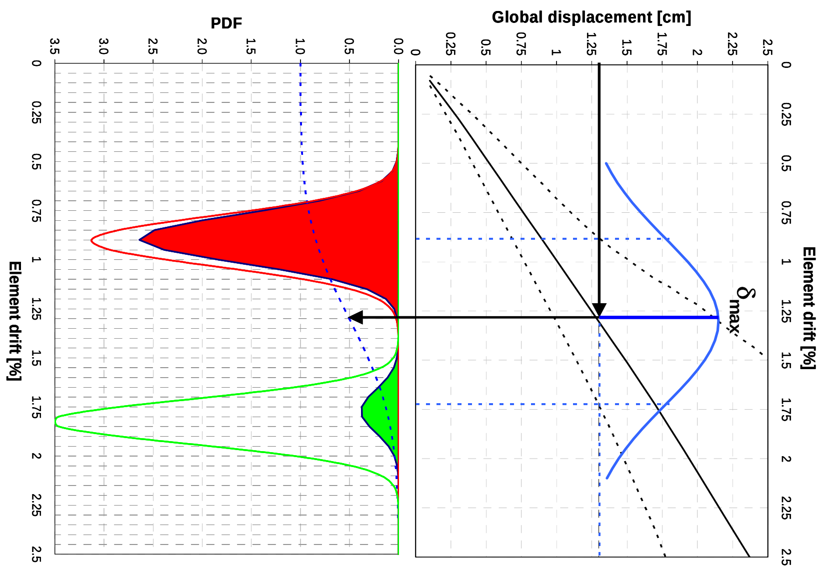


Figure 4 Scheme of the procedure used for the identification of drift-dependent limit state probabilities

For each value of global displacement, the probability distribution of the maximum value of element drift is known. The probability that this value of drift exceeds the drift threshold associated to the considered damage states must be defined. The drift threshold of each damage level is also defined in terms of a probability distribution. To obtain the final probability distribution relating DS1 and DS2 to the global structural displacement, it is necessary to perform the convolution of the two described probability distributions. Figure 4 qualitatively describes the procedure followed: for each global displacement value, the drift demand on the elements is evaluated and represented by a probability distribution of values (right part of Figure 4). The complementary cumulative distribution function of such values (blue dashed line in the left part of the figure) is then convolved with the probability density function of the two limit state thresholds (red and green lines in the left part of the figure). The result of the convolution for each DS is represented by the coloured areas in the figure. Such areas represent the integral of the joint probability distributions.

6. NONLINEAR DYNAMIC (TIME HISTORY) ANALYSES

6.1. Selection of accelerograms

In order to carry out incremental dynamic analyses, an appropriate set of acceleration time histories is required. In this study, 7 real accelerograms have been selected from strong motion record databases (www.isesd.cv.ic.ac.uk/, peer.berkeley.edu/smcat/, db.cosmos-eq.org/), with the constraint of the spectrum-compatibility with the target response spectrum of the EC8, which corresponds to the one of the Italian seismic code [OPCM 3274, 2003], for a seismic zone 2, with an associated PGA of 0.25g.

Accelerograms have been selected through an algorithm that is described in detail in Dall'Ara *et al.* [2006]: it is based on a Monte Carlo random selection of the groups of accelerograms better fitting the target spectrum.

Notice that a minimum of 7 accelerograms must be applied to the structure, to be allowed to use average results instead of the most unfavourable ones, as suggested by several modern seismic codes [OPCM 3274, 2003; UBC, 1997; EC8-1, 2005] and by research works [e.g. Bommer *et al.*, 2003].

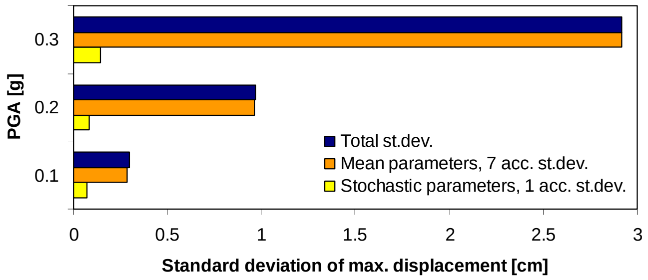
Finally, the selected accelerograms have been scaled linearly in order to match their PGA with the target PGA of the selected response spectrum. Notice that the scale factors applied to the accelerograms were all quite close to one, within 0.6 and 1.1.

6.2. Results of time history analyses

A set of incremental time history analyses has been performed by applying to the structural model the accelerograms defined in the previous section. A value of Rayleigh critical damping equal to 2% has been assumed for all the analyses. In order to take into account both the variability due to different ground motions (7 selected accelerograms) and the variability of mechanical parameters in the structural model, a very large number of analyses would be required. Since each nonlinear time history analysis requires a significant computational time, it is necessary to reduce the number of analyses, by making some assumptions.

A first set of time history analyses has been carried out using the mean value of each mechanical parameter and applying all the 7 accelerograms scaled so that the mean PGA matches 0.05, 0.1, 0.15, 0.2, 0.25 and 0.3g. These analyses will be referred in the following as the deterministic set. A second set of analyses has been carried out, taking into account the variability due to material parameters and neglecting the variability induced by the different ground motion records. In this case, 100 realisations of each material parameter have been generated through Monte Carlo simulations and a corresponding number of analyses has been carried out, for the values of PGA = 0.1, 0.2 and 0.3g and applying only the first accelerogram.

In order to check the assumption that the variability related to ground motion has a stronger influence on displacement demand than the one due to mechanical parameters, for each considered PGA level, the total standard deviation associated to displacement demand has been calculated as the square root of the sum of the squares of the two standard deviations associated respectively to ground motion and mechanical parameters variability. The results are summarised in Figure 5, where the total standard deviation obtained is compared to the standard deviations corresponding to the deterministic case (mean mechanical parameters and 7 accelerograms) and the probabilistic case (stochastic mechanical parameters and a single accelerogram). Observation of the plot shows that, for this prototype and with the selected ground motions, the variability due to mechanical properties is significantly smaller then the variability associated to different ground motions. Hence, it seems acceptable to neglect the variability associated to different values of material parameters and work with the deterministic hypothesis. However, further analyses, with different ground motion records, would be needed to confirm the results obtained.



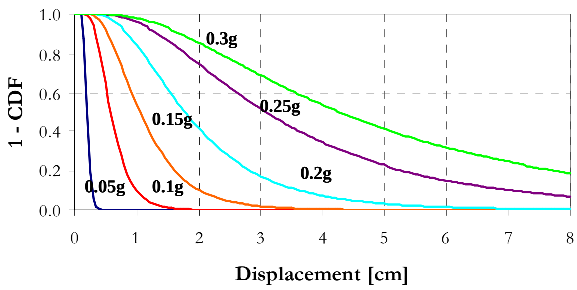
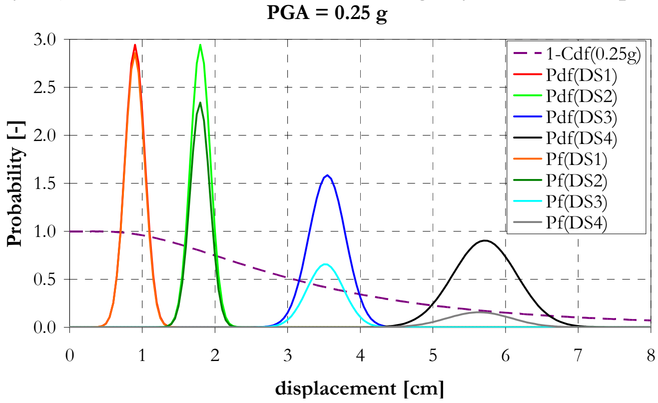


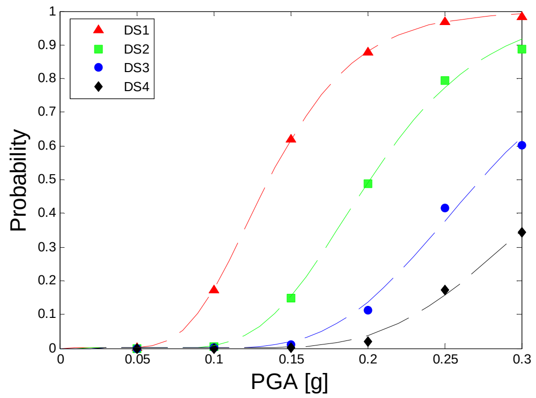
Figure 5 Left: comparison of standard deviations obtained with mean mechanical parameters and 7 accelerograms, with stochastic parameters and only 1 accelerogram and combining the two cases; right: complementary cumulative distribution functions of the displacement demand at various PGA levels.

At this point, the probability distribution functions of displacement demand at different values of PGA are needed, since they will be convolved with the distributions of each damage state, in order to derive fragility curves. In particular, the complementary cumulative distributions, reported in the right part of Figure 5 will be used for such purpose. Such complementary cumulative distributions of maximum displacement demand have been obtained from time history analyses with the 7 accelerograms at each value of PGA. From the results of such analyses, the parameters of the corresponding lognormal distributions have been obtained, by identifying on the hysteretic cycles obtained from time history analyses with the 7 accelerograms scaled to the different PGA values, the mean and the standard deviation of the maximum displacement demand.

7. DERIVATION OF FRAGILITY CURVES

The convolution of the probability distributions of the different damage states (defined from pushover analyses) and the displacement demand imposed on the building by different levels of ground motion (evaluated from time history analyses) can be convolved to obtain fragility curves. The procedure is graphically illustrated in Figure 6 (left).





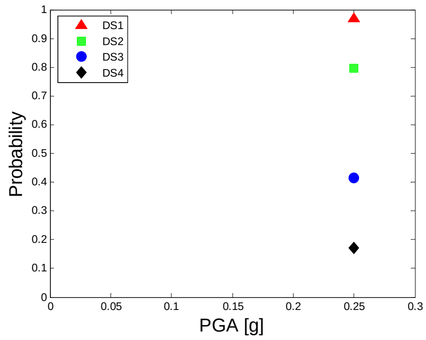


Table 7.1 Parameters of the lognormal distributions fitting the analytical fragility points

DS	μ	σ
DS1	-2.026	0.362
DS2	-1.645	0.273
DS3	-1.351	0.218
DS4	-1.169	0.175

Figure 6 Left: Derivation of the fragility points (bottom) for a PGA = 0.25g, from convolution of probability distributions of limit states and complementary CDF of demand (top); right: lognormal fragility curves fitting the analytically derived points (top) and corresponding parameters (bottom).

The top left part of the figure shows the complementary cumulative distribution function (purple dashed line) of the demand and the probability distributions of the various damage states, for a PGA of 0.25g. These curves are convolved and the obtained curves are shown, for each damage state, with a similar but slightly different colour (lower curves). The integral of the area below these curves gives the probability of exceedence of that damage state, for a PGA of 0.25g, i.e. the fragility points for this PGA level (reported in the bottom part). This procedure is then repeated for all the considered PGA values and the various fragility points obtained have been fitted using a lognormal probability distribution function. The points obtained and the lognormal curves fitting them are plotted in the top right part of Figure 6, while the parameters (μ and σ) of the lognormal curves are summarised below.

8. CONCLUSIONS

For the first time, a methodology has been presented for the derivation of analytical fragility curves for classes of

masonry buildings, based on detailed 3D nonlinear dynamic analyses of entire structures. This is significantly different from previously existing approaches, often based on very simplified models of the buildings and approximate analysis types (linear or nonlinear static analyses). The proposed approach is based on the results of nonlinear stochastic analyses of a prototype building, considered to be representative of a building typology. All the mechanical properties of the structure are assumed to be random variables, to which normal probability distributions are associated based on realistic ranges of variation. Even the association of mechanical parameters to each structural element is treated as a random variable. Monte Carlo simulations are then used to extract the input parameters from such distributions and then advanced nonlinear analyses of the whole structure are performed. The probability distributions of selected damage thresholds are determined from pushover analyses and then convolved with the probability distribution of displacement demand obtained from nonlinear time history analyses. Lognormal fragility curves are hence obtained, by fitting the fragility points representing the probability of exceeding different damage levels for discrete PGA levels.
Clearly the proposed approach is still at a methodological stage and needs to be further tested and improved by applying it to different building prototypes representative of the same typology and also to structures belonging to different types.

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