# BEHAVIOR OF SQUARE CONCRETE-FILLED TUBULAR COLUMNS UNDER ECCENTRIC COMPRESSION WITH DOUBLE CURVATURE DEFLECTION 

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#### Abstract

: Objective of this paper is to present experimental information about the behavior of square CFT columns subjected to eccentric compression with double curvature deformation, to investigate validity of the current design formula for slender CFT columns, meanwhile to study its applicability to the middle-length columns.

Forty-three square CFT columns were fabricated and tested. The experimental variables among the tests are, 1 ) the buckling length to depth ratio of the column, 2) the concrete strength, 3 ) the moment gradient which deforms the column into double curvature, and 4) the eccentricity of the axial load.

Experimental results have indicated that the flexural strength of slender square CFT columns increases with the concrete strength and the moment gradient. Comparison between the test results and the theoretical ones predicted by the current design code has shown that for the slender square CFT columns with the buckling length to depth ratio of 20 , the current design formula could satisfactorily well predict the flexural strength in spite of the degree of moment gradient. For the middle-length square CFT columns with the buckling length to depth ratio of 10 , however, the current design formula overestimates the flexural strength when the column was under eccentric compression without moment gradient, i.e. under pure bending. On the other hand, for the middle-length columns under eccentric compression with moment gradient, the current design formula could be applied to evaluate their flexural strength with satisfactory accuracy.


## KEYWORDS:

Concrete-filled tubular column, Slenderness ratio, Double curvature deformation, Eccentric compression, Square column

## 1. INTRODUCTION

In the current design standards for steel reinforced concrete structures (hereafter refereed to as SRC standards) and the design guideline for concrete-filled tubular structures (hereafter refereed to as CFT guideline) by the Architectural Institute of Japan, the ultimate capacity of a steel-concrete composite slender column is calculated by simply superimpose the capacities of the concrete column and the steel column (AIJ, 2001; AIJ 1997). This simple superimposition method has such advantages as to keep continuity in the calculation of capacity for the short and the slender columns, and to contain the up-to-date information on each component, while the design formula becomes inevitably complicated. In addition, for the columns under concentric compression, the current design standards not only provides calculation equations for short and slender columns with slenderness ratio $L_{k} / D$ less than and equal to 4 and larger than and equal to 12 , respectively, but also recommendation for the calculation for the middle-length column with $L_{k} / D$ varying between 4 and 12 . However, for the columns under combined flexure and axial compression, the current SRC standards only shows capacity design formulae for short and slender column, and approximates the calculation for the middle-length column by applying the equation for the short column, hence resulting discontinuity in the design equation.

As to the capacity design equation for the middle-length columns with $L_{k} / D$ varying between 4 and 12 , Chung et al [2004] haves proposed a method to obtain the ultimate capacity via linear interpolation between the short and the
slender column. This method, however, involves tedious calculation procedure, while its concept is clear and mathematical expression is simple. Kido et al [2005] have also proposed a theoretical approach for the calculation for the middle-length columns, but in their proposal there is still discontinuity in the calculation.

Fujinaga et al [2005] have recently proposed a simple method for the middle-length CFT column. In this method, the ultimate capacity for the middle-length columns is calculated by linear interpolation between the short and the slender column. Nevertheless, validity of their proposed method has not yet been verified with experimental results, neither the details of the moment modification factor $C_{M}$ were mentioned in the method.

On the other hand, experimental study on effects of the moment gradient in the columns under double curvature on the ultimate capacity is scarce. While Kilpatrick et al [1997] have conducted systematic tests on the circular CFT columns to investigate the effect of the moment gradient; there are no experimental results of square CFT columns available. Furthermore, the moment amplification factor $C_{M}$ due to the moment gradient for the middle-length column has not yet been investigated.

Objectives of the research is to obtain experimental information for the behavior of square CFT slender and middle-length columns under double curvature deformation, and to investigate validity of the previous calculation methods through comparing the experimental results with the theoretical predictions.

## 2. OUTLINES OF THE CURRENT DESIGN FORMULAE

### 2.1. Capacity Equations for the CFT Columns under Combined Flexure and Axial Compression

According to the current SRC standards, ultimate flexural strength of the CFT columns under combined flexural and axial load can be obtained as follows:
(1) For the column with $L_{k} / D$ ratio less than and equal to 12

$$
\left.\begin{array}{l}
N_{U}={ }_{c} N_{U}+{ }_{s} N_{U}  \tag{2.1}\\
M_{U}={ }_{c} M_{U}+{ }_{s} M_{U}
\end{array}\right\}
$$

(2) For the column with $L_{k} / D$ ratio larger than 12
(i) When $N_{U}<{ }_{c} N_{c U}$ or $M_{U}>{ }_{s} M_{U 0}\left(1-{ }_{c} N_{c U} / N_{k}\right) / C_{M}$

$$
\begin{align*}
& N_{U}={ }_{c} N_{U}  \tag{2.2}\\
& \left.M_{U}=\left\{{ }_{c} M_{U}+{ }_{s} M_{U 0}\left(1-\frac{{ }_{c} N_{U}}{N_{k}}\right)\right\} \frac{1}{C_{M}}\right\}
\end{align*}
$$

(ii) When $\quad N_{U}>{ }_{c} N_{c U}$ or $M_{U}<{ }_{s} M_{U 0}\left(1-{ }_{c} N_{c U} / N_{k}\right) / C_{M}$

$$
\left.\begin{array}{l}
N_{U}={ }_{c} N_{c U}+{ }_{s} N_{U}  \tag{2.3}\\
M_{U}={ }_{s} M_{U}\left(1-\frac{{ }_{c} N_{c U}}{N_{k}}\right) \frac{1}{C_{M}}
\end{array}\right\}
$$

where ${ }_{c} N_{U},{ }_{c} M_{U},{ }_{s} N_{U},{ }_{s} M_{U}$ are ultimate compressive strength and ultimate flexural strength of the filled concrete section, and those of the steel tube, respectively, ${ }_{s} M_{U 0}$ is ultimate flexural strength of steel tube under pure bending, $N_{k}$ is Euler's buckling load of the CFT column, ${ }_{c} N_{c U}$ is ultimate compressive strength of the filled concrete section, and $C_{M}$ is the moment amplification factor.

### 2.2. Moment Amplification Factor $\boldsymbol{C}_{M}$

In the current SRC standard, the formula recommended in the plastic design guideline by the AIJ [AIJ, 1975] is applied to calculate the moment amplification factor in the form of

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$$
\begin{equation*}
C_{M}=1-0.5\left(1-\frac{M_{2}}{M_{1}}\right) \sqrt{\frac{N_{U}}{N_{k}}} \geq 0.25 \quad, \quad N_{k}=\frac{\pi^{2}}{l_{k}^{2}}\left(\frac{{ }_{c} E \cdot{ }_{c} I}{5}+{ }_{s} E \cdot{ }_{s} I\right) \tag{2.4}
\end{equation*}
$$

where $M_{1}, M_{2}$ are the larger absolute end moment and smaller end moment, respectively, $E$ and ${ }_{s} E$ are the Young's modulus of concrete and of steel, respectively, ${ }_{c} I,{ }_{s} I$ are moment inertia of concrete and of steel, respectively, $l_{k}$ is buckling length of the column.

### 2.3. Strength of Middle-length Column Calculated by Linear Interpolation

In the AIJ design formula, the boundary of short column is $L_{k} / D=4$, and the boundary of slender column is $L_{k} / D=12$.

Table 1 Experimental Condition and Result

| Specimen | Type | $L_{k} / D$ | $\begin{gathered} e_{1} \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{array}{\|c\|} \hline e_{2} \\ (\mathrm{~mm}) \end{array}$ | $\begin{gathered} \beta \\ \left(=e_{2} / e_{1}\right) \\ \hline \end{gathered}$ | $\begin{array}{c\|} \hline{ }_{c} \sigma_{B} \\ \left(\mathrm{~N} / \mathrm{mm}^{2}\right) \\ \hline \end{array}$ | $\begin{gathered} N \\ (\mathrm{kN}) \end{gathered}$ | $\begin{gathered} \hline \hline M=N \square e_{1} \\ (\mathrm{kNm}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R20-27-C | B | 20 | 0 | 0 | - | 29.1 | 835 | 0.0 |
| R20-27-10-1.0(+) |  |  | 10 | 10 | 1.0 | 31.6 | 585 | 5.8 |
| R20-27-10-0.5(+) |  |  |  | 5 | 0.5 |  | 627 | 6.3 |
| R20-27-10-0.0 |  |  |  | 0 | 0.0 | 33.9 | 671 | 6.7 |
| R20-27-10-0.333(-) |  |  |  | -3.33 | -0.333 |  | 711 | 7.1 |
| R20-27-10-0.667(-) |  |  |  | -6.67 | -0.667 | 33.3 | 738 | 7.4 |
| R20-27-10-1.0(-) |  |  |  | -10 | -1.0 |  | 823 | 8.2 |
| R20-27-30-1.0(+) |  |  | 30 | 30 | 1.0 | 36.7 | 440 | 13.2 |
| R20-27-30-0.5(+) |  |  |  | 15 | 0.5 |  | 479 | 14.4 |
| R20-27-30-0.0 |  |  |  | 0 | 0.0 |  | 552 | 16.6 |
| R20-27-30-0.333(-) |  |  |  | -10 | -0.333 |  | 574 | 17.2 |
| R20-27-30-0.667(-) | C |  |  | -20 | -0.667 | 31.5 | 620 | 18.6 |
| R20-27-30-1.0(-) |  |  |  | -30 | -1.0 |  | 665 | 19.9 |
| R20-27-100-1.0(+) | B |  | 100 | 100 | 1.0 | 32.1 | 222 | 22.2 |
| R20-27-100-0.5(+) |  |  |  | 50 | 0.5 |  | 265 | 26.5 |
| R20-27-100-0.0 |  |  |  | 0 | 0.0 | 35.5 | 307 | 30.7 |
| R20-27-100-0.333(-) |  |  |  | -33.3 | -0.333 |  | 328 | 32.8 |
| R20-27-100-0.667(-) | C |  |  | -66.7 | -0.667 | 35.8 | 325 | 32.5 |
| R20-27-100-1.0(-) |  |  |  | -100 | -1.0 |  | 336 | 33.6 |
| R20-60-C | B |  | 0 | 0 | - | 61.3 | 1106 | 0.0 |
| R20-60-30-1.0(+) |  |  | 30 | 30 | 1.0 |  | 550 | 16.5 |
| R20-60-30-1.0(-) |  |  | 30 | -30 | -1.0 | 63, | 878 | 26.3 |
| R20-60-100-1.0(+) |  |  | 100 | 100 | 1.0 | 65.5 | 250 | 25.0 |
| R20-60-100-0.0 |  |  |  | 0 | 0.0 |  | 354 | 35.4 |
| R20-60-100-0.667(-) |  |  |  | -66.7 | -0.667 | 66.4 | 372 | 37.2 |
| R20-60-100-1.0(-) | C |  |  | -100 | -1.0 |  | 385 | 38.5 |
| R10-27-C | A | 10 | 0 | 0 | - | 35.9 | 1016 | 0.0 |
| R10-27-30-1.0(+) |  |  | 30 | 30 | 1.0 | 35.4 | 606 | 18.2 |
| R10-27-30-0.5(+) |  |  |  | 15 | 0.5 |  | 650 | 19.5 |
| R10-27-30-0.0 |  |  |  | 0 | 0.0 | 31.6 | 695 | 20.9 |
| R10-27-30-0.333(-) |  |  |  | -10 | -0.333 |  | 722 | 21.7 |
| R10-27-30-0.667(-) |  |  |  | -20 | -0.667 | 31.1 | 737 | 22.1 |
| R10-27-30-1.0(-) |  |  |  | -30 | -1.0 | 37.1 | 776 | 23.3 |
| R10-27-100-1.0(+) |  |  | 100 | 100 | 1.0 |  | 281 | 28.1 |
| R10-27-100-0.5(+) |  |  |  | 50 | 0.5 |  | 328 | 32.8 |
| R10-27-100-0.0 | B |  |  | 0 | 0.0 | 35.9 | 351 | 35.1 |
| R10-27-100-0.333(-) |  |  |  | -33.3 | -0.333 | 34.1 | 372 | 37.2 |
| R10-27-100-0.667(-) | C |  |  | -66.7 | -0.667 |  | 372 | 37.2 |
| R10-27-100-1.0(-) |  |  |  | -100 | -1.0 | 34.7 | 385 | 38.5 |
| R10-60-C | A |  | 0 | 0 | - | 68.4 | 1452 | 0.0 |
| R10-60-100-1.0(+) | B |  | 100 | 100 | 1.0 | 68.4 | 324 | 32.4 |
| R10-60-100-0.0 |  |  |  | 0 | 0.0 | 67.6 | 406 | 40.6 |
| R10-60-100-1.0(-) |  |  |  | -100 | -1.0 |  | 430 | 43.0 |

[^0]${ }_{c} \sigma_{B}$ : Compressive strength of concrete,
$N$ : Experimental maximum strength Then the flexural strength for middle-length column ( $4<L_{k} / D<12$ ) can be easily calculated by using linear interpolation method.

## 3. ECCENTRICALLY COMPRESSIVE TESTS UNDER DOUBLE CURVATURE DEFORMATION

### 3.1. Outlines of Specimen

A total of forty-three columns were fabricated and tested. Figure 1 shows details of the test columns. All of the specimens were made of square steel tube with dimensions of $125 \times 125 \times 3.2 \mathrm{~mm}$ (STKR400) and filled with concrete having targeted compressive strength of $27 \mathrm{~N} / \mathrm{mm}^{2}$ or 60 $\mathrm{N} / \mathrm{mm}^{2}$. The experimental variables are; 1) the $L_{k} / D$ ratio (20 and 10), the concrete strength, the moment gradient expressed in term of $M_{2} / M_{1}$ ratio, and the initial eccentricity $e_{1}(100,30,10 \mathrm{~mm})$.


Figure 1 Specimen (mm)

Table 2 Mixture of Concrete

| Depth | Thickness | Tension test |  | Compression test |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Yield stress | Tensile strength | Yield stress | Compressive strength <br> $(\mathrm{mm})$ |
| $(\mathrm{mm})$ | $\left.\mathrm{N} / \mathrm{mm}^{2}\right)$ | $\left.\mathrm{N} / \mathrm{mm}^{2}\right)$ |  |  |  |
| $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$ | $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$ |  |  |  |  |



Figure 2 Stress - strain relationship


Figure 3 Loading condition


Photo 1 Loading situation


Photo 2 Details of loading equipment


Figure 4 Comparison of strength (continue)

## 4. EXPERIMENTAL RESULTS AND COMPARISON WITH THEORETICAL PREDICTIONS

### 4.1 Moment Versus Axial Load Relationship

Figure 4 shows experimental flexural strengths along with several theoretical predictions. In Figure 4, the black circles express the experimental results, while the theoretical results are represented by several curves; the black solid line corresponds to the flexural strength of the CFT column calculated by the SRC standard, where effect of the slenderness has been taken into consideration, and the dotted line expresses the ultimate flexural strength of the CFT section. For the middle-length columns, a black line is further plotted as shown in Figure 4 (b) and (d) to
represent the flexural strength computed by linear interpolation between the flexural strengths of short and slender columns by SRC standard.

As obvious from Figure 4 (a) and (c), the theoretical results calculated by current SRC standard agree well with the experimental results of the slender columns in spite the concrete strength, while the theoretical predictions tend to underestimate the experimental ones as the moment gradient becomes larger.

For the middle-length columns, however, the SRC standard seems to overestimate the experimental result for the columns under pure bending,, i.e. without moment gradient. This discrepancy is mainly due to that in the SRC standard, the flexural strength of the middle-length column with $L_{k} / D$ ratio varying between 4 and 12 is computed by using the equation for the short column. It is apparent, on the other hand, that the theoretical results calculated by linear interpolation could conservatively predict the test results in spite of the moment gradient.

From the above-mentioned comparisons and observation, one can see that flexural strength of the middle-length square CFT column can be reasonably predicted by linear interpolation between the flexural capacities of the short and slender columns calculated by the current SRC standard.

To better understand difference between the experimental and theoretical flexural strengths, relationships between the excess ratios and the excess angles of flexural strength, which are defined in Figure 5, are plotted in Figure 6. For the slender columns with $L_{k} / D$ ratio of 20 , the ratio of the experimentally measured strength to the theoretical results varies between 1.02 and 1,24, having an average of 1.13 . For the middle-length columns with $L_{k} / D$ ratio of 10 , the flexural strength ratio varies between 0.90 and 1.29 when the SRC equation for short column is directly applied to calculate the flexural strength. On the other hand, the linear interpolation method can give a more reasonable prediction to the

(c) $L_{k} / D=20$ (Slender column), ${ }_{c} \sigma_{B}=60 \mathrm{~N} / \mathrm{mm}^{2}$



(d) $L_{k} / D=10$ (Middle-length column), $\sigma_{B}=60 \mathrm{~N} / \mathrm{mm}^{2}$

Figure 4 Comparison of strength (the rest)


Figure 6 Comparison about excess ratio
flexural strength for the middle-length CFT columns, with the strength ratio varying between 1.05 and 1.29 and an average of 1.18 .

### 4.2 Moment Amplification Factor $\boldsymbol{C}_{M}$

In order to obtain experimental results for the moment amplification factor, the experimental moment versus axial load curves were at first approximated by conducting regression analysis on the experimental flexural strengths for each moment gradient as shown in Figure 7. Then the experimental moment amplification factors were calculated as the difference between the experimentally measured strength and the strength drawn from the approximated moment-axial load curve shown in Figure 7.

Figure 8 shows experimental moment amplification factors along with the theoretical curves recommended by the AIJ plastic design guideline for steel structures [AIJ, 1975]. The bold solid lines and dotted lines in Figure 8 represent equation for the moment amplification factor based on elastic theory and the approximate equation, respectively. Axial force is normalized by Euler's load (see Eqn. (2.4)).

For the slender columns with $L_{k} / D$ ratio of 20 , in the case of $F_{c}=27 \mathrm{~N} / \mathrm{mm}^{2}$, the approximate equation agreed well with the test results as the moment ratio $M_{2} / M_{1}$ is equal to -1.0 . As becomes larger, i.e. the moment gradient becomes smaller, the discrepancy between the experimental and the theoretical results becomes wider, but the approximate equation could still trace the tendency of the moment amplification factor along with the axial load. In the case of $F_{c}=60 \mathrm{~N} / \mathrm{mm}^{2}$, although the experimental data is few, the approximate formula traced the tendency of moment amplification factor $C_{M}$ as well as in the case of $F_{c}=27 \mathrm{~N} / \mathrm{mm}^{2}$.

For the middle-length columns with $L_{k} / D$ ratio of 10 , the experimental moment amplification factors were less than the calculated results obtained either by the elastic theoretical equation or by the approximate equation, which means that the design formula for the slender CFT columns recommended in the current standards would underestimate the axial capacity of the middle-length CFT columns.

(a) $L_{k} / D=20$

(b) $L_{k} / D=10$

Figure 8 Comparison of moment modified factor $C_{M}$

## 5. CONCRUDING REMARKS

Experiments of slender and middle-length square CFT columns under eccentrically compression with doublecurvature deflection were conducted to examine the effect of moment gradient on the flexural strength of column. Comparing the experimental results with the theoretical predictions leads to the following conclusions:

1) For the slender square CFT columns with the buckling length to depth ratio of 20 , the current design formula could satisfactorily well predict the flexural strength in spite of the degree of moment gradient.
2) For the middle-length square CFT columns with the buckling length to depth ratio of 10 , the current design formula overestimated the flexural strength when the column was under eccentric compression without moment gradient, i.e. under pure bending. On the other hand, for the middle-length columns under eccentric compression with moment gradient, the current design formula could be applied to evaluate their flexural strength with satisfactory accuracy.
3) Moment amplification factor, which was calculated backward from experimental result, compared well with the elastic theoretical and approximate equations. Approximate equation predicts well the results for the columns with $L_{k} / D=20$ and $M_{2} / M_{1}=-1.0$. However, for the columns with $L_{k} / D=10$, accuracy is much lower regardless of the moment gradient.

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[^0]:    $e_{1}, e_{2}$ : Eccentricity of upper end and bottom end, respectively,

