# Forcing Mode Superposition Method for Analysis of Seismic Response of Lighting Steel Adding Storey Frame

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**ABSTRACT :**

Light steel adding storey frame is compounded structure, which is composed with two kinds of damping materials (reinforced concrete and steel). The structural dynamic equation can't be decoupled in real number field because of the non-proportional damping matrix; The traditional real mode superposition method can't be used on account of the non-proportional damping. The damping ratio of the reinforced concrete and the steel are all little damping, so, on the basis of theoretical analysis, this paper presents the forcing mode superposition method and writes a program suitable for engineering convenience. The accuracy and feasibility of the proposed methods is later verified through case studies on two sample structures.

**KEYWORDS:** Light steel adding storey frame, forcing mode superposition method, non-proportional damping

## 1. INTRODUCTION

Light steel adding storey frame is compounded structure composed with concrete and steel, which is composed with two different kinds of damping materials. The structural dynamic equation can't be decoupled in real number field because of the non-proportional damping matrix, so, the traditional real modes superposition method can't be used. Han Jianping[1] have resolved the non-proportional damped model of the base isolating vibration structures by partition Rayleigh damping model; Gui Guoqing[2][3] have made a research on how to solve the non-proportional damping problem by the complex-mode method and the approximate decoupling method and established an improved matrix perturbation method to solve the complex mode eigenvalue of the non-proportional damping structural systems; Zhou Xiyuan[4] have studied the analytical method on seismic response of the non-proportional damping structure under different earthquakes; Du Yongfeng[5] have studied the engineering algorithm of the dynamic responses of structure with non-proportional damping isolated systems with arbitrary multiply degree of freedom in time domain by the combined application with the real mode superposition method and the Laplace transform method. This paper solves the seismic responses of the light steel adding storey frame by the forcing mode superposition method.

## 2. THE FORCING MODE SUPERPOSITION METHOD OF THE LIGHT STEEL ADDING STOREY FRAME

### *2.1 The Basic Principle of The Forcing Mode Superposition Method*

The motion equation of the structure with n degrees of freedom under different earthquakes is:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=-[M]\{1\}\ddot{x}_g \tag{1}$$

Where $[M]$、$[K]$ and $[C]$ are, respectively, the $n\times n$ mass, stiffness and damping matrix. $\{\ddot{x}\}$、$\{\dot{x}\}$ and $\{x\}$ are, respectively, the $n\times 1$ column vector of displacement、velocity and acceleration. $\{1\}$ is a column vector with all its elements equal to 1.The non-damping natural vibration frequency and all modes of vibration can be obtained on the basis of the mass matrix $[M]$ and the stiffness matrix $[K]$:

$$\varpi=\{\varpi_1,\varpi_2,\ldots\omega_n\}\,,\quad [\Phi]=\{\{\phi\}_1,\{\phi\}_2,\ldots\{\phi\}_n\}$$

Where $[\Phi]$ which is composed with every orders of mode vibration is regard as the modal matrix. From the transformation

$$\{x\}=[\Phi]\{q\} \tag{2}$$

Where $q$ is generalized coordinates. Through left multiplication of modal matrix $[\Phi]^T$ on both sides, Eq. (2) can be written as:

$$[\Phi]^T[M][\Phi]\{\ddot q\}+[\Phi]^T[C][\Phi]\{\dot q\}+[\Phi]^T[K][\Phi]\{q\}=-[\Phi]^T[M]\{1\}\ddot x_g \tag{3}$$

On account of:

$$\{\phi\}_i[M]\{\phi\}_j=\begin{cases}\overline{m}_i(i=j)\\0(i\neq j)\end{cases}\qquad \{\phi\}_i[K]\{\phi\}_j=\begin{cases}\overline{k}_i(i=j)\\0(i\neq j)\end{cases}$$

Eq.(3) can be written as:

$$[\overline{M}]\{\ddot q\}+[\overline{C}]\{\dot q\}+[\overline{K}]\{q\}=-[\Phi]^T[M]\{1\}\ddot x_g \tag{4}$$

Where $[\overline{M}]$、$[\overline{K}]$ and $[\overline{C}]$ are , respectively, the modal mass matrix, modal stiffness matrix and modal damping matrix, and

$$[\overline{M}]=\begin{bmatrix}\overline{m}_1&0&\cdots&0\\0&\overline{m}_2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\overline{m}_n\end{bmatrix}\qquad [\overline{K}]=\begin{bmatrix}\overline{k}_1&0&\cdots&0\\0&\overline{k}_2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\overline{k}_n\end{bmatrix}$$

The light steel adding storey frame is composed with two kinds of materials. The modal damping matrix $[\overline{C}]$ is not orthogonal matrix.because of its off-diagonal element. The dynamic equation can not be decoupled on account of the non-proposition damping.

The basic principle of the forcing mode superposition method is to neglect the off-diagonal element of the modal damping matrix because of its little material damping. Then, one can suppose that:

$$\{\phi\}_i[C]\{\phi\}_j=\begin{cases}\overline{c}_i(i=j)\\0(i\neq j)\end{cases}, \text{ so,}$$

$$[\overline{C}]=\begin{bmatrix}\overline{c}_1&0&\cdots&0\\0&\overline{c}_2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\overline{c}_n\end{bmatrix},\quad \begin{cases}\overline{m}_j=\{\phi\}_j^T[M]\{\phi\}_j=\overline{m}_j\\ \overline{k}_j=\{\phi\}_j^T[K]\{\phi\}_j=\varpi_j^2\overline{m}\\ \overline{c}_j=\{\phi\}_j^T[C]\{\phi\}_j=2\overline{\xi}\varpi_j\overline{m}\end{cases}\quad \text{(j=1 ,2 , …n) .}$$

Where $\overline{\xi}_j$ is the jth mode damping ratio, which is not the concrete damping ratio and is also not the steel damping ratio ,but the commuted damping ratio of the concrete and the steel.

Eq.(4) can be simplified as $n$ SDOF dynamic equations independent of each other in the generalized coordinate after approximate treatment. And the jth generalized coordinate motion equation is:

$$\overline{m}_j \ddot{q}_j + 2\overline{\xi}_j \varpi_j \overline{m}_j \dot{q}_j + \overline{k}_j q_j = -\{\phi\}_j^T [M]\{1\}\ddot{x}_g \tag{5}$$

From the transformation $\gamma_j = \frac{\{\phi\}_j^T [M]\{1\}}{\overline{m}_j}$, where $\gamma_j$ is the jth mode related coefficient , Eq.(5) can be written as

$$\ddot{q}_j + 2\overline{\xi}_j \varpi_j \dot{q}_j + \varpi_j^2 q_j = -\gamma_j \ddot{x}_g \tag{6}$$

$q_j(t)$ can be written as:

$$q_j(t) = -\frac{\gamma_j}{\varpi'_j}\int_0^x \ddot{x}_g(\tau) e^{-\overline{\xi} j \varpi j (t-\tau)} \sin \varpi'_j (t-\tau) d\tau = \gamma_j \delta_j(t) \tag{7}$$

Where $\varpi'_j = \varpi_j \sqrt{1-\overline{\xi}_j^2}$ , $\delta_j(t) = -\frac{\gamma_j}{\varpi'_j}\int_0^x \ddot{x}_g(\tau) e^{-\overline{\xi} j \varpi j (t-\tau)} \sin \varpi'_j (t-\tau) d\tau$ . $\varpi'_j$ is the jth natural vibration frequency of the structure. Eq.(2) from Eq.(7) can be written as:

$$\{x(t)\} = \sum_{j=1}^{n} \{\phi\}_j q_j(t) = \sum_{j=1}^{n} \{\phi\}_j \gamma_j \delta_j(t) \tag{8}$$

### *2.2 Forcing Mode Superposition Method*

For a MDOF system, the seismic response of the ith particle of the jth mode in real modal theory can be obtained by the following formula: $F_{ji} = \alpha_j \gamma_j \phi_{ji} G_i$ . where $F_{ji}$ is the standard value of the horizontal seismic response of the ith particle of the jth mode ; $\alpha_j$ is coefficient of seismic effect; $\gamma_j$ is the jth mode related coefficient; $\phi_{ji}$ is the horizontal relative displacement of the ith particle of the jth mode; $G_i$ is the gravity standard value of the ith particle.

The effect of the horizontal seismic response can be written as: $S_{EK} = \sqrt{\sum S_j^2}$ . Where $S_{EK}$ is the standard value of the effect of the horizontal seismic response; $S_j$ is the standard value of the effect of the horizontal seismic response of the jth mode.

## 3. ALGORITHM FEATURES

The basic principle of the forcing mode superposition method is to neglect the non-diagonal elements of the modal damping matrix and to retain only the main diagonal elements, which make the dynamic equation decoupled in real region. the essence of this method is to use the mode damping ratio in the calculation of seismic response. This method is a proximate method because of the neglect of the non-diagonal elements. The precision can be got through the analysis of the examples below.

## 4. EXAMPLES

### *4.1Algorithm Selection*

As we all know, Newmark method can directly make a numerical solution to Eq. (1) without thinking the damping matrix is a orthogonal matrix or not. This paper proposed a Newmark procedure to solving the exact solution of the dynamic equation of the seismic response with MATLAB language. The forcing mode superposition method is an approximate solution. One can use the time-history method based on vibration mode superposition to calculate the approximate solution. However, Newmark method and modal time-history superposition is two different algorithm. This paper established a proportional damping matrix on the basis of all mode damping ratio to reduce the error between two different algorithm and the approximate solution were also obtained by Newmark method.

### *4.2Calculated Process*

#### *4.2.1 Calculation of the initial value*

(1) Make sure the value of the matrix $[M]$、$[K]$、$[C]$ and the vector of $\{x\}_0$、$\{\dot{x}\}_0$、$\{\ddot{x}\}_0$ ,and determine time step $\Delta t$ and algorithm adjustment parameter $\gamma$ , $\beta$ ;

(2) Calculation of the coefficient as follows: $a_0=\dfrac{1}{\beta\Delta t^2}$ , $a_1=\dfrac{\gamma}{\beta\Delta t^2}$ , $a_2=\dfrac{1}{\beta\Delta t}$ , $a_3=\dfrac{1}{2\beta}-1$ , $a_4=\dfrac{\gamma}{\beta}-1$ , $a_5=\dfrac{\Delta t}{2}\left(\dfrac{\gamma}{\beta}-2\right)$ , $a_6=\Delta t(1-\gamma)$ , $a_7=\gamma\Delta t$ ;

(3) Establish the equivalent stiffness matrix: $[\overline{K}]=[K]+a_0[M]+a_1[C]$.

#### *4.2.2 For each time step*

(1) Calculation of the equivalent load:

$$\{\overline{Q}\}_{T+\Delta T}=\{Q\}_{T+\Delta T}+[M](a_0\{x\}_t+a_2\{\ddot{x}\}_t)+[C](a_1\{x\}_t+a_4\{\dot{x}\}_t)+a_5\{\ddot{x}\}_t$$

(2) Solution of the displacement vector: $[\overline{K}]\{x\}_{t+\Delta t}=\{\overline{Q}\}_{t+\Delta t}$ ;

(3) Solution of the velocity and acceleration vector:

$$\{\dot{x}\}_{t+\Delta t}=\{\dot{x}\}_t+a_6\{\ddot{x}\}_t+a_7\{\ddot{x}\}_{t+\Delta t},\quad \{\ddot{x}\}_{t+\Delta t}=a_0(\{x\}_{t+\Delta t}-\{x\}_t)-a_2\{\dot{x}\}_t-a_3\{\ddot{x}\}_t$$

### *4.3Examples*

Three-storey reinforced concrete frame structure added with two-storey light steel structure .The model parameters of concrete are: $\xi_1=5\%$ , $m_1=m_2=m_3=2\times10^4(kg)$, and the steel model parameters are: $\xi_2=2\%$ , $m_1=m_2=0.7\times10^4(kg)$ , $k_1=k_2=0.7\times10^7(N/m)$. The seismic wave is EL-Centro wave. The maximum of the interlaminal displacement and interlaminal shear force using the two methods above are listed in table 1.

Table 1 The maximum of the interlaminal displacement、interlaminal shearing force and error

| | | Layer 1 | Layer 2 | Layer 3 | Layer 4 | Layer 5 |
|---|---|---|---|---|---|---|
| Displacement(mm) | Exact values | 4.35 | 8.03 | 10.61 | 13.22 | 13.05 |
| | Calculated values | 4.35 | 8.03 | 10.61 | 13.23 | 13.05 |
| | Errors (%) | 0.04 | 0.05 | 0.04 | 0.04 | 0.03 |
| Interlaminal shear (kN) | Exact values | 65.25 | 55.16 | 38.62 | 18.30 | 9.70 |
| | Calculated values | 65.29 | 55.19 | 38.61 | 18.30 | 9.70 |
| | Errors (%) | 0.07 | 0.06 | 0.02 | 0.01 | 0.06 |

This method is proved to be feasible by the calculation of several other examples"9+1"、"27+3". One can also draw the conclusion that the precision of this method is very high.

## 5. CONCLUSION

The basic principle of the forcing mode superposition method is to neglect the non-diagonal elements of the modal damping matrix and to retain only the main diagonal elements. The damping matrix of the light steel adding storey frame is non-proportional damping matrix, but, the non-proportional character is relatively weak on account of the little damping of the concrete and the steel. In other words, the non-diagonal elements of the modal matrix multiplied by mode matrix on both sides was little than the corresponding main diagonal elements. The method of neglecting non-diagonal element is reasonable. The calculation results have also proved that the errors of displacement、velocity and acceleration are all very small. The forcing mode superposition method has a high degree of accuracy for the seismic response analysis of the light steel adding storey frame, which is feasible for the calculation of the seismic response.

## REFERENCES

[1] Han Jianping, Du Yongfeng, Li Hui.(2002) Analysis and design of base isolation structure based on non-proportional damping. *Engineering Mechanics (Supplement)* 353-356.

[2] Gui Guoqing, He Yuao.(1994) Study on dynamic analysis methods for non-proportionally damped structural systems [J]. *Journal of Tongji University* **22:4**,505-510.

[3] Gui Guoqing, He Yuao.(1996). Matrix perturbation methods for solving the complex modal problem of non-proportionally damped structures. *Journal of Tongji University* **24:6**,613-618.

[4] Zhou Xiyuan, Dong Di, Su Youpo.(2003). New method for linear systems with non-classical damping under ground motion .*Journal of Civil Engineering* **36:6**,30-36.

[5] Du Yongfeng, Li Hui, Su Panshi, Zhao Guofan.(2003). Real mode superposition method for analysis of Seismic response of non-proportionally damped isolated structures. *Engineering Mechanics* **20:4**,24-32.