

A COMPARATIVE STUDY OF THE SEISMIC ANALYSIS OF RECTANGULAR TANKS ACCORDING TO DIFFERENT CODES

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ABSTRACT :

As known from very upsetting experiences, liquid storage tanks were collapsed or heavily damaged during the earthquakes all over the word. Damage or collapse of the tanks causes some unwanted events such as shortage of drinking and utilizing water, uncontrolled fires and spillage of dangerous fluids. Due to this reason numerous studies done for dynamic behavior of fluid containers; most of them are concerned with cylindrical tanks. But very few studies on the seismic response of rectangular containers exist. Therefore this study is to be mostly about rectangular tanks. Comparative seismic analyses are carried out for a rectangular tank considering requirements and expressions given in seismic codes prepared for USA, European Community and New Zealand. Turkish seismic code is also discussed. Hydrodynamics pressures, sloshing displacements, base shears, lateral displacements, bending and overturning moments are determined for considered codes and specifications. Finally, results obtained this study are evaluated and some conclusions related to the encountered differences and similarities for seismic analysis and design of rectangular tanks.

KEYWORDS: Rectangular tank, seismic analysis, codes, hydrodynamic pressure, base shears

1. INTRODUCTION

It is known that, some of the fluid containers are damaged in many earthquakes. Damage or collapse of these containers causes some unwanted events such as shortage of drinking and utilizing water, uncontrolled fires and spillage of dangerous fluids. Even uncontrolled fires and spillage of dangerous fluids subsequent to a major earthquake may cause substantially more damage than the earthquake itself (Priestley et al. 1986). Due to these reasons this type of structures which are special in construction and in function from engineering point of view must be constructed well to be resistant against earthquakes. There have been numerous studies done for dynamic behavior of fluid containers; most of them are concerned with cylindrical tanks. But very few studies on the dynamic response of rectangular containers exist (Rammerstorfer et al 1990).

Hoskins and Jacobsen gave the first report on analytical and experimental observations of rigid rectangular tanks under a simulated horizontal earthquake excitation (Hoskins and Jacobsen 1934). Graham and Rodriguez used spring-mass analogy for the fluid in a rectangular container (Graham and Rodriguez 1952). Housner proposed a simple procedure for estimating the dynamic fluid effects of a rigid rectangular tank excited horizontally by earthquake (Housner 1963). An extended application of Housner's concept in the sense of a practical design rule is given by Epstein (Epstein 1976). After these important studies some researchers are carried out about rectangular fluid containers. These researches may be summarized as below:

- Studies related to flexibility of walls (Minowa 1980; Priestley et al 1986; Doğangün et al. 1996, 2004; Kim et al 1996; Koh et al 1998)
- Studies related to seismically induced bending moments in walls (Haroun 1984)
- Studies related to sloshing (Bauer and Eidel 1987; Lepettier and Raichlen 1988; Haroun and Chen 1989, Livaoğlu 2008)
- Studies related to soil-structure interaction (Kim et al 1998)
- Studies related to seismic isolation (Park et al 2000)
- Experimental studies (Minowa 1984, Koh et al 1998)



European Committee for Standardization prepared a new code named Eurocode-8 (2006). Part 4 of this code is related to tanks, silos and pipelines. But, requirements for rectangular tanks are very limited according to cylindrical tanks in this code. There is a statement related to rectangular storage tanks as "studies on the behavior of flexible rectangular tanks are not numerous, and the solutions are not amenable to a form suitable for direct use in design" in this code. Therefore, it is explained that the method suggested by New Zealand Code (Priestley et al. 1996) may be used as an approximation for design.

2. DYNAMIC MODEL

The tank-liquid system may be modeled by two single-degree-of-freedom systems, one of them represents the impulsive component, moving together with the flexible wall, and the other corresponding to the convective component as shown in Figure 1a. Furthermore sometimes tanks categorized as shallow and deep tanks. For deep tanks (H/L>1,5) impulse mass may consisting of two masses as shown in Figure 1b. Using this equivalent dynamic model, it is possible to calculating the resultant seismic forces acting on a ground-based liquid container with rigid walls. This model has been accepted by the profession for the past over 30 years (ACI 350 2001). The impulsive and convective masses m_i and m_c are given in the figure as fractions of the total liquid mass m, along with the heights from the base of the application point of the resultant of the impulsive and convective hydrodynamic wall pressure, h_i and h_c .



Figure 1 Dynamic model of liquid-containing tank rigidly supported on the ground.

2.1. Eurocode-8 requirements

Values of the dynamic model parameters for cylindrical tank are given in Table 1. But it is stated in Eurocode 8 that these values may be adopted for the design of rectangular tanks as well (with L replacing R), with an error less than 15%.

Table 2.1 Coefficients C_i and C_c for the natural periods, masses m_i and m_c and heights h_i and h_c from the base of the point of application of the wall pressure resultant, for the impulsive and convective components (h_i and h_c excluding base pressure; h'_i and h'_c including base pressure)

H/L	C_i	C_c (s/m ^{1/2})	m_i/m	m_c/m	h_i/H	h_c/H	h'_i/H	h'_c/H
0.3	9.28	2.09	0.176	0.824	0.4	0.521	2.64	3.414
0.5	7.74	1.74	0.3	0.7	0.4	0.543	1.46	1.517
0.7	6.97	1.6	0.414	0.586	0.401	0.571	1.009	1.011
1	6.36	1.52	0.548	0.452	0.419	0.616	0.721	0.785
1.5	6.06	1.48	0.686	0.314	0.439	0.69	0.555	0.734
2	6.21	1.48	0.763	0.237	0.448	0.751	0.5	0.764
2.5	6.56	1.48	0.81	0.19	0.452	0.794	0.48	0.796
3	7.03	1.48	0.842	0.158	0.453	0.825	0.472	0.825

The total base shear is:

$$V = (m_i + m_w + m_r)S_e(T_{imp}) + m_cS_e(T_{con})$$



where: m_w mass of the tank wall, m_r mass of tank roof; $S_e(T_{imp})$ impulsive spectral acceleration, obtained from an elastic response spectrum for a value of damping consistent with the limit state; $S_e(T_{con})$ convective spectral acceleration, from a 0,5%-damped elastic response spectrum. Eurocode 8 recommends equations for T_{imp} and T_{con} for cylindrical tanks not rectangular tanks.

For flexible rectangular tank walls, the period of vibration for the first impulsive storage tank-fluid horizontal made is given approximately by:

$$T_f = 2\pi \sqrt{d_f / g} \tag{2.1}$$

Where d_f is the deflection of the wall on the vertical centre-line and at the height of the impulsive mass, when wall is loaded by a load uniform in the direction of the ground motion and of magnitude $m_i g / (4BH)$. Where B is the half with perpendicular to the direction of loading (earthquake direction) and m_i is the impulsive mass. This mass can be obtained from the equivalent cylindrical tank results and should include the wall mass (Eurocode-8 2006). For thanks without roofs the deflection d_f may be calculated assuming the wall to be free at the top and fixed on the other three sides.

The period of oscillation of the first sloshing mode for rectangular tank is:

$$T_1 = 2\pi \sqrt{\frac{L/g}{\frac{\pi}{2} \tanh\left[\frac{\pi}{2}\frac{H}{L}\right]}}$$
(2.2)

The overturning moments immediately above (M) and below (M') the base plate are

$$M = (m_i h_i + m_w h_w + m_r h_r) S_e(T_{imp}) + m_c h_c S_e(T_{con})$$
(2.3)

$$M' = (m_i h_i + m_w h_w + m_r h_r) S_e(T_{imp}) + m_c h_c S_e(T_{con})$$
(2.4)

2.1. ACI 350 requirements

Equivalent masses (m_i and m_c) and heights (h_i , h_c , h'_i and h'_c) of accelerating liquid can be determined from Figure 2 and 3 depending on 2L/H ratios.



Figure 2 Factors m_i/m and m_o/m versus ratio 2L/H for rectangular tanks (ACI 350 2001).





Figure 3 Factors h_i/H and h_o/H ; h'_i/H and h'_o/H versus ratio 2L/H for rectangular tanks (ACI 350 2001).

The combined natural period of vibration of the containment structure and the impulsive component of the stored liquid can be determined below equation:

$$T_i = 2\pi \sqrt{\frac{(m_w + m_i)}{k}} \tag{2.5}$$

For fixed-base, open-top tanks, flexural stiffness k and in which h may be determined the following equations:

$$k = \frac{E_c}{4x10^6} \left(\frac{t_w}{h}\right)^3; \quad h = \frac{(h_w \cdot m_w + h_i \cdot m_i)}{(m_w + m_i)}$$
(2.6)

Period of the vibration of the convective component of the stored liquid shall be estimated as;

$$T_c = \frac{2\pi}{\sqrt{3.16g \cdot \tanh[3.16(H/2L)]}} \sqrt{2L}$$
(2.7)

The dynamic lateral forces and moments shall be determined using following equations:

$$P_{w} = Z \cdot S \cdot I \cdot C_{i} \frac{\varepsilon \cdot W_{w}}{R_{wi}} \quad \text{where} \quad \varepsilon = \left[0.0151 \left(\frac{2L}{H}\right)^{2} - 0.1908 \left(\frac{2L}{H}\right) + 1.021 \right] \le 1.0; \quad M_{w} = P_{w} \cdot h_{w} \quad (2.8)$$

$$P_r = Z \cdot S \cdot I \cdot C_i \frac{W_r}{R_{wi}} \quad ; \quad M_r = P_w \cdot h_r \tag{2.9}$$

$$P_i = Z \cdot S \cdot I \cdot C_i \frac{W_i}{R_{wi}} \quad ; \quad M_i = P_i \cdot h_i \quad ; \quad M'_i = P_i \cdot h'_i \tag{2.10}$$

$$P_c = Z \cdot S \cdot I \cdot C_c \frac{W_c}{R_{wc}} \quad ; \quad M_c = P_c \cdot h_c \quad ; \quad M_c' = P_c \cdot h_c' \quad (2.11)$$

Where Z seismic zone factor, S soil profile coefficient, I importance factor, C_c and C_i period-dependent spectral amplification factors for the horizontal motion of the convective component (for 0.5% of critical damping) and the impulsive component (for 5% of critical damping), R_w response modification factor

The base shear due to seismic forces applied at the bottom of the tank wall shall be determined by the following equation:

$$V = \sqrt{(P_i + P_w + P_r)^2 + P_c^2}$$
(2.12)



Bending moment (M_b) on the entire tank cross section just above the base of the tank wall and overturning moment (M_o) at the base of the tank, including the tank bottom and supporting structure (IBP) are given as:

$$M_b = \sqrt{(M_i + M_w + M_r)^2 + M_c^2}$$
(2.13)

$$M_o = \sqrt{(M'_i + M_w + M_r)^2 + M'_c^2}$$
(2.14)

3. HYDRODYNAMIC PRESSURES

Static and dynamic pressure distributions for rectangular tank wall are shown in Figure 4.



Figure 4 Static and dynamic pressures acting on tank wall

The total hydrodynamic pressure (*p*) is given by the sum of an impulsive (p_i) and a convective (p_c) contribution: for the tanks whose walls can be assumed as rigid can be determined below equations:

$$p(z,t) = p_i(z,t) + p_c(z,t)$$
(3.1)

$$p_i(z,t) = q_0(z) \cdot \rho \cdot L \cdot A_g(t) \tag{3.2}$$

$$p_{cn}(z,t) = q_{cn}(z)\rho \cdot L \cdot A_n(t)$$
(3.3)

where *L* is the half-with of the tank in the direction of the seismic action, $q_0(z)$ is the function plotted in Figure 5 and $A_g(t)$ is the ground acceleration. In this figure *H* is depth of stored liquid, $q_{cn}(z)$ is shown Figure 6 for first and second sloshing modes and $A_n(t)$ is the acceleration response function of the *n*. mode



Figure 5. Dimensionless impulsive pressure on rectangular fluid container wall (Priestley et al. 1986)





Figure 6. Dimensionless convective pressure on rectangular fluid container wall (Priestley et al. 1986)

It is concluded in Eurocode-8 that wall flexibility produces generally a significant increase of the impulsive pressures while leaving the convective pressures practically unchanged. So, for flexible rectangular storage tanks, an approximation which is suggested in Priestley et al (1986) is to use the same pressure distribution valid for rigid walls. But ground accelerations $A_g(t)$ in Eq. (15) replaced with the response acceleration of a simple oscillator having the frequency and the damping factor of the first impulsive tank-fluid mode.

ACI 350 recommends the following equation for impulse and convective pressure distribution for rectangular tanks.

$$p_{i} = \frac{\frac{ZSIC_{i}W_{i}}{2R_{wi}} \left[4H - 6h_{i} - (6H - 12h_{i})\frac{z}{H} \right]}{H^{2}} ; \quad p_{c} = \frac{\frac{ZSIC_{c}W_{c}}{2R_{wc}} \left[4H - 6h_{c} - (6H - 12h_{c})\frac{z}{H} \right]}{H^{2}} \quad (3.4)$$

4. NUMERICAL EXAMPLE

In this study, a rectangular storage tank with two different wall thicknesses is considered as shown in Figure 7. In the example, density of fluid and wall material are taken to be 1000 kg/m^3 , 2400 kg/m³ respectively. Damping ratio for fluid is taken to be 0.5% for convective component and the 5% for impulsive component.



Figure 7 Plan and elevation of the sample rectangular tank



In this solution; density, Young's modulus and Poisson's ratio of structural material are taken to be 2400 kg/m³, 21x10⁹ N/m² and 0.17, respectively. Wall thickness is taken to be t_w =0.5m and the wall is fixed to the ground. It is assumed that the tank is high seismic zone 0.4g. Taken values for ACI 350: Z=0.4, I=1.25, S=1.2, R_{wt} =2.75, R_{wc} =1.0. Similar values are taken suitable with ACI 350 as: A_{Ek} =0.4g, γ_I =1.25, S=1.2, q=(2.75 for impuls, 1.0 for convective). Type 1 spectrum is selected.

Impulsive mass, convective mass and equivalent heights related to these masses, periods, base shear, bending and overturning moments and wave height are given Table 4.1.

Table 2.1 Determined dynamic parameters and resultant forces and moments considering requirements in Eurocode 8 and ACI 350

Codes	m_i	m_c	h_i	h_c	h'_i	h'_c	T_{imp}	T_{con}	V	M_b	M_o	d_{max}
	(kg)	(kg)	(m)	(m)	(m)	(m)	(sec)	(sec)	(kN)	(kNm)	(kNm)	(m)
Eurocode 8	4527000	4473000	3.717	5.409	7.353	7.743	0.52	5.37	37808	161603	259988	-
ACI 350	4482068	4693941	3.375	5.131	7.913	8.405	0.59	5.35	45858	162462	352260	1.26

The hydrodynamic pressure distributions acting on tank wall are given Figure 8. As seen from this figure hydrodynamic pressure distribution obtained for rigid wall assumption is smaller than the code defined pressure distributions along the whole wall height.



Figure 8 Hydrodynamic pressure distributions obtained this study

5. CONCLUSIONS

ACI 350 and Eurocode 8 Part 4 main requirements for rectangular tank were presented. Analyses were carried out for sample rectangular tank.

ACI 350 gives smaller impulsive mass and bigger convective mass than those obtained for Eurocode 8. Smaller equivalent heights related to these masses for bending moment, but larger heights for overturning moment obtained for ACI 350.

The most difficult parameter to determine for engineers is impulsive period of the rectangular tank. Eurocode 8



gives practical formulation for impulsive period for cylindrical tanks. Lateral displacement should be determined to estimate this period.

The hydrodynamic pressure distribution and magnitude obtained by Eurocode 8 and ACI 350 are generally in agreement. The differences between hydrodynamic pressures were occurred top and bottom of the tank wall.

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