THE ROLE OF EPSILON FOR THE IDENTIFICATION OF GROUPS OF EARTHQUAKE INPUTS OF GIVEN HAZARD

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ABSTRACT :

Response spectra have been used in the years not only to obtain estimations of the seismic response for given dynamic system in response spectral analysis, but also as a useful reference for the identification of design time-history inputs (when time-history analysis are required). This second use is somehow improper. Indeed, uniform hazard response spectra are capable of providing information regarding the value of the spectral acceleration (characterized by a given probability of exceedance) for a given precise system period, but may prove to be misleading when the overall shape of the design response spectra is used to identify groups of time-history inputs. In this paper, a tool which can be used for the identification of groups of design time-history earthquake inputs for Performance-Based Seismic Design applications is developed.
First, a peculiar Probabilistic Seismic Hazard Analysis, which allows to explicitly handle (exclude/include and numerically quantify) the epistemic uncertainty due to the error of the ground-motion prediction (attenuation) model and to clearly distinguish it from the space-time aleatory variability, is developed.
Second, grounded on this Probabilistic Seismic Hazard Analysis, a relationship is obtained between a given hazard level and the statistical characteristics of the ensemble of response spectral ordinates, as computed at multiple periods. This allows to identify the characteristics that must be possessed by the groups of design earthquake inputs, by means of a newly introduced tool, herein defined as “spectral cloud”.

KEYWORDS: ground motion prediction model, aleatory variability, epistemic uncertainty, probabilistic seismic hazard analysis, spectral acceleration

1. INTRODUCTION

In any sound seismic engineering design, it is of prime importance the correct identification of the acceleration time-histories to be used as inputs for the dynamic analyses. Within a Performance Based Seismic Design framework [SEAOC Vision 2000, 1995], the choice of the reference design seismic input (commonly referred to as "earthquake bins") is deeply rooted upon their probabilistic identification. Typically, the earthquake bins are identified by means of earthquake "intensity measures" (*IM*s). *IM*s consisting of a scalar or vector-valued combination of selected ground motion parameters (*GMP*s) associated to a given probability. In recent years, many research works [Giovenale et. al, 2004, Trombetti et al , 2007] have focused on the identification of the optimal *IM* for earthquake bin creation. In this paper, a specific procedure is developed for the identification of the characteristics that groups of earthquake inputs must posses in order to represent a sound engineering choice of the design input. In particular, the procedure allows to identify the properties which must be satisfied by the design time-histories through the use of a newly introduced tool, herein defined as "spectral" cloud. The spectral cloud being capable of providing a probabilistic condition upon the spectral ordinates at multiple periods.

2. THE GROUND MOTION PREDICTION MODEL

A central role, in the procedure for the identification of the "spectral cloud", is played by the ground motion prediction model. The attenuation law provides an estimate of a selected ground motion parameter *GMP* given the magnitude *M*, the site-epicentre distance *R* and the local soil characteristics. In general, attenuation laws are obtained by means of a linear regression of the available data in logarithmic scale, so that they may be written in the following general form:

$$\log_b GMP = prediction + residual \tag{2.1}$$

The *prediction* generally depends on the magnitude *M*, the site-epicentre distance *R* and the local soil characteristics (which are fixed for a selected site), as given by:

$$prediction = g(M, R) \tag{2.2}$$

where *g* is a deterministic function of *M* and *R*. In common hazard calculations, the magnitude *M* and the distance *R* are assumed as random variables which involve the space- and time-related sources of randomness (to which the attenuation law does not contributes at all), so that the *prediction* may be seen as a deterministic function of two random variables; the *residual* is an additional random variable which takes into account the randomness of the error associated with the prediction provided by the attenuation law or, using the words of Berge-Thierry *et al.* [2003], "*the intrinsic variability of the data, and the error due to the attenuation model*".
To better explain the relationships between these random variables, Equations 2.1 and 2.2 can be rewritten as $Y = Y' + Z = g(M,R) + Z$, $Y' = g(M,R)$, where $Y = \log_b GMP$ and $Z = residual$
It is possible to distinguish two basic different sources of randomness. The first is related to the randomness in space and time of occurrence of events and pertains to *M* and *R*. The second is related to the error in the attenuation law in providing a prediction for the ground motion parameter for a given event (model-dependent source) and pertains to *Z.* According to the unambiguous terminology suggested by Abrahamson [1988], it is possible to refer to the first randomness as "aleatory variability" (given that it is the natural variability of the physical phenomenon), and to the second one as "epistemic uncertainty" (given that it is the scientific uncertainty in the simplified model we use to describe the effects of the physical phenomenon). Without wanting to get to the heart of the matter, the terms "aleatory variability" and "epistemic uncertainty", which are commonly used in generic ways and often mixed up, are here used to underline the separation between the two distinct sources of randomness.
To explicitly take into account this separation, it is here defined a new variable as the prediction, GMP', of the ground motion parameter *GMP*:

$$GMP' = b^{Y'} \tag{2.3}$$

so that we can also express the prediction Y' of the base-*b* logarithm of the ground motion parameter as $Y' = \log_b GMP'$. It is possible to write $\log_b GMP = \log_b GMP' + Z$.

Note that, by definition, the prediction Y' tries to capture both the mean value, $\mu_{\log_b GMP}$, and the median, $(\log_b GMP)_{median}$, of the base-b logarithms of the observed values of the ground motion parameter, so that $\log_b GMP' \cong \mu_{\log_b GMP}$ and $\log_b GMP' \cong (\log_b GMP)_{median}$.
From the theory of functions of random variables, and given that the logarithm function is monotonic, the median of the *GMP*, $(GMP)_{median}$, may be obtained as $(GMP)_{median} = b^{(\log_b GMP)_{median}}$, that is $GMP' = (GMP)_{median}$. Note that the same conclusion does not apply to the mean value.
As per the above considerations, the GMP' (which does not account for random variable *Z*) is associated to the aleatory variability only, while the *GMP* is associated to both the aleatory variability and the epistemic uncertainty. It is thus fundamental to clearly distinguish between *GMP* and GMP'.
To quantitatively account for this epistemic uncertainty, it is generally made reference to the standard error of estimate.

3. THE "PSHA" PROCEDURE

In order to associate given values of the ground motion parameter *GMP* to corresponding probabilities of exceedance, a PSHA procedure is generally carried out. The PSHA procedure leads to the identification of the Probability Density Function (PDF), $f_{GMP}(gmp)$, and the Cumulative Distribution Function (CDF), $F_{GMP}(gmp)$, of the *GMP*, as computed over a given observation time *t*. The procedure (based upon the consolidated approach suggested by Cornell [1968], is composed of the 8 steps summarized below.
Step 1: identification of the reference earthquake catalogue (same hypothesis as per Cornell's approach).
Step 2: the recurrence rate λ of seismic events and the seismic magnitude *M* is assumed to follow Gutenberg-Richter relationship. For each *i*-th seismic source zone, the event rate $\bar{\lambda}(m_i)$ is given by:
$\bar{\lambda}(m_i) = \hat{p}_i \cdot \exp(-\hat{q}_i \cdot m_i)$ with $\hat{p}_i = \exp(p_i \ln 10)$ and $\hat{q}_i = q_i \ln 10$.
Step 3: the occurrence of seismic events is modeled as a Poisson arrival process. Consequently, the probability, $P[X = x]$, that exactly *x* events characterized by magnitude M_i strictly larger than m_i occur within the *i*-th seismic source zone over a given observation time *t*, is given by:

$$P[X = x] = \frac{\left(\bar{\lambda}(m_i) \cdot t\right)^x}{x!} e^{-\bar{\lambda}(m_i)t} \tag{3.1}$$

Step 4: the PDF of the magnitude *Mi* for each seismic source zone over a given observation time *t* is obtained as

$$f_{M_i}(m_i) = \sum_{j=1}^{J} \alpha_{ij} \cdot \hat{p}_i \cdot \hat{q}_i \cdot t \cdot e^{-\hat{q}_i \cdot m_{ij} - t \cdot \hat{p}_i \cdot \exp(-\hat{q}_i \cdot m_{ij})} \tag{3.2}$$

with $\alpha_{ij} = S_{ij} / S_i$, S_{ij} representing the area of the *j*-th sub-division of the *i*-th seismic source zone and S_i representing the area of the *i*-th seismic source zone.
Step 5: for a given site, the ground model prediction gives:

$$\log_b GMP' = b_1(\bar{T}) + b_2(\bar{T}) \cdot M + b_3(\bar{T}) \cdot \log_b D + b_4(\bar{T}) \cdot D + b_5(\bar{T}, \bar{s}) \tag{3.3}$$

Where *D* represents the distance (either epicentral or hypocentral) between the location of the event and the site of interest, $\bar{s}$ represents the local soil characteristics (which are fixed for a selected site), $b_1(\bar{T})$, $b_2(\bar{T})$, $b_3(\bar{T})$, $b_4(\bar{T})$ and $b_5(\bar{T}, \bar{s})$ are appropriate coefficients which may depend on the structural period of vibration $\bar{T}$ and on the local soil characteristics $\bar{s}$.
Step 6: once $f_M(m)$ and $f_R(r)$ are known, it is possible to obtain:

$$F_{GMP'_i}\left(gmp'_i\right)=\sum_{j=1}^{J}\alpha_{ij}\exp\left[-K_{2,ij}\,gmp'^{\,-K_{1,i}}_i\right] \quad (3.4)$$

$$f_{GMP'_i}\left(gmp'_i\right)=\sum_{j=1}^{J}\alpha_{ij}K_{1,i}K_{2,ij}\,gmp'^{\,-(K_{1,i}+1)}_i\exp\left[-K_{2,ij}\,gmp'^{\,-K_{1,i}}_i\right] \quad (3.5)$$

with:

$$K_{1,i}=\frac{\hat{q}_i}{b_2\left(\bar{T}\right)\cdot\ln b} \quad (3.6)$$

$$K_{2,ij}=t\cdot\hat{p}_i\cdot\exp\left[K_{1,i}\cdot\left(b_1\left(\bar{T}\right)+b_3\left(\bar{T}\right)\cdot\log_b\sqrt{R_{ij}^2+\bar{h}^2}+b_4\left(\bar{T}\right)\cdot\sqrt{R_{ij}^2+\bar{h}^2}+b_5\left(\bar{T},\bar{s}\right)\right)\cdot\ln b\right] \quad (3.7)$$

Step 7: assuming that the seismic activities of all seismic source zones are independent from each other:

$$F_{GMP'}\left(gmp'\right)=\prod_{i=1}^{I}F_{GMP'_i}\left(gmp'_i\right) \quad (3.8)$$

and:

$$f_{GMP'}\left(gmp'\right)=\frac{\partial F_{GMP'}\left(gmp'\right)}{\partial gmp'} \quad (3.9)$$

Step 8: Finally the PDF of the *GMP* can be expressed as:

$$f_{GMP}\left(gmp\right)=\int_0^\infty f_{GMP|gmp'}\left(gmp\right)f_{GMP'}\left(gmp'\right)dgmp' \quad (3.10)$$

where $f_{GMP'}\left(gmp'\right)$ is known, while $f_{GMP|gmp'}\left(gmp\right)$ is to be determined as:

$$f_{GMP|gmp'}\left(gmp\right)=\int_0^\infty\int_0^\infty f_{GMP|gmp',m,r}\left(gmp\right)\cdot f_{M,R|gmp'}\left(m,r\right)\cdot dm\cdot dr \quad (3.11)$$

In general, most attenuation laws provides, together with the prediction equation, the standard error, $SE_{\log_b GMP}$, of the base-*b* logarithm of the observed values of the ground motion parameter with respect to the base-*b* logarithm of the prediction gmp', as given by:

$$SE_{\log_b GMP}=\sqrt{\frac{\sum_{i=1}^{N}\left(\log_b gmp_{i,a.d.}-\log_b gmp'\right)^2}{N}} \quad (3.12)$$

where $gmp_{i,a.d.}$ represent the *i*-th value of the ground motion parameter among the available data used to define the prediction equation and *N* is the number of the available data.
It is then common practice, especially for spectral acceleration laws, to define the normalised logarithmic residual (epsilon), ε_i, as follows:

$$\varepsilon_i=\frac{\log_b gmp_i-\log_b gmp'}{SE_{\log_b GMP}} \quad (3.13)$$

where gmp_i is the *i*-th observed value of the ground motion parameter.
If, in Equation (3.13),, the magnitude *m* and distance *r* used to compute the prediction gmp' are equal to those associated to the gmp_i data entry (and if the $SE_{\log_b GMP}$ is that associated to the prediction equation used to compute the gmp'), the normalised logarithmic residual (epsilon) is a random variable E characterised by the standard normal distribution:

$$E=N\left(0,1\right) \quad (3.14)$$

From Equation (3.13), it is then possible to express $GMP\,|\,gmp',m,r$ as a function of the random variable E

(epsilon) only, as given by:

$$GMP \mid gmp', m, r = b^{E \cdot SE_{\log_b GMP} + \log_b gmp'} \quad (3.15)$$

which, together with Equation (3.14) leads to:

$$GMP \mid gmp', m, r = LN(\lambda, \zeta) \quad (3.16)$$

with:

$$\lambda = \ln gmp' \quad (3.17)$$

and

$$\zeta = SE_{\ln GMP}, \quad (3.18)$$

For many attenuation laws, $SE_{\ln GMP}$ is independent from both *M* and *R*. As a consequence, $f_{GMP|gmp',m,r}(gmp)$ does not depend on *M* and *R*, so that Equation (3.12) becomes:

$$f_{GMP|gmp'}(gmp) = f_{GMP|gmp',m,r}(gmp) \quad (3.19)$$

which gives the following fundamental result:

$$GMP \mid gmp' = LN(\lambda = \ln gmp', \zeta = SE_{\ln GMP}) \quad (3.20)$$

which characterises the distribution of the *GMP* (due to the epistemic error) associated with a fixed probability of exceedance of the prediction gmp'. Eq. (3.20) allows to obtain the moments/parameters which characterise $GMP \mid gmp'$:

$$\mu_{GMP|gmp'} = gmp' \cdot e^{\left(\frac{1}{2}\zeta^2\right)} \quad (3.21)$$

$$\sigma_{GMP|gmp'} = \mu_{GMP|gmp'} \cdot \sqrt{e^{\zeta^2} - 1} \quad (3.22)$$

$$\delta_{GMP|gmp'} = \sqrt{e^{\zeta^2} - 1} \quad (3.23)$$

4. UNIFORM HAZARD INPUT GROUP

For the development of practical seismic design, the structural engineer needs to have at hand groups of earthquake inputs characterised by a given seismic hazard level. The identification of earthquake inputs of given hazard is generally obtained through a two-step association.

The first step encompasses the association between an exceedance probability, $\overline{P}$, and a threshold value, $\overline{w}$, of a given physical quantity, *w*, which identifies the intensity of the seismic event (often called Intensity Measure *IM*):

$$\overline{P} \rightarrow \overline{w} \quad (4.1)$$

This association can be expressed in words as follows: $\overline{P}$ = “*probability that a give physical quantity w which identifies the intensity of the seismic event exceeds a specified threshold value* $\overline{w}$ *, for a selected site over a given observation time* $\overline{t}$ ”.

The second (and subsequent) step sees the identification of the design seismic records on the basis of the threshold value $\overline{w}$. In the light of the observations drawn from the study of the residuals developed in the previous sections, the hazard association problem expressed by Equation (4.1) may assume different meanings depending on the way the quantity *w* is associated to the probability of exceedance: indeed, this association may be obtained either (A) in terms of the ground motion parameter itself (*gmp*) or (B) in terms of its prediction (gmp'), as given in detail below.

4.1 Approach A

The approach A associates, to a given probability $\overline{P}$, a specific threshold value $\overline{gmp}$ of the *gmp*, as obtained from:

$$\bar{P} \rightarrow \overline{gmp} \quad \text{as per} \quad \bar{P} = \int_0^{\overline{gmp}} f_{GMP}\left(gmp\right) dgmp \tag{4.2}$$

The association provided by Equation (4.2) accounts simultaneously for both the intrinsic hazard of the physical phenomenon (i.e. the space-time aleatory variability) and the error in the evaluation of the parameter which identifies the hazard (i.e. the epistemic uncertainty due to the error of the attenuation model), as they are grouped in the *gmp*.

4.2 Approach B

The approach B associates, to a given probability $\bar{P}$, a specific threshold value $\overline{gmp'}$ of the gmp', as obtained from:

$$\bar{P} \rightarrow \overline{gmp'} \quad \text{as per} \quad \bar{P} = \int_0^{\overline{gmp'}} f_{GMP'}\left(gmp'\right) dgmp' \tag{4.3}$$

The association provided by Equation (4.3) accounts only for the intrinsic hazard of the physical phenomenon (i.e. the space-time aleatory variability), as contained in the gmp'. Note that Equation (4.3) may be seen as the result of a PSHA which does not take into account the ground motion variability.
The epistemic uncertainty due to the error of the attenuation model may be then conveniently accounted for through the use of appropriate tools (e.g. given percentile values of the *EDP* to obtain desired confidence levels) capable of handling the distribution of the $GMP \mid \overline{gmp'}$:

$$\overline{gmp'} \rightarrow GMP \mid \overline{gmp'} \quad \text{as per} \quad f_{GMP\mid\overline{gmp'}}\left(gmp\right) \tag{4.4}$$

5. APPLICATION TO THE SPECTRAL ACCELERATION

Let us here specialize the analysis of above with reference to spectral acceleration, $S_A\left(T_j\right)$, in what follows, the notations *GMP*, *gmp*, GMP' and gmp' thus specialise into $S_A\left(T_j\right)$, $s_a\left(T_j\right)$, $S_A'\left(T_j\right)$ and $s_a'\left(T_j\right)$, respectively.

5.1 Uniform hazard spectrum

Following approach A for the identification of the seismic hazard, Equation (4.2) specialises as follows:

$$\bar{P} \rightarrow \overline{s_a}\left(T_j\right) \quad \text{as per} \quad \bar{P} = \int_0^{\overline{s_a}\left(T_j\right)} f_{S_A\left(T_j\right)}\left(s_a\left(T_j\right)\right) ds_a\left(T_j\right) \tag{5.1}$$

As far as the identification of groups of seismic records of given hazard level is concerned, approach A is the common approach used in current performance based seismic design with its limitations. It is known that the ensemble of the spectral ordinates thus obtained (often referred to as the "*uniform hazard spectrum*", defined as "*the locus of points such that the spectral acceleration value at each period has an exceedance probability equal to the specified target probability*") does not represent the spectrum of any single earthquake, given that each PSHA, for each specified T_j, is independent from the others. Consequently, each piece of information collected in the uniform hazard spectrum must be used one at a time.

5.2 The spectral cloud

It is known that the response spectrum is the curve that connects the *m* spectral ordinates, computed at the *m* multiple reference periods. When a group of *n* seismic records is considered, *n* response spectra are obtained.
We here define as the "*spectral cloud*" the ensemble of the $n \cdot m$ spectral ordinates, computed at the *m* multiple reference periods, of a group of *n* seismic records. Note that, in addition to the complete knowledge of the characteristics of the distribution of the *m* random variables, were the autocorrelation functions of the response

spectra known, the *n* response spectra could be seen as a set of *n* signals of a stochastic process.

5.3 Uniform hazard spectral cloud

Following approach B for the identification of the seismic hazard, Equations (4.3) and (4.4) specialise as follows:

$$\bar{P} \rightarrow \overline{s_a}'(T_j) \quad \text{as per} \quad \bar{P} = \int_0^{\overline{s_a}'(T_j)} f_{S_A'(T_j)}\left(s_a'(T_j)\right) ds_a'(T_j) \tag{5.2}$$

$$\overline{s_a}'(T_j) \rightarrow S_A(T_j) \mid \overline{s_a}'(T_j) \quad \text{as per} \quad f_{S_A(T_j)|\overline{s_a}'(T_j)}\left(s_a(T_j)\right) \tag{5.3}$$

The seismic records of the group are characterised by values of the ground motion parameter $s_{a,i}(T_j)$ which, on the whole, fit the distribution $f_{S_A(T_j)|\overline{s_a}'(T_j)}\left(s_a(T_j)\right)$ of Equation (5.3). The advantages of approach B over approach A become clear when more than one reference periods are considered. Indeed, both Equations (5.2) and (5.3) can be easily applied simultaneously for different periods T_j and T_k ($\forall k \neq j$).

Note that the simultaneous application of Equations (5.2) and (5.3) at different reference periods T_j implicitly determine a "*uniform hazard spectral cloud*". Introducing the uniform hazard spectral cloud, approach B enables to overtake the uniform hazard spectrum concept. The application of Equation (5.3) involves the explicit identification of $f_{S_A(T_j)|\overline{s_a}'(T_j)}\left(s_a(T_j)\right)$. This can be done in the light of the "epsilon" parameter, commonly used to identify the dispersion of the spectral acceleration attenuation law.

The epsilon parameter evaluated, at a specified period T_j, for the *i*-th seismic record, $\varepsilon_i(T_j)$, is given by:

$$\varepsilon_i(T_j) = \frac{\ln s_{a,i}(T_j) - \ln \overline{s_a}'(T_j)}{SE_{\ln S_A(T_j)}} \tag{5.4}$$

where the numerical values of $SE_{\ln S_A(T_j)}$ are generally provided along with the formulations of the spectral acceleration attenuation laws.

The statistical distribution of epsilon is generally considered to be well represented by the standard normal distribution, as it was assumed in the analytical developments of previous in sections. Consequently, all conclusions drawn starting from Equation (3.13) and elucidated throughout section 2 still hold for the case of $S_A(T_j)$ assumed as *GMP*.

Thus, the fundamental result provided by Equation (3.19) leads to:

$$f_{S_A(T_j)|\overline{s_a}'(T_j)}\left(s_a(T_j)\right) = \frac{1}{SE_{\ln S_A(T_j)} \cdot s_a(T_j) \cdot \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln s_a(T_j) - \ln \overline{s_a}'(T_j)}{SE_{\ln S_A(T_j)}}\right)^2}, \quad \forall T_j \tag{5.5}$$

i.e.:

$$S_A(T_j) \mid \overline{s_a}'(T_j) = LN(\lambda_j, \zeta_j), \quad \forall T_j \tag{5.6}$$

In words, $S_A(T_j) \mid \overline{s_a}'(T_j)$ has a lognormal distribution, with the following parameters and moments:

$$\lambda_j = \ln \overline{s_a}'(T_j) \tag{5.7}$$

$$\zeta_j = SE_{\ln S_A(T_j)} \tag{5.8}$$

$$\mu_{S_A(T_j)|\overline{s_a}'(T_j)} = e^{\left(\lambda_j + \frac{1}{2}\zeta_j^2\right)} = e^{\left(\ln \overline{s_a}'(T_j) + \frac{1}{2}SE^2_{\ln S_A(T_j)}\right)} = \overline{s_a}'(T_j) \cdot e^{\frac{1}{2}SE^2_{\ln S_A(T_j)}} \tag{5.9}$$

$$\sigma_{S_A(T_j)|\overline{s_a}'(T_j)} = \mu_{S_A(T_j)|\overline{s_a}'(T_j)}\sqrt{e^{\zeta_j^2}-1} = \left(\overline{s_a}'(T_j)\cdot e^{\frac{1}{2}SE^2_{\ln S_A(T_j)}}\right)\sqrt{e^{SE^2_{\ln S_A(T_j)}}-1} \tag{5.10}$$

$$\delta_{S_A(T_j)|\overline{s_a}'(T_j)} = \sqrt{e^{\zeta_j^2}-1} = \sqrt{e^{SE^2_{\ln S_A(T_j)}}-1} \tag{5.11}$$

The condition given by Equation (5.6), as applied simultaneously at any reference period T_j, characterises the uniform hazard spectral cloud and indirectly identifies the group of uniform hazard earthquake inputs.

6. "FULL" CONDITION ON THE SAMPLE OF SPECTRAL ORDINATES

The group can be identified imposing that, at each period T_j, the ensemble of the n sample values $\{s_{a,1}(T_j) \quad s_{a,2}(T_j) \quad \dots \quad s_{a,i}(T_j) \quad \dots \quad s_{a,n}(T_j)\} | \overline{s_a}'(T_j)$ follows the lognormal distribution, as given by:

$$\{s_{a,1}(T_j) \quad s_{a,2}(T_j) \quad \dots \quad s_{a,i}(T_j) \quad \dots \quad s_{a,n}(T_j)\} | \overline{s_a}'(T_j) = LN(\lambda_j, \zeta_j) \tag{6.1}$$

7. CONCLUSIONS

The research work presented in this paper identifies the statistical characteristics of the ensemble of the spectral ordinates, as computed at multiple periods, for groups of earthquake inputs characterized by given hazard level (here defined as the "uniform hazard spectral cloud"). The statistical characterisation of the spectral cloud (which can be assimilated to a lognormal random process) as here proposed allows to:

- treat separately and independently the epistemic uncertainty due to the error of the attenuation model from all other time- and space-related sources of aleatory variability;
- identify earthquake inputs which retain their significance independently from the period range considered;
- obtain groups of design earthquake inputs which can be used for different structures and for structures with substantial variations in vibration periods;
- link the identification of the seismic hazard strictly to the site, without involving the structure.

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REFERENCES

SEAOC Vision 2000 Committee. Performance-based seismic engineering for buildings. Report prepared by Structural Engineers Association of California, Sacramento, CA, 1995.
Giovenale P, Cornell CA, Esteva L. Comparing the adequacy of alternative ground motions intensity measures for the estimation of structural responses. Earthquake Engineering and Structural Dynamics 2004; 33: 951-979.
Trombetti T, Silvestri S, Malavolta D, Gasparini G. A methodology for determination of efficient earthquake bins for Performance Based Seismic Design. Proceedings of the Tenth International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP10), Tokyo, Japan, July 31 – August 3, 2007.
Abrahamson NA. Statistical properties of peak ground accelerations recorded by the Smart 1 array. Bulletin of the Seismological Society of America 1988; 78: 26–41.
Berge-Thierry C, Cotton F, Scotti O, Griot-Pommera DA, Fukushima Y. New empirical response spectral attenuation laws for moderate European earthquakes. Journal of Earthquake Engineering 2003; 7(2): 193-222.
Cornell CA. Engineering seismic risk analysis. Bulletin of the Seismological Society of America 1968; 58: 1583-1605.