The Anti-seismic Reliability Analyses for Network System with Intermediate State

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ABSTRACT: In the common engineering network system, the transformation of the working state, from safety to failure, of unit and system is gradual. There is an intermediate state between the states of being completely safety and completely failing. On the basis of "safety-intermediate-failure" three working modes, this paper divides it into two second modes: "safety-non safety" and "non failure-failure", and realizes the calculation of connective reliability for network system by traditional disjoint algorithm. In order to solve the non-polynomial increase hard problem of reliability calculation for large network system with intermediate state, this paper, according to the breadth first search technology of graph theory, proposes a method for the determination of multi-source and multi-terminal network connection which can by applied with Monte Carlo simulation to the calculation of reliability for super-large system. Calculating example in the paper confirms the validity of this method.

KEYWORDS: network system, intermediate state, reliability vector, connective reliability, Anti-seismic

Network system reliability refers to the probability that network completes the predetermined function in limited conditions and time. When the predetermined function refers to connectivity of network system, network system connective reliability forms. The anti-seismic reliability analysis of network system is to calculate the probability that sources and terminals keep connecting after obtaining the reliability vector of various unit construction under the earthquake loading function.

1. DEFINITION OF THTEE WORKING MODES AND RELIABILITY VECTOR OF NETWORK SYSTEM

For the complex multi-source and multi-terminal network system, under the supposed conditions of
1) The arc unit has the safety, intermediate, and failure three states;
2) The arc units which compose the system have the failure independence.

Based on the anti-seismic design problem of network system, considering different connecting situations of the sources and the terminals, this paper concludes that three kinds of incompatible working states of network system usually have the following two groups of definition:
The first group:
(1) Safety state— between at least one terminal and the sources there is a minimum path with all arc units safe. The logical relation between various system units can be described by the structure function

$$R=\bigcup_{k=1}^{N_t}\left(\bigcup_{j=1}^{N_s}\left(\bigcup_{i=1}^{m_{jk}}A_{ijk}\right)\right)=\bigcup_{k=1}^{N_t}\bigcup_{j=1}^{N_s}\bigcup_{i=1}^{m_{jk}}\bigcap_{x_{it}\in A_{ijk}}x_{it} \quad (1)$$

In the formula, A_{ijk} is the minimum path set i between sources j and terminals k. m_{jk} is the assemblage of minimum path sets between sources j and terminals k. N_s and N_t are separately the sum of sources and that

of terminals. x_{it} represents the arc unit in minimum path. In $t \le n-1$, n represents the sum of nodes in the system.
(2) Failure state — each terminal at least has one cut set with a source. Every arc unit is at failure state. The logic relation of units in system can be described by structure function.

$$F = \bigcap_{k=1}^{N_t}\left(\bigcup_{j=1}^{N_s}\left(\bigcup_{i=1}^{m_{jk}} C_{ijk}\right)\right) = \bigcap_{k=1}^{N_t}\bigcup_{j=1}^{N_s}\bigcup_{i=1}^{m_{jk}}\bigcap_{x_{it}\in A_{ijk}} x_{it} \qquad (2)$$

In the formula, C_{ijk} is the minimum path i between sources j and terminal point k. x_{it} represents the arc unit in minimum cut set.
(3) Intermediate state— other situations except（1.1）（1.2）.
The reliability analysis consistent with three working modes is to evaluate the probability of various working states: reliable probability $\psi_R = P(R)$, failure probability $\psi_F = P(F)$, intermediate probability $\psi_M = 1-\psi_R-\psi_F$. They constitute reliability vector $\psi_s = [\psi_R, \psi_M, \psi_F]$, obviously

$$\psi_R + \psi_F + \psi_M = 1 \qquad (3)$$

Likewise, the following is the second group of definition of system working states:
(1) Safety state —— there exists one minimum path with all arc units which are safe between terminal and at least one source
(2) Intermediate state ——other situations except (1) (2);
(3) Failure state ——in the system at least one cut set exists between one terminal point and the sources. Each arc unit is at the mode of failure.

The network system “safety-intermediate-failure” mode is consistent with anti-seismic design's third-level standards, namely, the safety state means “No damage in small earthquake ” (basically complete and slight damage). The intermediate state means “Repairable under moderate earthquake ” (moderate damage), and the mode of failure corresponds with “No collapsing with strong earthquake ” (serious damage and collapsing). The mode is accordant with the spirit and regulation of anti-seismic design; the proposed reliable vectors are the function of fortification intensity and site basic intensity, thus graphs can be established and applied directly to the engineering design; The establishment of three working modes in the analysis of reliability not only consider the intermediary uncertainty, but also avoid the complexity of “generalized reliability theory”. Also this method carries on a more comprehensive estimation to the randomness of network working states, and facilitates in the project design and the decision-making.

2. THE ANALYSIS OF NETWORK REBLIABILITY WHEN CONSIDERING INTERMEDIATE STATE

In the network structural function which is established by equation (1) and (2), each arc unit has three states “safety, intermediate and failure”, so the analysis of system reliability becomes quite complex. The characteristics of analysis of reliability in network system with the intermediate state lie in the existence of intermediate probability, which causes $\psi_R + \psi_F \ne 1$. Therefore, ψ_R and ψ_F in the system must be calculated separately.

2.1 Calculation Based on “Two States” Mode [1]-[4]

Considering the completeness of “safety”, “intermediate”, “failure” of arc unit and network system and normalizing of probability, this paper divides three states mode of the unit and the system with two levels into "safety - non-safety (including intermediate and failure)" and "non-failure (including intermediate and safety) - failure" . And it also establishes correspondingly "two states structure functions". Under the condition of known reliability Vector of each unit ψ_i =[ψ_{Ri} , ψ_{Mi} , ψ_{Fi}], i =1,2,⋯, N, reliability vectors of network system ψ_s =[ψ_R , ψ_M , ψ_F] can be calculated in terms of the following strategies.

Algorithm 1

1) Transform the mode "safety-intermediate-failure" of units and system into "safety—non-safety. Calculate "reliable probability" ψ_R of system by means of minimum path method or disjoint minimum path method.

2) Transform the mode "safety-intermediate-failure" of units and system into "non-failure—failure". Calculate "failure probability" ψ_F by means of minimum cut algorithm or disjoint minimum cut algorithm.

3) Calculate intermediate probability ψ_M in terms of formula (3).

Algorithm 2

1) Transform the mode "safety-intermediate-failure" of units and system into "safety—non-safety. Calculate "reliable probability" ψ_R of system by means of minimum path method or disjoint minimum path method.

2) Transform the mode "safety-intermediate-failure" of units and system into "non-failure—failure. Calculate "non-failure probability" $\psi_{\bar{F}}$ by means of minimum path method or disjoint minimum path method.

3) Calculate "failure probability" ψ_F of the system in terms of 1—$\psi_{\bar{F}}$.

4) Calculate "intermediate probability" ψ_M of the system in terms of equation (3).

Algorithm 3

1) Transform the mode "safety-intermediate-failure" of units and system into "non-failure-failure. Calculate "failure probability" ψ_F by means of minimum cut algorithm or disjoint minimum cut algorithm.

2) Transform the mode "safety-intermediate-failure" of units and system into "safety—non-safety. Calculate "unreliable probability" $\psi_{\bar{R}}$ by means of minimum cut algorithm or disjoint minimum cut algorithm.

3) Calculate "reliable probability" ψ_R in terms of 1—$\psi_{\bar{R}}$.

4) Calculate "intermediate probability" ψ_M in terms of equation(3).

Generally speaking, when the number of minimum path sets is basically equal with the number of the minimum cut sets, algorithm 1 is acceptable; When the number of minimum path sets is smaller than the number of the minimum cut sets, algorithm 2 is suitable; When the number of minimum path sets is bigger than the number of the minimum cut sets, algorithm 3 is adoptable.

2.2 Obtaining Solution Directly Based on "Three States"

The Boolean cubic matrix $R_{m\times n}$ and $F_{m\times n}$ represent the logical relation [5] of equation (1) and (2). The Boolean cube matrix addition operator ($\oplus$) and the multiplication operator ($\otimes$) can separately realize operations of union and intersection of the minimum path. The rules of multiplication and addition algorithm of the Boolean cube matrix $A_{m_1\times n}$ and $B_{m_2\times n}$ are shown as the following:

S1: $\forall\ a_i \in A$, $1 \le i \le m_1$, $\forall\ b_j \in B$, $1 \le j \le m_2$, then

$$R_{m\times n} = A_{m_1\times n} \oplus B_{m_2\times n} = \begin{bmatrix} f_1 \\ \vdots \\ f_k = \begin{cases} a_k & k \le m_1 \\ b_{k-m_1} & k > m_1 \end{cases} \\ \vdots \\ f_{m_1+m_2} \end{bmatrix} \quad 1 \le k \le m, \quad m = m_1 + m_2 \tag{4}$$

$$F_{m\times n} = A_{m_1\times n} \otimes B_{m_2\times n} = \begin{bmatrix} f_1 \\ \vdots \\ f_k = a_i \cap b_j \\ \vdots \\ f_{m_1\times m_2} \end{bmatrix} \quad 1 \le k \le m, \quad m = m_1 \times m_2 \tag{5}$$

S2: $\forall k$, if $f_k = \phi$, delete f_k, $m = m-1$.
S3: $\forall k, l, l \ge k+1$, if $f_k \subseteq f_l$, delete f_k, $m = m-1$.
S4: repeat operation in S2~S3 until there is no deletion.

In the formula（4）（5）, $f_1, f_2 \cdots f_m$ is Boolean cubic of m minimum logic functions; a, b are two cubes, m is the number of rows in matrix $F_{m\times n}$; n is the column number.
The disjoint algorithm of the Boolean cube matrix(#)can be seen in literature [6]. After the disjoint of $R_{m\times n}$, $F_{m\times n}$, the Boolean logic function in $\# R_{m\times n}$, $\# F_{m\times n}$ has been independent. The reliable probability of network system is

$$\psi_S = P(R) = \bigcap_{k=1}^{N_t} \bigcup_{j=1}^{N_s} \bigcup_{i=1}^{m_{jk}} \bigcap_{x_{it}\in A_{ijk}} P(x_{it}) = \sum_{i=1}^{m} P(\mathrm{dis}f_i) = \sum_{i=1}^{m} \prod_{x_{it}\in \mathrm{dis}f_i} P(x_{it}) \tag{6}$$

In the formula, m is the Boolean cubic volume of $\# R_{m\times n}$ matrix under the condition of calculation of reliability, $\mathrm{dis}f_i$ is the Boolean cubic in $\# R_{m\times n}$ matrix, namely disjoint minimum path.
By similar method ψ_F can be evaluated, and ψ_M can be evaluated from (3).

3. MONTE-CARLO ALGORITHM FOR CALCULATION OF RELIABILITY BASED ON BFS POLYMORPHISM NETWORK

Monte-Carlo algorithm is used to avoid the complexity of equation (1), (2). Its basic philosophy aims to approximately reproduce the destruction condition of network units by means of the destruction probability, through the massive random simulations. Finally, calculate the frequency of connectivity of sources and terminals, and then replace the precise probability analysis by this approximate frequency.
The literature [7] simulates the destruction condition of network, and forms the adjacent matrix of the destruction. Find the forest which grows from various sources, and determines the connectivity of various terminals and the sources; if the terminal is not in the forest, then this spot is at the seriously unreliable state. If the terminal is in the forest, it is at the reliable and intermediate state. How to distinguish these two conditions depends on the color of the node. The concrete steps can be summarized:

1）Write down the adjacent matrix A of the network, $A=[a_{ij}]$ is $n\times n$ order matrix， n is the vertex number in the network graph. If nodes i、 j are connected， a_{ij} =1. Or else, a_{ij} =0. The adjacent matrix A contains the original topological structural information of the network in the computer.
2）In terms of the evaluation of arc unit probability, determine the reliable probability, intermediate probability, and failure probability ψ_R， ψ_M， ψ_F of the network edges(units).
3）Reflect the three states probability of arc units in a real number zone. Produce a set of random numbers that are evenly distributed among [0, 1] by means of random number generator, and these numbers match the network arcs.
4）Compare the random number r_i on each edge with the corresponding zone of the probability of three states, then determine the working state of the arc unit. Thus, a simulation of network destruction condition is achieved.
5）According to one of the simulated destruction conditions from the sample, the adjacent matrix of the destruction D^* of the network forms. D^* is adapted from the adjacent matrix of the network A. If edge e_{ij} is seriously damaged, adapt a_{ij} by 0. If e_{ij} is at the intermediate state, adapt a_{ij} by x。
6）In one simulated network, a directed tree grows from a source. Search outward gradually from the sources by means of BFS（Breadth First Search），When searching the node i, according to D^*, if $a_{ij}=1$, dye the following node j blue; if $a_{ij}=x$, when j has not been dyed，dye it yellow. If j has been dyed，then make no change. After finishing searching all the linked edges adjacent to i, go ahead to the next node; repeat the above steps until the forest forms from all the sources. Obviously, the final results are: the blue node represents reliable state. The yellow represents the intermediate state between reliable and failure states. The node without color represents seriously unreliable state.
7）Repeat steps in（3）~（6）, calculate the frequency of connectivity of nodes and sources, that is to say, count the times to be dyed and the colors of the nodes in the simulation. Then divide the times to be dyed by the times of simulation. Take the color frequency of the state of the coloring node as the approximate value of the corresponding probability. Count the times of each node to be dyed the same color in the simulation. Then count the times of at least one terminal which is dyed a certain color. Thus obtain the reliable probability, intermediate probability and failure probability of the network system.

4. CALCULATING EXAMPLES

The general edge weight engineering network is shown in Fig 1. The vertex set is $V=\{v_1,v_2,\ldots,v_9\}$, the edge set is $E=\{e_1,e_2,\ldots,e_{13}\}$， v_7， v_9 are sources， v_4， v_8 are terminals. The anti-seismic reliable vector of the edge can be seen in Table 1. Try to evaluate the anti-seismic reliability of the network system. The corresponding calculating results can be seen in table 2- table 3.

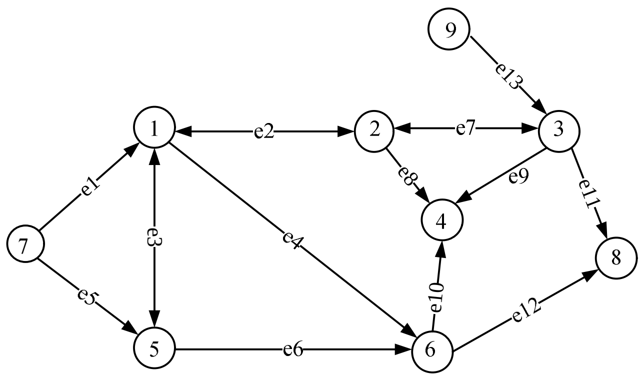

Figure 1 Directed edge weight network chart with intermediate state

Table 1 Anti- seismic reliability vector of arc section in the network

Arc section No.	Reliability Vector		
	Reliable probability	Intermediate probability	Failure probability
e1	0.90	0.05	0.05
e2	0.85	0.05	0.10
e3	0.90	0.07	0.03
e4	0.80	0.10	0.10
e5	0.85	0.05	0.1
e6	0.95	0.02	0.03
e7	0.80	0.05	0.15
e8	0.85	0.10	0.05
e9	0.90	0.07	0.03
e10	0.80	0.10	0.10
e11	0.80	0.10	0.10
e12	0.85	0.05	0.10
e13	0.85	0.05	0.10

Table 2 Connective reliability vector of node and sources

Node No.	Type	Reliability	Intermediate state	Unreliability
1	Node	0.99002	0.00182	0.00816
2	Node	0.9471	0.02062	0.03228
3	Node	0.94874	0.02044	0.03082
4	Terminal	0.98838	0.00164	0.00998
5	Node	0.97884	0.0041	0.01706
6	Node	0.98122	0.00422	0.01456
7	Source	1	0	0
8	Terminal	0.95804	0.01092	0.03104
9	Source	1	0	0

Table 3 Calculating results of reliability vector of the network system

Definition of system working states	Analytical Method			Monte-Carlo simulation		
	ψ_R	ψ_M	ψ_F	ψ_R	ψ_M	ψ_F
Group 1	0.99637	0.00298	0.00065	0.99664	0.00258	0.00058
Group 2	0.95370	0.03207	0.01423	0.94978	0.03844	0.01198

Analysis and discussion:

1) This paper makes description and division to the working states of the network system. It has given corresponding logical structural function, defined the system reliable vector with intermediate condition, and avoided the confusion of two concepts--- unit reliability and system reliability. The reliable vector of terminals 4, 8 in Table 2 have substantive disparity with network system reliable vector in Table 3. Although both the two vectors have the intermediate state, they are on two different kinds of levels--- unit and system. Therefore, some literatures which evaluate network system's reliability by calculating the average of the connective reliability of all the terminals and sources, cannot reflect the real situation of network system reliability, because they expand calculation of unit reliability to system reliability, that is to say, from the low level to the high level[8].

2) For the units and systems with intermediate condition, definitions of their respective working states have incompatibility, and the calculated system reliability of three states have the normalizing.

3) The reliable vector of network system depends on the divisions of "safety - intermediate - failure" three conditions and the construction of corresponding structure function. For different network systems, the division standard is different from each other, and the structure function is different, too. The computed results have

disparity, thus the system reliability has the multiplicity, see Table 3.
4) The system reliable vectors which are gained separately by Applied analytical method and Monte-Carlo are in consistence in table 3.

5 CONCLUSION

This paper has expanded the two working modes “failure-reliable” in reliable theory into “safety — intermediate - failure” three working modes. “Safety” probability and “failure” probability have been under the analysis of the traditional reliable thought that merges the fuzzy elements into the intermediate state. Thus, this paper proposes a general method for analysis of anti-seismic reliability vector, which is used to differentiate the influence of the clear elements and fuzzy elements to system, so as to evaluate the anti-seismic reliability of the multi-source and multi-terminal system with intermediate state. So a new kind of effective and practical method is offered for the network system's anti-seismic optimization design and the disaster forecast, and will be promoted to the optimization design of large-scale complex structure and systems in other professions.

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