INFLUENCE OF DYNAMIC DISTURBANCE ON EVOLUTION AND STABILITY OF DEEP UNDERGROUND ROCK MASS

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ABSTRACT :

According to the self-organizing characteristic of rock system during evolution, synergetics is applied to investigate the evolution process by taking dynamic disturbance into account, and a generalized dynamical equation is established. And then a potential function equation is derived from analysis of rock mass structure. Furthermore, by using potential function method, we analyzed the influence of dynamic disturbance on evolution and stability of rock system. The relationship between critical point of instability and parameters of control parameter and disturbance intensity is also discussed and a formula which can be used to forecast critical point of instability is derived. Results indicated that dynamic disturbance can not only decrease the stability of rock system but also induce its instability under certain conditions. Under same static states, stability and feasibility of disturbance induced instability might increase along with increasing of fluctuation intensity. The capability of destroying rock mass for a given disturbance is also effected by the state or control parameters of rock mass.

KEYWORDS: dynamic disturbance, rock mass, stability, induced instability, evolution

1. INTRODUCTION

Rock burst and shock bump are typical dynamic disasters in deep mining, to seek for their generation mechanism is the basis of hazards prediction and control. In recent years, more and more evidence show that deep underground rock mass is sensitive to dynamic disturbances, including earthquake vibration and blasting vibration, which are usually the main reason for dynamic disasters. Evidently, to find out the influence of these disturbances on rock failure and instability is beneficial to disaster prevention of deep mining.
Due to the complexity of rock structure and mechanical response, many non-liner methods have been applied to investigate the evolution mechanism and stability of deep rock mass or other disorder materials [*e.g*., Rundle, 1988; Lu, 1996; Silberschmidt, 1996; Yang, 1997; Andersen *et al*, 1997; Zhao, 1998; Main, 1999; Leung and Neda, 2000; Xie *et al*., 2001; Kapiris *et al*., 2004; Jiménez *et al.*, 2007; Shao *et al*., 2007, 2008]. Especially, constructive attempts based on self-organization theory have caught many researchers' attention. In this paper, based on synergetics theory and analysis of rock mass characteristic, we first establish a generalized dynamical equation to describe the evolution process by taking dynamic disturbance into account. And then, we investigate the mechanism of disturbance induced dynamical instability using potential function method. Furthermore, the criticality of dynamic disturbance induced catastrophic failure of rock mass is discussed.

2. GENERALIZED DYNAMICAL EQUATION OF ROCK MASS

2.1. Generalized Langevin Equation

Assuming that $q_1(u,t)$, $q_2(u,t)$, …, $q_n(u,t)$ are space-time variables of a system, then the states of this system can be expressed using vector form $\vec{q}=\{q_1,q_2,\dots,q_n\}$. The change rate of variables is

$$q_1'=K_1+F_1(q_1,q_2,\ldots,q_i,\ldots,q_n),\ \ldots,\ q_n'=K_n+F_n(q_1,q_2,\ldots,q_i,\ldots,q_n) \quad (2.1)$$

In terms of synergetics[Haken, 1983], to describe the behavior of the nonlinear dynamical system expediently, the slaving principle can be used to simplify the high-dimensional equations to be low-dimensional equations, which are called order parameter equations. When only one order parameter q_1 in the system, by inserting $q_2'=\ldots=q_i'=\ldots q_n'=0$ in Eqn. 2.1 we obtain

$$q_2=H_2(q_1),\ \ldots,\ q_i=H_i(q_1),\ \ldots,\ q_n=H_n(q_1) \quad (2.2)$$

Substituting Eqn. 2.2 in Eqn. 2.1 we have

$$q_1'=Kq_1+g_1(q_1,q_2,\ldots,q_i,\ldots,q_n)=f(q_1) \quad (2.3)$$

where K is a control parameter and $f(q_1)$ is the nonlinear function of q_1.
To deal with the problem of system evolution using synergetics, a common method is to establish a proper Langevin Equation which can reflect the characteristics of the system state. According to synergetics theory, when outside stochastic disturbance is considered, the generalized Langevin Equation used to describe the evolution process of a system can be expressed as

$$q_1'=f(q_1)+q_1\eta(t) \quad (2.4)$$

where q_1 is the order parameter, $f(q_1)$ is a function of q_1 and $\eta(t)$ denotes the outside stochastic fluctuation with properties given by

$$<\eta(t)>=0,\ <\eta(t)\cdot\eta(t')>=2D\lambda(t-t') \quad (2.5)$$

where λ is a correlation function and D denotes fluctuation intensity. So the idiographic form of $f(q_1)$ should be defined for further discussion.

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2.2. Dynamical Equation of rock mass

To deep underground engineering, excavation will cause the variation of initial stress in rock mass. For brittle rock, the typical fracture mode due to stress redistribution is vertical split, which may form rock block structure along rock wall, as shown in Figure 1.
In order to obtain the evolution rule, the fractured structure is simplified to be a block with both ends built-in. Thus, its bending deformation under stress in rock mass can be expressed as

$$v=M_e[1-\cos(2\pi x_p/L)]/P \quad (2.6)$$

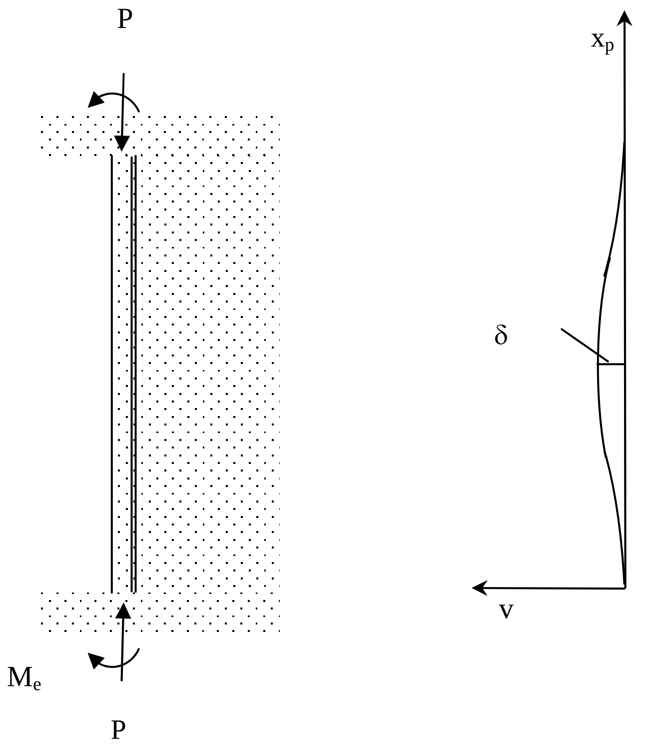
where v is the flexure deformation of arbitrary point along vertical direction of rock block, M_e is moment of couple, P is the external force and L is the length of rock block. Because M_e /P in Eqn. 2.6 equals to half of the maximal deflection δ, Eqn. 2.6 is rewritten as follows

$$v=0.5\delta[1-\cos(2\pi x_p/L)] \quad (2.7)$$

Under compressive stress, the bending deformation of block will cause the increase of elastic potential in it. According to theory of elasticity, the increase of elastic potential V is

$$V=\int_0^L M(E,I,x_p)[\,v''^2/(1+v'^2)^{-3/2}]\,dx_p \quad (2.8)$$

where $M(E, I, x_p)$ is the bending moment of arbitrary point of rock block, E is elastic modulus and I is moment of inertia。



(a) rock block structure (b) simplified model

Figure 1 Deformation of rock block structure and analytical model

To obtain the general expression for V, we first use Eqn. 2.7 to calculate the second order derivatives of v and have

$$v''=2\delta(\pi/L)^2\cos(2\pi x_p/L) \tag{2.9}$$

Then do a Taylor-series expansion of $(1+v'^2)^{-3/2}$. Thus, if we substitute Eqn. 2.9 and the expansion equation in Eqn. 2.8, we have

$$V=\alpha\delta^2-\beta\delta^4 \tag{2.10}$$

where,

$$\alpha= 4(\pi/L)^4\int_0^L[M(E, I, x_p)\cos^2(2\pi x_p/L)]dx_p$$

$$\beta=1.5(\pi/L)^4\int_0^L M(E, I, x_p)[1-2\cos(2\pi x_p/L) + \cos^2(2\pi x_p/L)]\cos^2(2\pi x_p/L)\, dx_p$$

Since the vertical length of rock block is far larger than that of its transverse deformation, it is thought reasonable that the actual force in the whole rock block can be expressed as the applied force F at $x_p=L/2$ approximately. It is

$$F=-\partial V/\partial x_p=4\beta\delta^3-2\alpha\delta \tag{2.11}$$

On the other hand, for a nonlinear elasticity system, the law of motion obeys

$$F=Mx''+\alpha_1 x'+\alpha_2(x')^2+\ldots \quad (2.12)$$

where M is the mass of rock block, x′ and x″ denote the velocity and accelerated velocity at the centroid of rock block, respectively. In Eqn. 2.12, the parameter x″ is usually small and can be neglected. We can also take no account of the high order terms of x′ since the velocity of deformation is not high. If it is considered that x equals to δ at the centroid of rock block, we then compare Eqn. 2.11 and Eqn. 2.12 and obtain

$$x'= ax^3-bx \quad (2.13)$$

where a and b are dominated variables, and $a=4\beta$, $b=2\alpha$.
As mentioned above, the stability of rock mass is sensitive to dynamic disturbances. When the disturbances is considered, we compare Eqn. 2.4 with Eqn. 2.13 and obtain the dynamical equation of rock mass as follows

$$x'=f(x)+x\eta(x)= ax^3-bx+x\eta(x) \quad (2.14)$$

3. POTENTIAL FUNCTION ANALYSIS OF EVOLUTION AND STABILITY

3.1. Stochastic potential function and critical property

Potential function analysis is a popular method in the domain of nonlinear dynamical system. In general, to deal with outside disturbances induced phase transition, we should derive a stochastic potential function relating to the evolution of system. The expression of stochastic potential function can be derived from the deterministic potential function V(x). According to synergetics, the relationship between V(x) and function f(x) is

$$f(x)=-\partial V(x)/\partial x \quad (3.1)$$

Hence, by using Eqn. 2.14 and Eqn. 3.1, we have

$$V(x)=-0.25ax^4+0.5bx^2 \quad (3.2)$$

Furthermore, the relationship between deterministic potential function V(x) and stochastic potential function U(x) is given by

$$U(x)=\int[V'(x)/ g^2(x)]dx+D\ln| g(x)| \quad (3.3)$$

where V′(x) is the first derivative of V(x) and g(x) = x. By inserting Eqn.3.2 in Eqn. 3.3 we get

$$U(x)=-0.5ax^2+(b+D)\ln x \quad (3.4)$$

Setting the first derivative of U(x) equal to zero and making use of Eqn. 3.4, we find that

$$x_c=[(b+D)/a]^{1/2} \quad (3.5)$$

where x_c denotes the critical point of disturbance induced instability. Thus it can be seen that the stability of rock mass relates to not only its instantaneous evolution state but also the fluctuation intensity resulted from outside disturbances. The critical property of instability is determined by combined action of control parameters and fluctuation intensity. This property remarkably differs from that happened under static loading condition.

3.2. Analysis of dynamic disturbance induced instability

In Figure 2 we show results from calculations of U(x) using Eqn. 3.4 for four different fluctuation intensities, D=0.05, 0.15, 0.25 and 0.40, with the fixed values for a= -1 and b= -0.2. We find that the plots show single potential wells when D are small. The right-hand of curves descend and the bottom of wells tends to gentle gradually. These characteristics show the stability of system is on the decline along with increasing fluctuation intensity. But when the value of D is big enough, for example, D=0.25 and 0.40, the bottom of potential wells disappear and the structure of potential wells overturn drastically. In these cases, there are no bottoms of wells and instability of system happens. These results indicate that the stability of rock mass may be affected by the intensity of disturbance remarkably. Although the rock mass is steady under static circumstance, there is a great probability of instability if the outside disturbance is conspicuous.

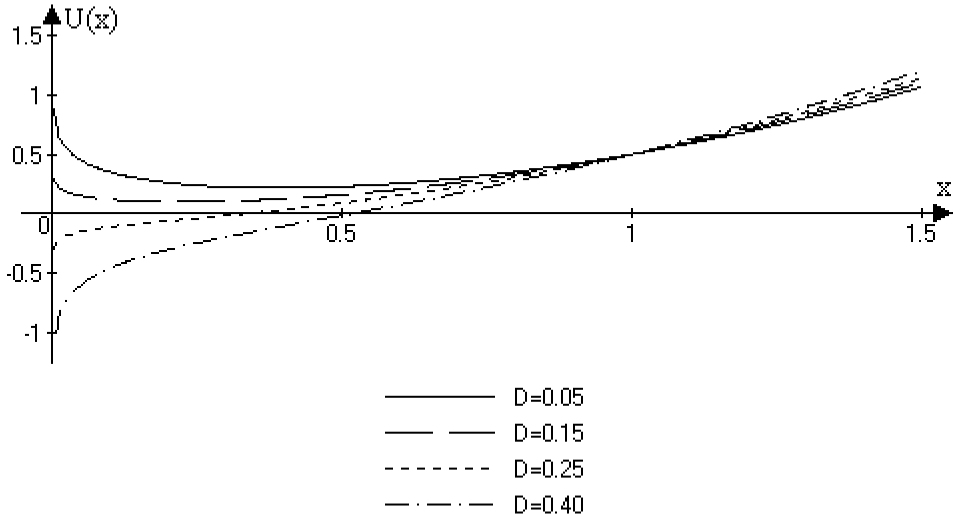


Figure 2 Curve of stochastic potential function for different values of D

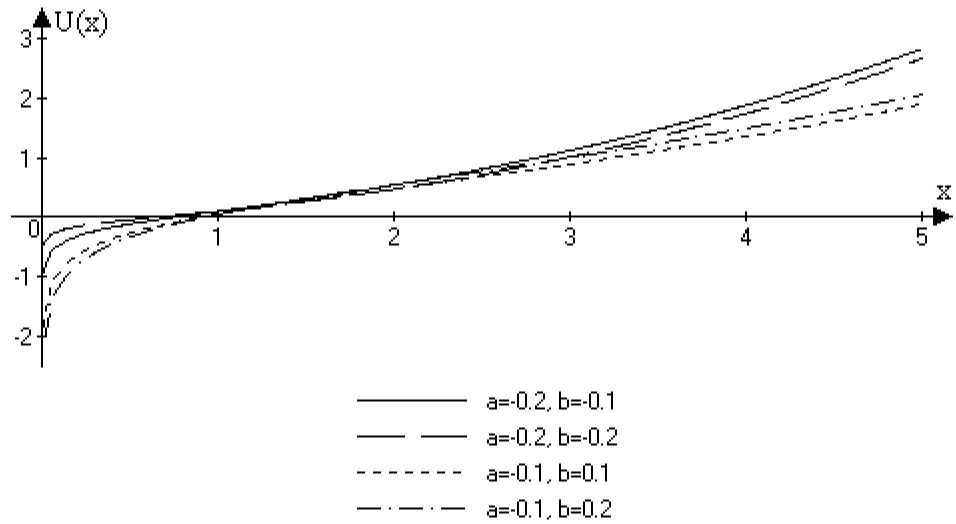


Figure 3 Curve of stochastic potential function for different control parameters

Figure 3 displays results from calculations of U(x) using Eqn. 3.4 for four different groups of control parameters. Each group of parameters represents a kind of static state of rock mass. As we can see， disturbance induced instability takes place in all the four given conditions, but the shapes of curves have some different. Near x tends to zero, the larger the numerical value of ordinate is, the easier the instability happens. So the capability of destroying rock mass for a given disturbance is effected by the state or control parameters of rock mass.

4. CONCLUSION

In this paper, we apply synergetics to research influence of dynamic disturbance on evolution and stability of deep underground rock mass based on the self-organizing characteristic of rock system. We have described in detail a new generalized dynamical equation which could simultaneously take deterministic and stochastic processes into account. Furthermore, a potential function equation is derived from analysis of rock mass structure. Using potential function method, the influence of dynamic disturbance on evolution and stability of rock system is analyzed. The relationship between critical point of instability and parameters of control parameter and disturbance intensity is also discussed and a formula which can be used to forecast critical point of instability is derived. Results indicated that dynamic disturbance can not only decrease the stability of rock system but also induce its instability under certain conditions. Under same static states, stability and feasibility of disturbance induced instability might increase along with increasing of fluctuation intensity. Under same fluctuation intensity, the larger the numerical value of ordinate is, the easier the instability happens. The capability of destroying rock mass for a given disturbance is effected by the state or control parameters of rock mass.

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