RESERVOIR LENGTH EFFECT IN CALCULATION ACCURATE OF DAM-RESERVOIR INTERACTION

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ABSTRACT :

Today's, many of studying have been started about dam building. Considerable effort has been devoted in recent years to understanding and developing numerical models for the behavior of dam and reservoir. Dam-reservoir interaction in earthquake poses problems of design and analysis in fields of dam engineering.
It is clear now to accurately predict, it is necessary to locate of boundary conditions. One of this boundary condition is infinite boundary. In this paper, we'll try to find efficient length of dam reservoir (near field) to safety design in earthquake with finite element method. With this length, we can cut the dam reservoir, and after that, do dam-reservoir interaction. With using of a program provided specifically for this purpose, we can predict reservoir length effect in dam.

KEYWORDS: gravity dam, reservoir, interaction earthquake, boundary condition.

1. INTRODUCTION

An important factor in the design of dams in seismic regions is the effect of hydrodynamic pressure exerted on the face of the dam as a result of earthquake ground motions. For an accurate analysis of hydrodynamic pressure on the dam having irregular geometries, the reservoir is generally treated as an assemblage of finite elements.
The fluid coupling problem of a dam was originally considered by Westergaard (1933); his theoretical work was the rigid body acceleration of a vertical plane wall will produce a hydrodynamic pressure. It was pointed out that the hydrodynamic pressure can be expressed in terms of a body of water, which is forced to move back and forth with the dam, whilst the rest of the reservoir remains inactive. Westergaard's study was based on the rigid body-movement points of the foundation and dam have the same instantaneous displacement, velocity and acceleration, the reservoir is infinitely long and the water is incompressible and in viscid. The above assumption may be, valid when the period of harmonic excitation is greater than the fundamental period of the reservoir-dam system, otherwise considerable negative pressure will be calculated and hence the added mass concept will lose its meaning. On the basis of a simple linear momentum-balance principle, Von Karman (1933) obtained distributions of the hydrodynamic pressure force and the total load on a rigid dam with a vertical upstream face, which were very close to the Westergaard results.
Chopra (1967) published an analytical solution of the hydrodynamic pressure on a vertical rigid dam. Chwang (1978) presented an exact solution of the hydrodynamic pressure on a rigid dam with an inclined upstream face of constant slope by neglecting the compressibility of the fluid in the reservoir. Mei et al. (1979) Obtained an exact solution for the fluid-structure interaction in the frequency domain. Liu (1986) extended Chwang's work to obtain an exact solution for a rigid sloping dam with a triangular reservoir.
Tsai and Lee (1990) first developed an efficient time-domain semielliptical method to express the radiation condition in the far field of the fluid domain.
In this paper, time domain solution takes into consideration of reservoir length effect in calculation accurate of dam reservoir interaction by numerical example.

2. TIME HARMONIC VIBRATION OF THE BEAM

As a first step and also to verify the solution strategy for this simple example, the dynamic response of sheet

pile wall with fluid interaction effect taken into account is analyzed as there are analytical results of the modal properties of the dry structure available for comparison sake (for dam-reservoir system, these properties are obtained by vibration tests and data analysis only).
For such purpose the reservoir area is divided into two domains: a finite, close to the dam structure, and an infinite one, away from the first domain extending to infinity, and using boundary element method and an analytical method for analysis, respectively.
The normal modes and the natural frequencies of vibrating beams under dry conditions, i.e. undamped-vibration mode shapes and frequencies, can be found in textbooks for different end conditions; free, clamped, and ideally hinged (or simply supported).
Considering two-dimensional free flexural vibrations of uniform beam in free vibration case,

$$EI\frac{\partial^4 w}{\partial x^4}+m_b\frac{\partial^2 w}{\partial t^2}=0 \tag{2.1}$$

where E is Young's modulus of elasticity; I the moment of inertia of the cross-sectional area; m_b the mass per unit length=ρA and ρ is the beam density referred to cross-sectional area.
The solution of Eq. (1) can be found by the method of separation of variables as follows:

$$w(x,t)=\phi(x).q(t);q(t)=e^{iwt} \tag{2.2}$$

Evaluation of Eq. (1) with prescribed boundary conditions, leads to the natural frequency and normal mode equation, respectively. The frequency equation of cantilever beam is given by,

$$\cos(a_nL).\cosh(a_nL)+1=0 \tag{2.3}$$

where the solution of Eq. (3) then provides the values of a_nL which represents the vibration frequencies of the cantilever beam. Each root of Eq. (3) conforms a natural frequency,

$$w_n=(a_nL)^2\sqrt{\frac{EI}{m_bL}} \tag{2.4}$$

and a normal mode function,

$$\phi(x)=[\cosh(a_nx)-\cos(a_nx)]-\sigma_n[\sinh(a_nx)-\sin(a_nx)];n=1,2,3,.... \tag{2.5}$$

where,

$$\sigma_n=\frac{\cosh(a_nx)+\cos(a_nx)}{\sinh(a_nx)+\sin(a_nx)};n=1,2,3,.... \tag{2.6}$$

After the modal value of a_nL has been obtained by solving numerically the frequency equation, it may be substituted into this shape-function expression, Eq. (5), to obtain the corresponding mode shape. The first three vibration modes are shown in Fig. 1.

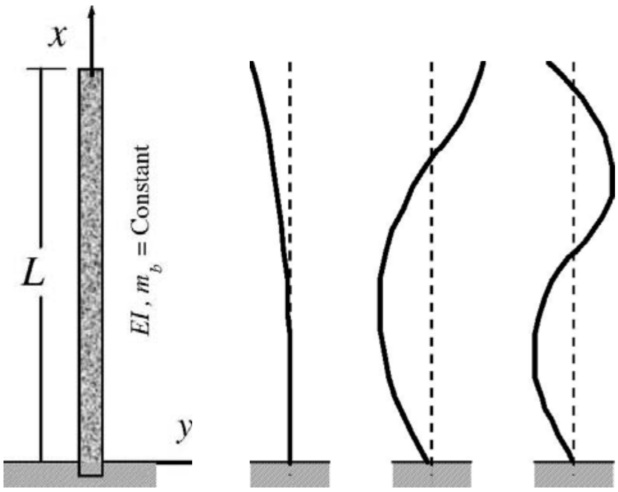

Figure 1 Cantiliever beam vibration analysis.

3. TIME HARMONIC VIBRATION OF COMPRESSIBLE FLUID DOMAIN WITH CONSTANT DEPTH

The beam-fluid system shown in Figure 2 is excited by a harmonic force in analogy to a dynamic in situ testing,

$$f(x,t) = f(x)e^{i.\nu.t} \tag{3.1}$$

where ν is the forcing frequency and f (x) is the time reduced force function.

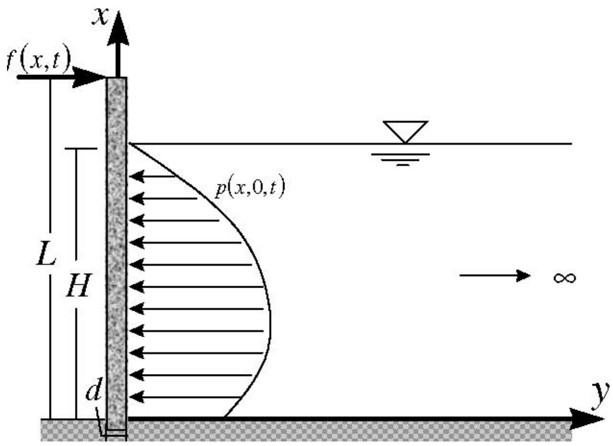

Figure 2 The Beam Fluid system model.

Displacement response of the beam, w(x; t), and the hydrodynamic pressure of linearly compressible response on the upstream face of the beam, p(x, 0, t), in their time reduced case, are represented as,

$$w(x,t) = w(x)e^{i.\nu.t} \tag{3.2}$$

and,

$$p(x,0,t) = p(x,0)e^{i.\nu.t} \tag{3.3}$$

respectively.

Applying the substructure synthesis method yields the solutions to the interactive problem. Further series expansion in the eigenfunctions of the undamped beam (e.g. arch dam in practice) is performed.

The time reduced hydrodynamic pressure (frequency response function) p(x,y,ν) is governed by the Helmholtz equation,

$$\frac{\partial^2 p(x,y,\nu)}{\partial x^2}+\frac{\partial^2 p(x,y,\nu)}{\partial y^2}+kp(x,y,\nu)=0 \tag{3.4}$$

where k is the wave number=(ν/c) and c is the sound velocity in water.
The boundary conditions for Eq. (9) are:
(i) Along beam-fluid interface (y=0),

$$\frac{\partial p(x,0,\nu)}{\partial y}=-\nu^2\rho w \tag{3.5}$$

(ii) At the bottom of fluid domain (x=0),

$$\frac{\partial p(0,y,\nu)}{\partial x}=0 \tag{3.6}$$

(iii) At the free surface of the fluid (x=H),

$$p(H,y,\nu)=0 \tag{3.7}$$

(iv) Radiation condition, waves propagate away from the beam to infinity (y=∞),

$$p(x,\infty,\nu)=0 \tag{3.8}$$

By considering the given boundary condition in Eqs. (10a) and (10d), the solution of Eq. (9) can be written into a Fourier cosine series [9] as:

$$p_j(x,0)=-\rho\nu^2\sum_{n=1}^{\infty}\left[\frac{C_{nj}}{\sqrt{\lambda_n^2-k^2}}\cos(\lambda_n x)\right] \tag{3.9}$$

where

$$C_{nj}=\frac{2}{H}\int_0^H \phi_j(x)\cos(\lambda_n x)dx \tag{3.10}$$

And

$$\lambda_n=\frac{2n-1}{2H};n=1,2,3,.... \tag{3.11}$$

4. EQUATION OF MOTION

In the dynamic analysis of fluid-structure (Figure 2), a beam structure is assumed to be governed by the equation of motion of (10),

$$EI\frac{\partial^4 u}{\partial z^4}+m\ \frac{\partial^2 u}{\partial t^2}=-m\ddot{u}_g-p(0,z,t) \tag{4.1}$$

Where E=elastic modulus; I=the moment of interia of the cross section about the axis of bending (y-axis); m=mass per unit length of the structure; u=displacement of the structure relative to the ground in the

x-direction: $\ddot{u}_g$ = ground accelerations in the upstream-downstream direction: and P = hydrodynamic pressure (in excess of the hydrostatic pressure). Both EI and m arc assumed to vary arbitrarily with z.
The response of the structure, including the fluid effects, can be expressed as a linear combination of mode shapes $\phi_n(z)$ and generalized coordinates $Y_n(z)$.

$$u(z,t) = \sum_{n=1}^{\infty} \phi_n(z) Y_n(t) \tag{4.2}$$

The acceleration of the structure is

$$\ddot{u}(z,t) = \sum_{n=1}^{\infty} \phi_n(z) \ddot{Y}_n(t) \tag{4.3}$$

Applying the modal-superposition analysis to (1), we obtain

$$M_n \ddot{Y}_n(t) + \omega_n^2 M_n Y_n(t) = -V_n(t) - P_n(t); n = 1,2,3,.... \tag{4.4}$$

The generalized mass M_n associated with the *n*th mode shape $\phi_n(z)$ is defined as;

$$M_n = \int_0^h \phi_n^2(z) m(z) dz \tag{4.5}$$

The generalized load induced from the hydrodynamic pressure is denoted by $P_n(t)$, that is;

$$P_n = \int_0^h \phi_n(z) P(0,z,t) dz \tag{4.6}$$

The generalized force resulting from the earthquake excitation is designated as $V_n(t)$, that is;

$$V_n = \ddot{u}_g(t) \int_0^h m(z) \phi_n(z) dz \tag{4.7}$$

The hydrodynamic pressure in the fluid domain of the structure-reservoir system is assumed to be governed by the pressure wave equation.

$$\nabla^2 P(x,z,t) = \frac{1}{C^2} \ddot{P}(x,z,t) \tag{4.8}$$

Where P(x,z,t) =hydrodynamic pressure distributions in excess of the hydrostatic pressure; C=velocity of sound in water; x,z=Cartesian coordinates; and ω_n =*n*th natural frequency of the structure without the reservoir.

5. FINITE ELEMENT METHOD FOR THE NEAR FIELD OF THE FLUID DOMAIN

For the case of dynamic analysis of the dam-reservoir system, the dam substructure can be discretized by using the finite element method with the equations of motion given by;

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) = -MB\ddot{u}_g(t) + L(t) \tag{5.1}$$

where $U(t)$= vector of nodal point displacements relative to the ground; $\dot{U}(t)$=vector of the nodal point velocities relative to the ground; $\ddot{U}(t)$= vector of the acceleration of the nodal points relative to the ground; M, C, K = symmetrical mass, damping, and stiffness matrices, respectively, for the darn structure; L(t)=vector of

hydrodynamic forces acting on the dam face arising from the hydrodynamic pressure of the reservoir; $\ddot{u}_g(t)$ = ground motion; and B=displacement transformation matrix.
The hydrodynamic pressure distribution in the reservoir of a dam-reservoir system is assumed to be governed by the pressure wave equation.
The hydrodynamic pressure distributions can be obtained by solving (2) with the following boundary conditions (see Figure. 1), and zero initial conditions.

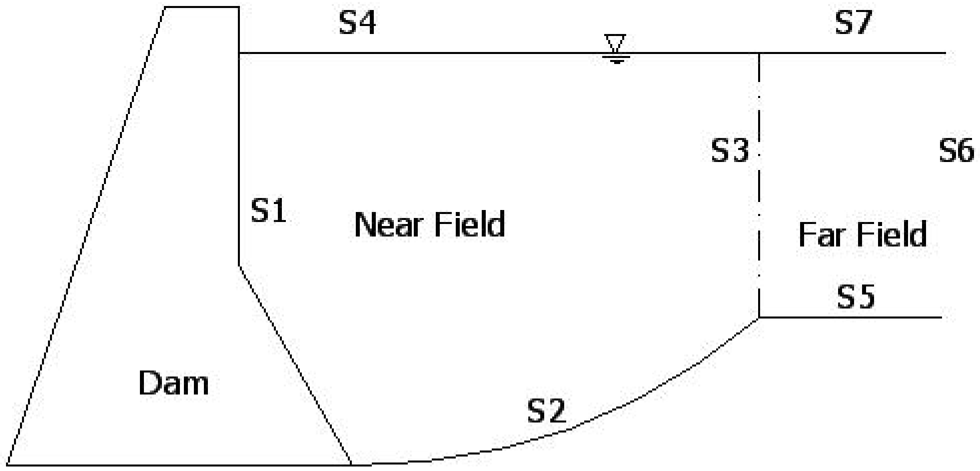

Figure 3 Dam-Reservoir System

1. Surface S1 at the interface of the dam and the reservoir ;

$$\frac{\partial P}{\partial n} = -\rho \ddot{r}^{t}_{ns1} \tag{5.2}$$

2. Surface S2 at floor of the reservoir ;

$$\frac{\partial P}{\partial n} = -\rho \ddot{r}^{t}_{ns2} \tag{5.3}$$

3. Surface S3 at the interface of the near field and far field;

$$\begin{aligned} \frac{\partial P}{\partial n} &= -\frac{\partial P^*}{\partial n} \\ P &= P^* \end{aligned} \tag{5.4}$$

4. Surface S4 at the free surface of the reservoir, if the surface wave effect of the fluid is neglected;

$$P = 0 \tag{5.5}$$

Assuming that the reservoir is initially at rest, the initial conditions are:

$$P\big|_{t=0} = 0 \tag{5.6}$$

and

$$\left.\frac{\partial P}{\partial n}\right|_{t=0} = 0 \tag{5.7}$$

where n denotes the outward normal direction to a boundary, ρ = the mass density of water; $\ddot{r}^{t}_{ns1}$ = the total normal acceleration of the particles on surface S1; $\ddot{r}^{t}_{ns2}$ = the total normal acceleration of the particles on surface S2; P* = the hydrodynamic pressure at the interface of the near field and the far field, $\frac{\partial P^*}{\partial n}$ = the normal derivative of the hydrodynamic pressure at the interface of the near field and the far field. (Positive denotes the outward normal direction to boundary of the far field.)
Using the finite element method for the near field of the fluid domain, the discretized formulation for (2) can be written in the matrix form as follows;

$$G\ddot{P}(t) + HP(t) = D(t) \tag{5.8}$$

here;

$$G_{ij} = \sum G^e_{ij}, H_{ij} = \sum H^e_{ij}, D_i(t) = \sum D^e_i(t) \tag{5.9}$$

The coefficients $G^e_{ij}, H^e_{ij}, D^e_i(t)$ for particular element are;

$$H^e_{ij} = \int_{V_e} \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\right)dV \tag{5.10}$$

$$G^e_{ij} = \frac{1}{C^2}\int_{V_e} N_i N_j dV \tag{5.11}$$

$$D^e_t = \int_{S_e} \frac{\partial P_i}{\partial n} dS \tag{5.12}$$

Where V_e and S_e denote the domain and external boundaries of an element, respectively; and N_t is the shape function.

6. NUMERICAL EXAMPLE

The numerical results of transient responses of the flexible vertical dam with different flexural rigidities are given. The prismatic vertical dam has reservoir of height h=180 m with a constant depth extending to infinity. The velocity of sound in water is taken as 1.438.656 m/s. The weight per unit length of the structure is 36 ton/in. Two different rigidities of the structure, EI= 9.8437x10^9 ton-m^2 and 9.84137 x 1010^7 ton - m1, are used to investigate the interactive behaviors of the entire system(Figure 4). The responses of the fluid-structure system subjected to 1940 EL Centro earthquake (Figure 5) ground motions are obtained.

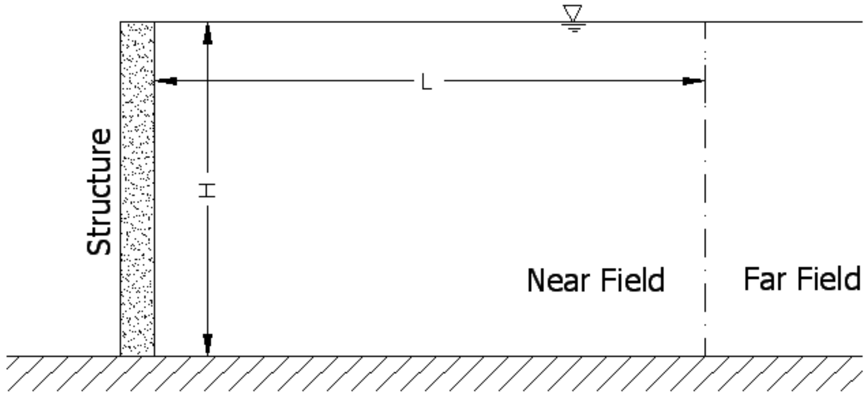

Figure 4 Numerical Modeling

For finding the location of near field, these process will be done one after another;
1-For the first step, the length of reservoir is H (L=H).
2-This model analyses and the displacement of top of the structure control with the results of EAGD84 program.
3-If the results were very different (max tan 1%), it shows that the length of reservoir is low and it should be increased.
4- If the results have less than 1% error, it shows that the length of reservoir is enough for cutting in fluid structure interaction. This length will be effective length in accurate of dam reservoir interaction.

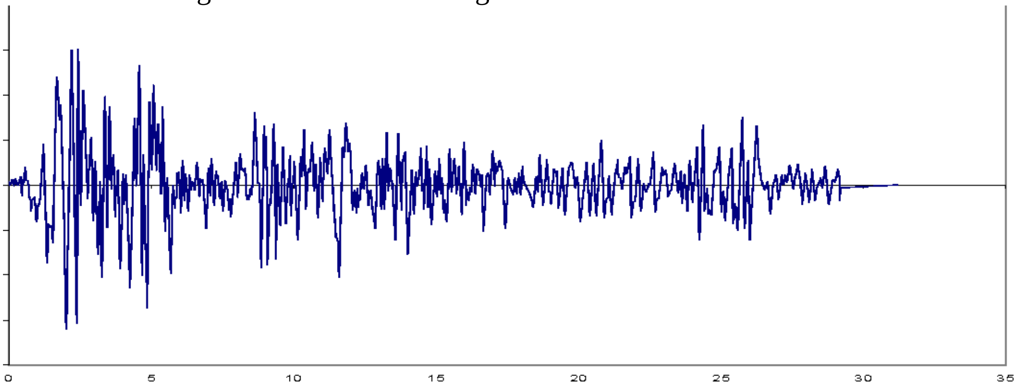

Figure 5 North-South Component og Ground Motion, El Centro (1940)

The difference between the result of fluid structure interaction with variable L/H with EAGD84 program output has been shown in figure 6.

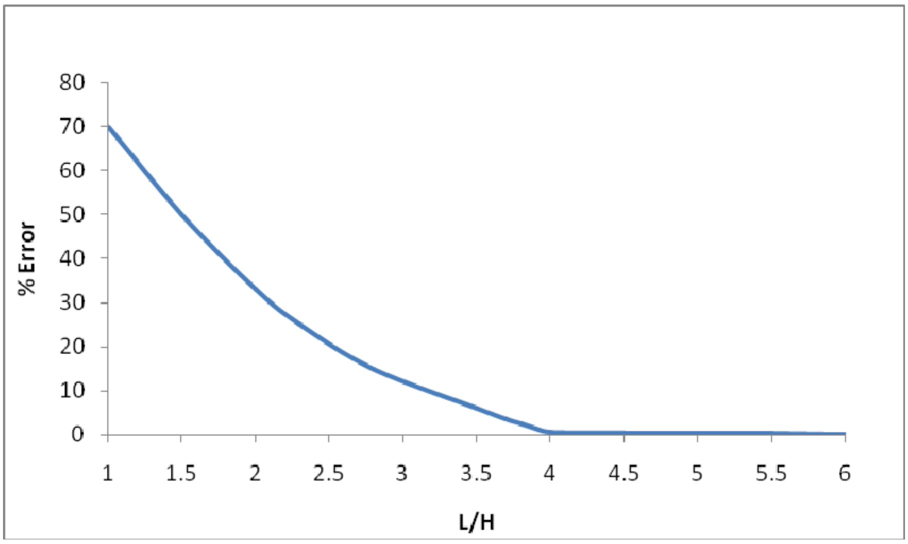

Figure 6 The difference between the result of fluid structure interaction with variable L/H with EAGD84 program output

7. CONCLUSIONS

An exact consideration of the transmitting boundary for the infinite fluid domain is included in a substructure formulation for the time-domain analyses of dam-reservoir systems. Judging from the numerical examples, the proposed method gave accurate results. Some numerical results in the given examples have indicated that the responses of the displacements and hydro-dynamic pressures of the dam-reservoir system are very dependent on the length of reservoir. This result shows that with L/H greater than about 4, the error is negligible and we can cut the reservoir in fluid structure interaction.

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