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UNIFORM HAZARD SPECTRA AND DEAGGREGATION PLOTS: A MONTE CARLO SIMULATION APPROACH

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SUMMARY

Earthquakes cause damage to engineering structures and often result in loss of life. Forecasting the exact time of an earthquake can at best reduce casualties, but at present seems to be an impossible task. Therefore structures need to be designed to withstand the impact of an earthquake and prevent collapse.

From the structural engineer's point of view, uniform hazard spectra and deaggregation plots are important to produce a safe structure and the process itself provides valuable insight into modelling of uncertainties.

Large magnitude earthquakes are low probability but high risk events. Design of structures for this rare event may prove to be expensive i.e. uneconomic, as many of these structures may not, in their lifetimes, experience this event. Thus a balance between safety and cost must be sought i.e. a decision made. Unfortunately, there are many areas of uncertainty in determining loads due to future earthquakes and this makes decision making difficult.

Seismic hazard analysis (evaluating design parameters of earthquake ground motion at site) provides useful guidelines for informed decision making.

Probabilistic treatment of seismic hazard analysis (SHA), first introduced by Cornell (1968) in a landmark paper, is now widely accepted. It provides a sound theoretical basis for representing various forms of seismic natural variability and allows treatment of uncertainties arising from incomplete knowledge. Typically, earthquakes can occur at any location along the fault line. This makes the distance from the source to the site a randomly variable quantity. The magnitude of the earthquake is also random in nature together with expected levels of ground shaking. The concept of uniform hazard spectra (UHS) follows directly.

This paper outlines the basic concepts of probabilistic modelling and the underlying assumptions. The probabilistic method as introduced by Cornell (1968) is discussed briefly and compared with the solution obtained from Monte Carlo simulation. This is followed up with a worked example illustrating the various steps for construction of the uniform hazard spectrum using Monte Carlo simulation. Discussion on deaggregation is followed by numerical steps for obtaining such plots with a worked example. Related topics of aleatory variability and epistemic uncertainty are also discussed.

1.0 INTRODUCTION TO PROBABILISTIC SEISMIC HAZARD ANALYSIS (PSHA)

1.1 Introduction

Earthquakes cause widespread disruption of human activity. A major earthquake is a disaster. It is a prime requirement in our society that all forms of man-made structures be protected during an earthquake. It is the structures and their collapse that cause the damage and loss of human lives and not the earthquake itself. The deterministic approach towards seismic hazard analysis (SHA) provides an easy-to-follow and transparent method for defining seismic loading at a site. The analysis makes use of discrete, single valued events or models. The maximum possible earthquake magnitude is identified for the 100 and 10,000-year (return periods) earthquakes. How likely or unlikely are these earthquakes to occur? The question posed before us is:

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‘*Is this approach satisfactory in dealing with uncertainties associated with practically all the elements that go towards defining the loading?’*

It is not possible to be certain about the exact magnitude of an earthquake, its location, let alone the complexities inherent in ground motion make-up arising from source, travel path, local site conditions etc. Probabilistic approaches provide a sound basis with a mathematical foundation for representing various forms of natural variability and more importantly the treatment of uncertainties arising from incomplete knowledge. Cornell’s (1968) probabilistic method is now the preferred method for all contemporary probabilistic seismic hazard analyses. Uniform hazard spectrum is very much a part of the probabilistic seismic hazard analyses (PHSA) carried out for a site. Hence, the basic elements of PSHA are discussed first before moving on to the construction of uniform hazard response spectrum (UHRS). A very good introduction to the subject has been provided by Mcguire and Arabasz (1995).

1.2 Outline of the Paper

In section 2, the steps associated with PSHA are discussed. Cornell’s (1968) probabilistic method is discussed in section 3. Cornell’s theoretical solution (example on line source problem Fig. 2) is compared next with the solution obtained from the Monte Carlo simulation process developed in this paper. This is covered in section 4. The steps associated with the construction of the UHRS are outlined in section 5. Finally, as part of further computational considerations, basic concepts of *deaggregation* and logic tree uncertainty model are introduced in section 6. An overview of the process and concluding remarks may be found in section 7.

2.0 BASIC STEPS IN PROBABILISTIC SEISMIC HAZARD ANALYSIS

The methodology being used in most contemporary probabilistic seismic hazard analysis, is that first proposed by Cornell in 1968. It has come to be accepted as the preferred approach to the problem of determining the design ground motion at the site. The steps are shown in Fig. 1.

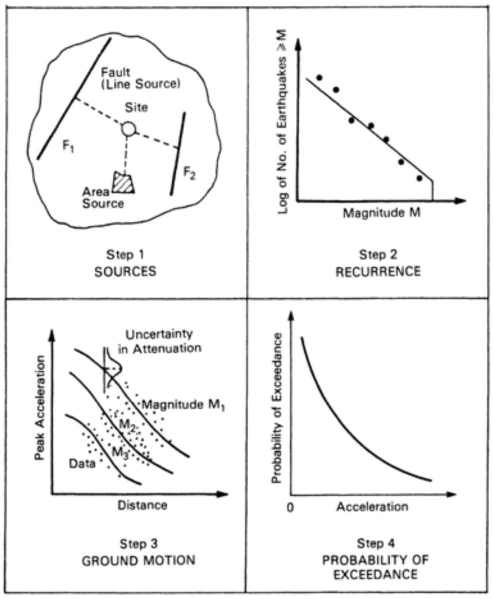

Figure 1: Basic Steps of Probabilistic Seismic Hazard Analysis (TERA Corp 1978, with permission)

With reference to Fig. 1, the following aspects need careful consideration:

a) Historical seismicity of the region.
b) Geology, tectonics, identification of earthquake source and the geometry.
c) Establishment of a recurrence law which specifies the *average rate* at which an earthquake of some size will be exceeded.
d) The ground motion parameter (peak acceleration, velocity etc) produced at a site with respect to earthquake size and distance expressed in terms of attenuation relationships.
e) Local soil conditions.
f) Finally, the temporal model. All the uncertainties of source geometry, distance, magnitude and ground motion parameter of interest are combined in this final step of

probability calculations (probability of exceeding a ground motion parameter at a site). This step is achieved by incorporating a temporal model (distribution of earthquake occurrence with respect to time, sometimes simply referred to as the arrival process).

2.1 Historical Seismicity of the Region

In general the region of interest around the site could be large, perhaps 5° x 5° or even greater. However, the idea is to establish and demarcate regions which could be termed 'homogeneous', meaning belonging to the same tectonic makeup. Data regarding earthquake size (magnitude, moment), location (epicentre), and focal depth if available are collected for earthquakes that have happened in the past. Instrumental recordings are of recent origin and thus the data would normally consist of historical data and recent instrumental recordings. The locations of pre-instrumental period earthquakes are obtained from historical records extending as far back as possible in time and vary from region to region.

2.2 Geology, Tectonics, Identification of Earthquake Source and the Geometry

The idea is to map the locations of all known active faults and if possible the depth at which they are located. Some areas are better documented than others. For example, along the Californian belt *inter-plate* boundaries, either active crustal faults or large-scale seismological features such as zones of plate subduction have been mapped. These are the possible locations where future earthquake energy release could take place. In the case of *intra-plate* regions it may not be that straightforward as sources of energy release may not be readily associated with identifiable faults. Moreover, some faults are located below the surface. It is customary to designate these regions as area sources.

Categorisation of source and identifying the geometry for the purpose of PSHA is an important step. It is still very subjective at present. It would depend to a great extent on the tectonic process involved.

2.3 Modelling Recurrence Laws

(*average rate* at which an earthquake of some size will be exceeded)

The next step in the process is to establish the recurrence relationship, or rather the sizes of earthquakes the region is likely to exhibit. How often do earthquakes occur? Does some kind of pattern exist in the time-scale at which earthquakes are known to occur? In a seismologically active region earthquakes occur at irregular intervals of time. It is fairly obvious that in order to extract a meaningful pattern, the length of the record must be reasonably large.

Gutenburg and Richter (1965) first described the general underlying pattern of magnitudes of earthquakes and their occurrence. The data (spread over a certain length of time) was organised in a manner to reflect the number of earthquakes that exceeded a certain magnitude. Also from the organised data, the *mean annual rate of being exceeded* λ_M of an earthquake magnitude M was defined as the number of occurrences greater than M divided by the length of the time period. Or in other words, the average rate at which an earthquake of some size will be exceeded. An empirical relation between magnitude and frequency was proposed:

$$\log \lambda_M = A - bM \quad \ldots \quad \ldots \qquad (2.2.1)$$

where A and b are seismic constants for a region.

Expressing the standard Gutenburg-Richter law (eqn. 2.2.1) in exponential form:-

$$\lambda_M = 10^{A-bM} = \exp(\alpha - \beta M) \qquad (2.2.2)$$

where $\alpha = 2.303A$ and $\beta = 2.303b$

From a practical engineering point of view, earthquakes of magnitude 4 or below are not of great interest to the designer as they do not cause much damage. Hence, bounded Gutenburg-Richter laws with lower cut-off limits have been developed. If M_o is denoted as the lower cut-off limit then the probability of its magnitude between M_o and M would be given by:

$$F(M) = \int_{M_o}^{M} f(M)dM \qquad (2.2.3)$$

A suitable expression for the *Cumulative Distribution Function* (CDF), $F(M)$, has been suggested [Epstein and Lomnitz 1966, Mcguire and Arabasz 1990] and is of the form:

$$F(M) = 1 - e^{-\beta(M - M_o)} \qquad (2.2.4)$$

and the probability density function:

$$f(M) = \beta e^{-\beta(M-M_o)} \quad (2.2.5)$$

The validity of Gutenburg-Richter law has been questioned by various investigators [Schwartz and Coppersmith 1984, Schwartz 1988, Youngs and Coppersmith 1985]. Although the Gutenburg-Richter law may have been questioned, available world-wide seismicity data ***does not*** support that other models be adopted.

Because of its simplicity and as it fits the data reasonably well in the lower magnitude range, this model has been found to be convenient and currently in general use today (italics due to author).

Other alternative models may be found in some of the related references [Youngs and Coppersmith 1985, Anderson 1979, Schwartz and Coppersmith 1986]. Another physical phenomenon needs to be pointed out very briefly. It has been observed that certain faults can exhibit a trend of generating similar magnitude earthquakes over time. Seismologists have referred to this phenomenon in the literature as '*characteristic earthquakes'*. The reader is referred to works by Wesnousky et al. (1984) and Wu et al. (1995).

2.4 The Ground Motion Parameter (peak acceleration, velocity etc.)

The uncertainty associated with predictive models of ground motion parameters needs to be addressed within PSHA. The ground motion parameters variation with M and R (magnitude and hypocentral distance) are log normally distributed (the log of the parameter is normally distributed). The regression laws themselves are not complicated, but it is worth noting that all authors have reported considerable scatter in the data. A certain amount of scatter is inevitable as not enough is known about the source, travel path and local site conditions. More sophisticated models of attenuation relationships are awaited.

2.5 Local Soil Conditions

It has been seen in many earthquakes that soft soils can amplify earthquake ground motion significantly. Local soil conditions are best treated separately for a more reliable prediction. FE (finite element) modelling of the actual soil strata at site still remains the best alternative for investigating local soil conditions.

2.6 Temporal Model (or the Arrival Process)

As noted earlier, the final step in our calculations is the incorporation of a mathematical model to account for distribution of earthquake occurrence with respect to time. The incorporation of this model within PSHA explicitly or implicitly assumes that the occurrence of one earthquake is not related to the previous one (the process is memory less). In other words, the time and occurrence and magnitude of the next earthquake are probabilistically independent. The memory less model is *inconsistent* with the physical concept of *elastic rebound theory* of continuous crustal strain accumulation and intermittent release of strain when the build-up exceeds the strain limits the faults can sustain. The release mechanism of this energy build-up is through movements on the faults.

From an engineering perspective, the reasons are published and outlined in the EPRI (1986) Report. Perhaps, the most important reason being is that *it is the simplest model that captures the basic elements of the entire concept* (italics due to author).

3.0 PSHA AS INTRODUCED BY CORNELL (1968)

Cornell (1968) first introduced this method of seismic risk evaluation at a site where an engineering project was to be located. Cornell's assessment of the requirements for seismic hazard analyses then may be summed up as '*seismologists have long recognised the need to provide engineers with their best estimates of seismic risk*'. The formulation put forward by Cornell (1968) is outlined first in this section and reproduced with minor annotations only. In section 4 the solution is compared with that obtained from Monte Carlo Simulation. The formulation is set in terms of Modified Mercalli Intensity, I .

4.0 MONTE CARLO SIMULATION TECHNIQUES

Monte Carlo simulation is an established technique for solving probabilistic models. This section explores how the line source illustrative problem worked out by Cornell (1968) may be solved by the Monte Carlo simulation process. It will be seen that this approach is different, but it is a transparent process, which the reader can follow easily. It is a powerful technique being used in many fields of science and engineering, notably nuclear science.

Construction of uniform hazard response spectra (UHRS) using MonteCarlo simulation techniques have been discussed in earlier paper (Sen, 2006).

5.0 UNIFORM HAZARD RESPONSE SPECTRUM (UHRS) METHODOLOGY

Basic steps of construction of UHRS are discussed below before discussing deaggregation.
The UHRS envelopes are contributions from a range of earthquakes. For example, the short period motion of UHRS is usually dominated by contributions from small nearby earthquakes while the long period response is dominated by contributions from large distant earthquakes. This is taken care of within the PSHA. Hence the plan for generating the UHRS:

Step 1: Calculation of peak ground accelerations for the remaining periods 0.2 – 4.0 sec are carried out first (see plot for T = 2 sec, Fig. 4) by modifying the *Mathcad* sheets.

Step 2: The concept and methodology are shown in Fig. 2.

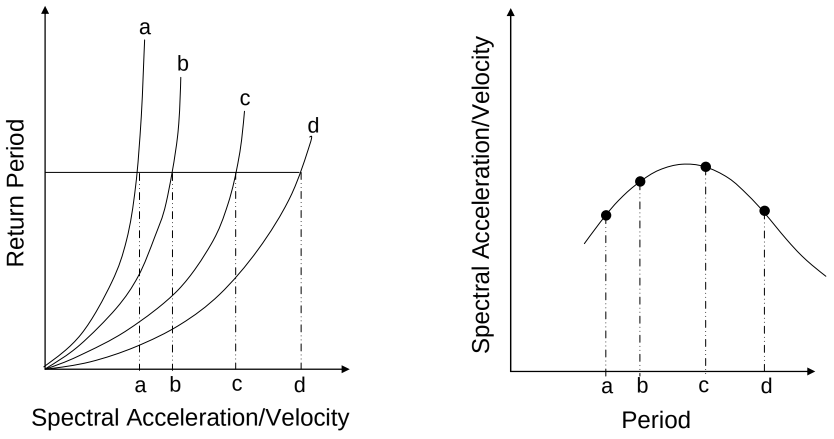

Figure 2: Procedure for Constructing Uniform Hazard Response Spectrum (UHRS)

The plot of UHRS for a return period is shown in Fig. 3.

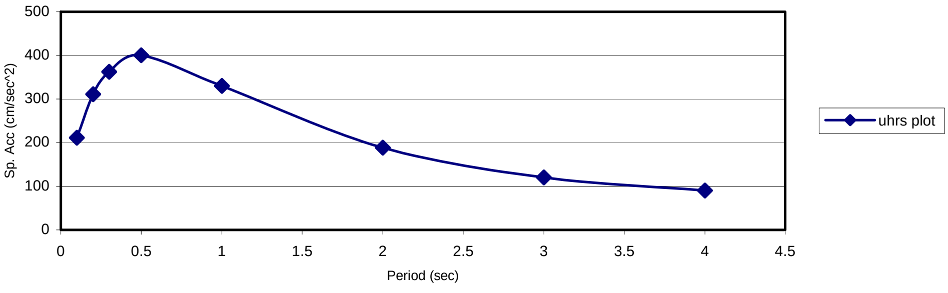

Figure 3: Plot of Uniform Hazard Response Spectrum (Return Period = 1000 years)

6.0 FURTHER COMPUTATIONAL CONSIDERATIONS

6.1 *Deaggregation* – An Introduction

There is usually a project requirement to estimate the most likely combination of earthquake magnitude and source distance contributing the most to the mean annual rate of the parameter being exceeded. In the process of *deaggregation*, we further analyse *individual* PSHA results to seek which combination contributes most. The reader may be wondering why we need *deaggregation*. Sooner or later there will be a requirement of dynamic analyses (at least all large projects do) for detailed assessment of local soil conditions and amplifications of seismic excitation.

The development of time histories require a knowledge of the magnitude and distances that dominate the calculated ground-shaking hazard at the return periods and structural periods of interest.

These estimates may be used to select existing ground motion records for dynamic analyses (for an example of this procedure see Chapter 5 (Sen, 2007) where European records were searched. Candidate time histories may be selected, for example, within say $\pm 0.3M$ (consistent with magnitude scale for PSHA) and $\pm 10km$.

Consider the line source problem (due to Cornell, 1968), analysed in section 6.6.1. The calculations that we have been performing following the Monte Carlo simulation technique allow computation of mean annual rate of a ground motion parameter being exceeded. The calculations considered all possible values of source to site distance and magnitude. Thus the mean annual rate is not associated with any particular combination of source to site distance and magnitude.

It is to be borne in mind that *deaggregation* can only be performed on individual PSHA results shown in the previous section. The information sought from *deaggregation* is lost when the combined uniform hazard response spectrum (UHRS) plot is produced. Thus, the *deaggregation* process may be carried out in the following manner:

i) the seismic hazard (ground motion parameter being exceeded) is partitioned into several magnitude-distance bins
ii) a return period is selected (say 1000 years)

At this point we have a further decision to make. We must decide on the most likely estimate of the natural period of the structure (if not known at this stage). The natural period must be away from the dominant period of seismic excitation.

iii) a target period is selected (say 2 seconds)

The bins with the largest relative contributions identify those earthquakes, which contribute the most towards the total hazard. Time histories (from existing records) are then selected which are consistent with the magnitude and distances of these design earthquakes.

The idea is similar to defining the dominant earthquake and has been explored by others (Chapman,1995, McGuire, 1995)

6.2 Computational Scheme for Deaggregation

Consider the Monte Carlo simulation plot for peak ground acceleration against return period for 100 years for a structural period of 2 seconds as shown in Fig. 4

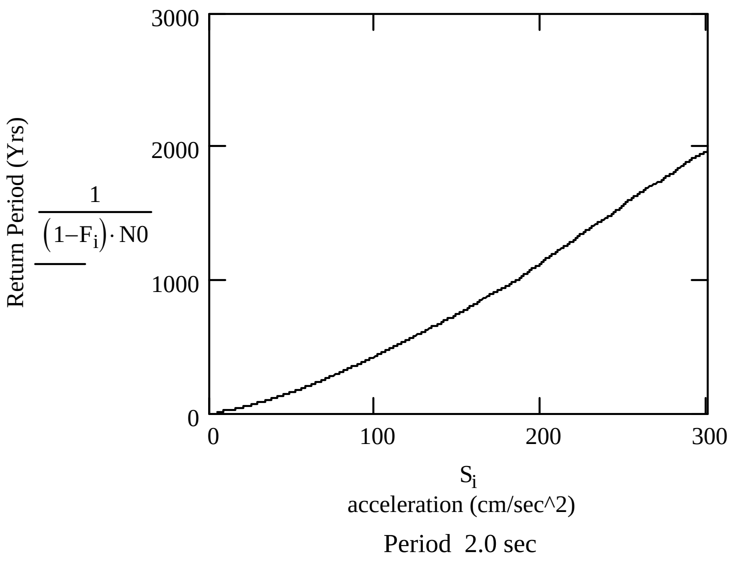

Fig. 4 Acceleration Vs Return Period (Period: 2 sec)
(Attenuation relationship: McGuire, 1974)

The Monte Carlo simulation was carried out for 200,000 samples. Each of the random number sampled produced *a pair of magnitude and distance data*. Thus we have 200,000 pairs of magnitude and distance data.

The return period of interest is 100 years. The acceleration corresponding to a return period of 100 years is approximately 30 cm/sec^2. From the samples, a subset with ground motion parameter (acceleration) corresponding to 100 years and above is created. Deaggregation is carried out on the subset.

The number of hits in each of the bins was counted. The relative contribution of each bin was calculated by dividing the number of hits in each bin by the total number in the subset.

The resulting plot with relative percentage contribution is shown in Fig. 5.

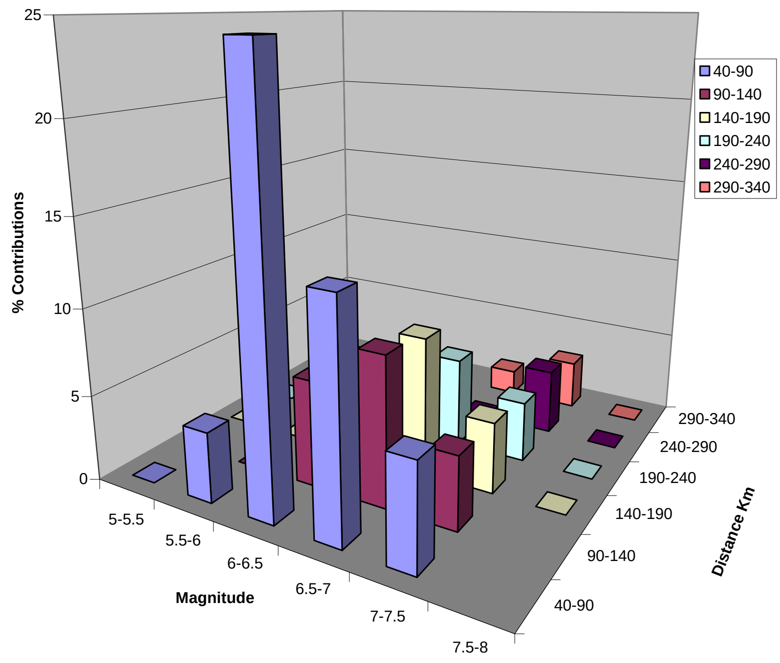

Fig. 5 Deaggregation Plot (Return Period 100Yrs)

Deaggregation Bin Size

Intuitively we are able to figure out that bin sizes would affect the deaggregation plot. In most hazard studies carried out for projects, there is little or no discussion on the selection of magnitude and distance bins. Usually, deaggregation is associated with the 'mode' and refers to the most likely scenario group. The mode has the advantage that it will correspond to a realistic source. But the mode would depend on grouping of scenarios. It has been pointed out by Abrahamson (2006) that with the grouping of the scenarios, the mode will change if the bin sizes are changed.

As pointed out earlier, it is often the case that the main use of the deaggregation analyses is selecting the appropriate time histories for further detailed analyses. In this case the characteristics of time histories of past earthquakes (duration, spectral shape etc) would influence the selection of bin sizes. The characteristics would change with magnitude and distance. Thus, the selection of bin sizes would be project specific. Abrahamson (2006). has provided a very informative discussion on bin sizes.

An alternative approach suggested by Abrahamson (2006) is to use fine deaggregation bins. The controlling scenarios are then identified. The mean magnitude and distance for each controlling scenario may then be computed. The approach is robust and will not be sensitive to bin size. It may get complicated if the scenarios overlap.

The bins with the largest relative contributions identify those earthquakes which contribute the most towards the total hazard. Time histories (from existing records) are then selected which are consistent with the magnitude and distances of these design earthquakes. The idea is similar to defining the dominant earthquake and has been explored by others [Chapman 1995, McGuire 1995].

6.2 Logic Tree Simulation – An Introduction

In the simulations carried out in the previous section, only one model-set was used. In the real world it is not possible to rely on one model-set. There are various options on recurrence laws; attenuation models etc. all well documented in the literature. A computational framework is needed to provide a systematic, clear and logical way of assimilation of various elements of hazard. Otherwise there is the risk of mixing the effects of various options and the consequence of not being able to provide the in-depth decision making required. The *logic tree* concept provides a systematic way forward.

The concept of a flow chart for logical sequence of activities is not new to engineering. The logic tree is essentially a decision flow-chart with branches and sub-branches. Each branch represents a discrete set of choices for the input parameter. For example, whether the attenuation model should be according to Boore, Joyner and Fumal (1997) or Campbell and Bozorgnia (2003). Similarly whether the magnitude, $M = 6.5, 7.0$ or 7.5. Each branch is also assigned the likelihood of being correct (weighting factor). Fig. 6 shows a simple *logic tree.* Uncertainties pertaining to attenuation model, recurrence law and magnitude are shown.

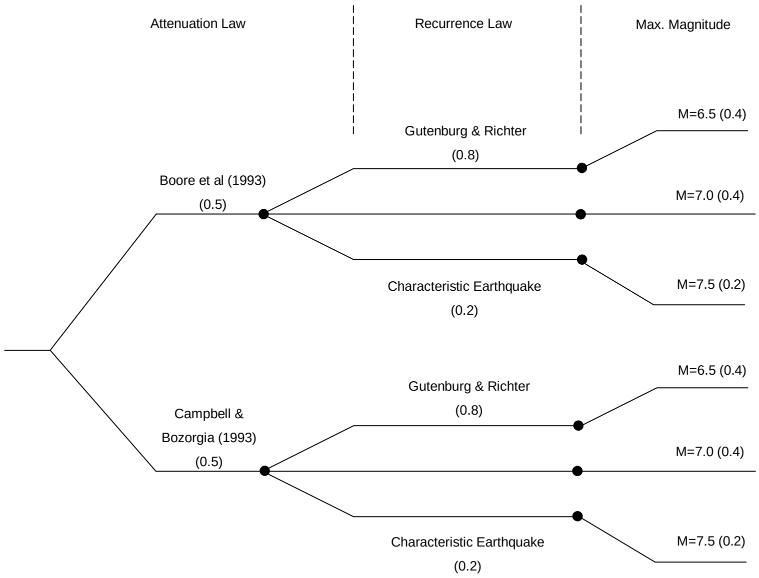

Figure 6: Logic Tree layout for model uncertainty

7.0 AN OVERVIEW

Probability calculations are important. However, it should be emphasised that seismic risk is not all about calculating probabilities. The input parameters themselves e.g. location, seismicity of faults, travel path, predicted magnitude etc. are all very uncertain quantities and continue to be so to the present day. Moreover, seismic probabilities have to be considered in the overall context of what is acceptable to society at large. What are the economic consequences of reducing risk? Can the society sustain this? The acceptable level of risk would depend on the importance of the structure. Probability calculations should be seen as one of the inputs to the wider decision making process.

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