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CRITERIA OF BUCKLING-RESTRAINED BRACES TO PREVENT OUT-OF-PLANE BUCKLING

Yuji KOETAKA [1], Tomohiro KINOSHITA [2], Kazuo INOUE [3] and Kunihiro IITANI [4]

[1] *Dept. of Architecture, Osaka Institute of Technology, Dr. Eng., Osaka, Japan*
[2] *Staff, Construction Materials & Services Center, JFE Steel Corporation, M. Eng., Tokyo, Japan*
[3] *Professor, Dept. of Architecture and Architectural Engineering, Kyoto University, Dr. Eng., Kyoto, Japan*
[4] *Staff Deputy Manager, Civil & Construction Sec., WEST JAPAN WORKS, JFE Steel Corporation, M. Eng., Kurashiki, Japan*
Email: koetaka@archi.oit.ac.jp

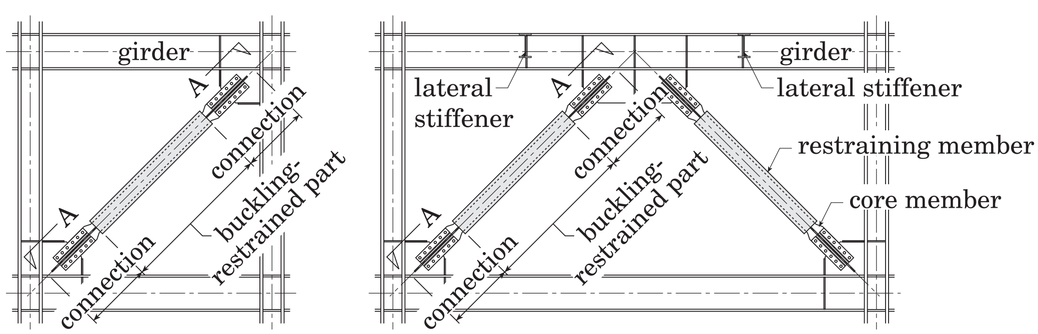
ABSTRACT :
Buckling-restrained brace (BRB), which satisfies criteria of preventing flexural buckling of restraining member, can form out-of-plane buckling mode if stiffness of either connections or a girder is small. In order to prevent out-of-plane buckling, the buckling load must be larger than the maximum axial force of the BRB. This paper addresses the required stiffness of both connections and the girder to prevent out-of-plane buckling. Based on observation of loading tests, buckling-restrained braces formed the buckling mode as torsional stiffness of the girder became less than the required stiffness. As a result, the required stiffness (Eq. (2.16)) is useful for design criteria of BRBs to prevent out-of-plane buckling.

KEYWORDS : Buckling-restrained brace, Out-of-plane buckling, Buckling load, Connection, Loading test

1. INTRODUCTION

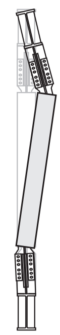
Buckling-restrained braces (BRBs) have been developed in Japan from 1970's, and have been used for seismic devices of low-rise buildings and hysteresis dampers of high rise buildings [Inoue, et al. (2001)]. In 2005, design of buckling-restrained braced frame (BRBF) is adopted in seismic provisions [AISC (2005)]. However, there are few researches about structural behavior or design criteria of BRB including its connections.

BRB consists of both buckling-restrained part, in which a core member is covered with a restraining member, and connections, which are connected to gusset plates by means of high-strength bolts, as shown in Fig. 1. BRB can form out-of-plane buckling mode (see Fig. 2) if either bending stiffness of connections or torsional stiffness of a girder where BRB is connected is small, even though it satisfies criteria of preventing flexural buckling of the restraining member [Takeuchi, et al. (2004), Tembata, et al. (2004), and Lin et al. (2006)]. When out-of-plane buckling occurs, not only connections and the girder are damaged instead of BRB's core member but also energy dissipation capacity degrades because the resisting force of BRB decreases. Meanwhile, in-plane buckling need not consider in design of BRB because in-plane stiffness of the gusset-plate, as shown in Fig. 1, is larger than that of out-of-plane stiffness. In order to prevent out-of-plane buckling, the buckling load must be larger than the maximum axial force of BRB. In this paper, design criteria which is given by the required



(a) Single-diagonal bracing (b) Inverted-V bracing (including chevron bracing)

Figure 1 Setup of BRBs



(A-A section in Fig. 1)

Figure 2 Out-of-plane buckling

stiffness of both connections and the girder to prevent out-of-plane buckling are described, based on theoretical research, and an experimental verification of BRBFs is reported to confirm the validity of the design criteria.
There are two kinds of setup of BRBs which are adopted in a moment resisting frame. One is called single-diagonal bracing (see Fig. 1 (a)) and another is called inverted-V bracing, which is including chevron bracing (see Fig. 1 (b)). The proposed design criteria in this paper are considered with these setup types, and lateral-torsional displacement of the girder is considered if lateral stiffeners of the girder are not set at center of the girder in case of inverted-V bracing or chevron bracing. Here, effects of floor slab to restrain lateral-torsional displacement of the girder [Usami, et al. (2005)] are not considered in order to underestimate the buckling load of the BRB. And if the ends of the core member are expanded and keep elastic, rotation at boundary between the restraining part and the connection is restrained. In this paper, however, in order to underestimate the buckling load as well as the effect of floor slab, it is considered that the ends of the core member agree with the ends of the restraining member. And additionally, strength of connections is large enough to keep elastic.

2. DESIGN CRITERIA TO PREVENT OUT-OF-PLANE BUCKLING

2.1 Analysis model

For the purpose of deriving out-of-plane buckling load, it is assumed that an analysis model consists of both the buckling-restrained part and connection parts, as shown in Fig. 3. Here, ξ means the ratio of the length of connection to the total length of BRB, and ξ of both connections are equal. The bending stiffness of connection is adopted lower value in both connections in order to underestimate the buckling load, and denotes by $r_J E I_B$ (where, r_J means the ratio of bending stiffness of the connection to that of the restraining member). In order to consider lateral-torsional displacement of the girder in case of inverted-V bracing, not only a lateral spring but also a rotational spring are arranged at the center of the girder, and these springs and upper connection are jointed by using rigid body whose length is d^* (=$0.5D/\sin\phi$; D is depth of the girder, ϕ is angle of between axis of the BRB and axis of the girder). As discussed below, inverted-V bracing is considered as the analysis model, however the analysis model can be applied to chevron bracing.
It is considered that the relationship between stress and strain of the core member is given by the perfect elasto-plastic model and the stress point is the singular point of *N-M* interaction curve. Rotation of the core member at the both ends can increase under bending moment keeps zero because the neutral axis of the core member is out of the section. Consequently, the both ends of the core member can be assumed pins, at which bending stiffness is zero, as shown in Fig. 3 (b).

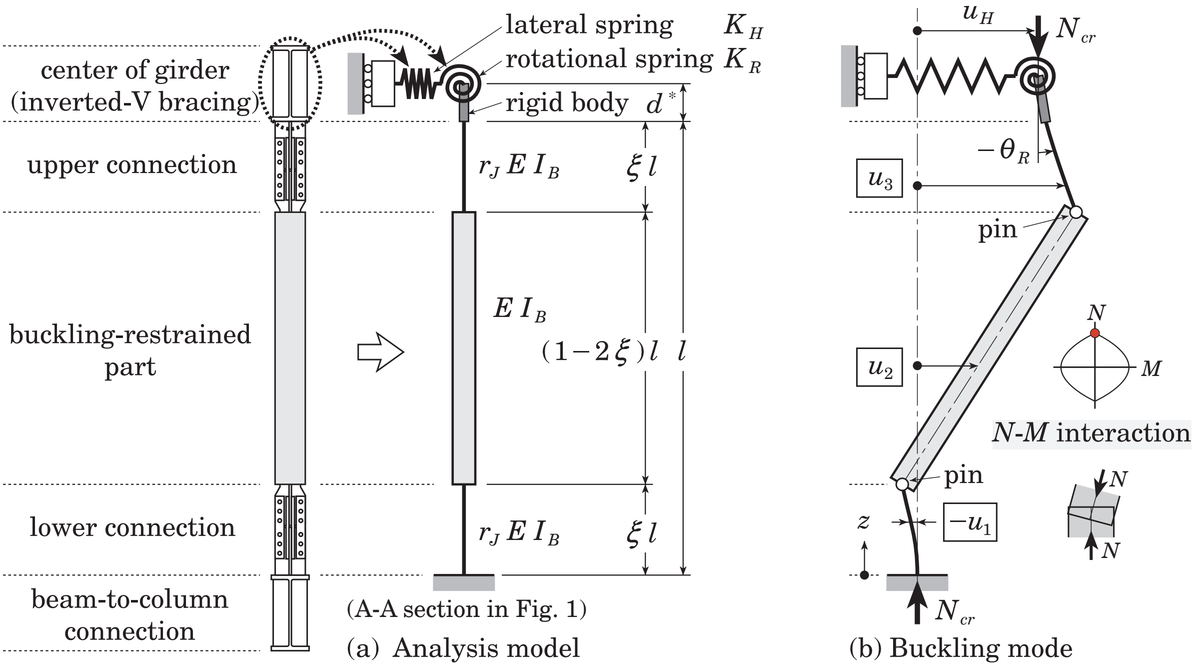


Figure 3 Analysis model

2.2 Out-of-plane buckling load

In general, an equilibrium condition at any section of a central compression member with under any boundary condition is given by Eq. (2.1).

$$E\,I\,\frac{d^4u}{dz^4}+N\,\frac{d^2u}{dz^2}=0 \tag{2.1}$$

Here, EI means bending stiffness of the central compression member. The displacement function of each part of the analysis model of BRB (see Fig. 3) are obtained from Eq. (2.1).

$$u_1={}_1C_1\sin\frac{\alpha_J\,z}{l}+{}_2C_1\cos\frac{\alpha_J\,z}{l}+{}_3C_1\,z+{}_4C_1 \tag{2.2.a}$$

$$u_2={}_1C_2\sin\frac{\sqrt{r_J}\,\alpha_J z}{l}+{}_2C_2\cos\frac{\sqrt{r_J}\,\alpha_J z}{l}+{}_3C_2\,z+{}_4C_2 \tag{2.2.b}$$

$$u_3={}_1C_3\sin\frac{\alpha_J\,z}{l}+{}_2C_3\cos\frac{\alpha_J\,z}{l}+{}_3C_3\,z+{}_4C_3 \tag{2.2.c}$$

Where, ${}_iC_j$ ($i=1,4$ and $j=1,3$) are the integral constants, and α_J in Eq. (2.2a), (2.2b), (2.2c) is given by the following equation.

$$\alpha_J=l\sqrt{\frac{N_{cr}}{r_J\,E\,I_B}}=\frac{\pi}{2\,\xi}\sqrt{\frac{N_{cr}}{N_E^J}} \tag{2.3}$$

N_E^J means Eular load of the connection if the buckling length is $2\xi l$, and N_E^J is given by Eq. (2.4).

$$N_E^J=\frac{\pi^2\,r_J\,E\,I_B}{(2\,\xi\,l)^2} \tag{2.4}$$

Referring to Fig. 3, boundary conditions of the analysis model are given by the following equations, which are represented from Eq. (2.5.a) to Eq. (2.5.n).

$z=0$:

$$u_1=0\,,\quad \frac{du_1}{dz}=0 \tag{2.5.a), (2.5.b}$$

$z=\xi\,l$:

$$u_1=u_2\,,\quad r_J\,E\,I_B\frac{d^2u_1}{dz^2}=0\,,\quad E\,I_B\frac{d^2u_2}{dz^2}=0\,,\quad r_J\,E\,I_B\frac{d^3u_1}{dz^3}+N_{cr}\left(\frac{du_1}{dz}-\frac{du_2}{dz}\right)=0$$

(2.5.c), (2.5.d), (2.5.e), (2.5.f)

$z=(1-\xi)\,l$:

$$u_2=u_3\,,\quad E\,I_B\frac{d^2u_2}{dz^2}=0\,,\quad r_J\,E\,I_B\frac{d^2u_3}{dz^2}=0\,,\quad r_J\,E\,I_B\frac{d^3u_3}{dz^3}+N_{cr}\left(\frac{du_3}{dz}-\frac{du_2}{dz}\right)=0$$

(2.5.g), (2.5.h), (2.5.i), (2.5.j)

$z=l$:

$$u_3=u_H-d^*\theta_R\,,\quad \frac{du_3}{dz}=\theta_R \tag{2.5.k), (2.5.l}$$

$$r_J\,E\,I_B\frac{d^3u_3}{dz^3}+N_{cr}\frac{du_3}{dz}=K_H\,u_H \tag{2.5.m}$$

$$r_J\,E\,I_B\frac{d^2u_3}{dz^2}+d^*\,r_J\,E\,I_B\frac{d^3u_3}{dz^3}=-K_R\,\theta_R \tag{2.5.n}$$

Where, K_H is stiffness of the lateral spring, K_R is stiffness of the rotational spring. Substituted Eq. (2.5) to Eq. (2.2), the integral constants, u_H, and θ_R are eliminated, and out-of-plane buckling loads are derived as following equations.

$$\left(\frac{K_H}{A_1}-1\right)\left(\frac{K_R}{A_2}-1\right) - A_3 = 0 \tag{2.6.a}$$

$$\cos\xi\alpha_J - \frac{2}{\alpha_J}\sin\xi\alpha_J = 0 \tag{2.6.b}$$

$$\cos\xi\alpha_J = 0 \tag{2.6.c}$$

Where, A1, A2, and A3 in Eq. (2.6.a) are given by next equations.

$$A_1 = \frac{\cos\xi\alpha_J}{\cos\xi\alpha_J - 2/\alpha_J \sin\xi\alpha_J}\cdot\frac{N_{cr}}{l} \tag{2.7.a}$$

$$A_2 = \frac{\{(l+d^*)\cos\xi\alpha_J - l/\alpha_J\sin\xi\alpha_J\}(d^*\cos\xi\alpha_J + l/\alpha_J\sin\xi\alpha_J)N_{cr}}{l\cos\xi\alpha_J(\cos\xi\alpha_J - 2/\alpha_J\sin\xi\alpha_J)} \tag{2.7.b}$$

$$A_3 = \frac{d^*\cos\xi\alpha_J + l/\alpha_J\sin\xi\alpha_J}{(l+d^*)\cos\xi\alpha_J - l/\alpha_J\sin\xi\alpha_J} \tag{2.7.c}$$

The minimum buckling load obtained from Eq. (2.6.a) corresponds with the buckling mode into which both the lateral spring and the rotational spring deform as illustrated in Fig. 4 (a). On the other hand, the minimum buckling loads obtained from Eq. (2.6.b) and Eq. (2.6.c) correspond with the buckling modes into which the lateral spring and the rotational spring do not deform and only connections deform as illustrated in Fig. 4 (b) and Fig. 4 (c). These bucking modes can be observed when the BRB is set in single-diagonal bracing or when lateral-torsional deformation of the center of the girder is restrained by lateral stiffeners in case of inverted-V bracing or chevron bracing. From Eq. (2.6.b) and Eq. (2.6.c), it is clear that N_{cr}^b is smaller than N_{cr}^c.

2.3 Design criteria

In order to prevent out-of-plane buckling of BRB, the buckling load which is the minimum value of N_{cr}^a and N_{cr}^b must be larger than the maximum axial force of BRB $N_{\max}$. Firstly, the design criterion to prevent out-of-plane buckling, as shown in Fig. 4 (b), is derived by means of approximating $\pi^2/8$ to 1, the first of Taylor series of the sine function, and the second of Taylor series of the cosine function.

$$N_{cr}^b = (1-2\xi)N_E^J > N_{\max} \tag{2.8}$$

Eq. (2.8) must be satisfied in case of both single-diagonal bracing and inverted-V bracing. And in case of inverted-V bracing, it is need to check of prevention of out-of-plane buckling as shown in Fig. 4 (a). The solid line in Fig 5 represents the relationship between K_H and K_R under $N_{cr}^a = N_{\max}$ in Eq. (2.6.a). Stiffness of both springs must be larger than those of the solid line in order to satisfy Eq. (2.6.a). In other words, both K_H and K_R should be laid in gray area (called safety area) in Fig. 5, and this criterion is represented as Eq. (2.9).

$$\left(\frac{K_H}{K_H^{req}}-1\right)\left(\frac{K_R}{K_R^{req}}-1\right) > \frac{d^*+a_N\xi l}{l+d^*-a_N\xi l} \tag{2.9}$$

Where, a_N in Eq. (2.9) means an amplitude factor and is given by Eq. (2.10).

$$a_N = \frac{1}{1-N_{\max}/N_E^J} \tag{2.10}$$

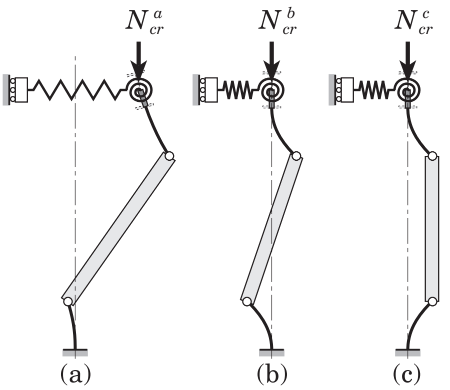


Figure 4 Out-of-plane buckling mode

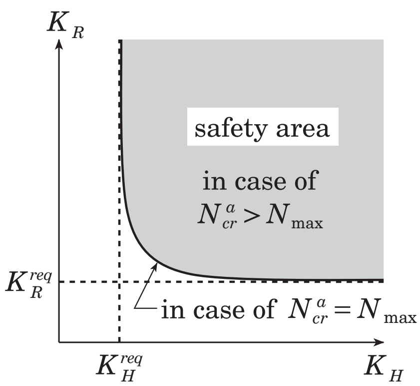


Figure 5 Required stiffness

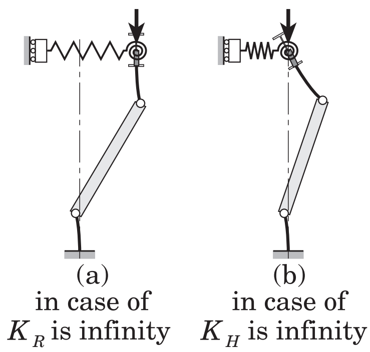


Figure 6 Buckling mode

And K_H^{req} in Eq. (2.9) means the required stiffness of the lateral spring if the rotational spring is rigid, as shown in Fig. 6 (a), and is given by Eq. (2.11.a). On the other hand, K_R^{req} in Eq. (2.9) means the required stiffness of the rotational spring if the lateral spring is rigid, as shown in Fig. 6 (b), and is given by Eq. (2.11.b).

$$K_H^{req} = \frac{N_{\max}}{(1-2a_N\xi)\,l} \tag{2.11.a}$$

$$K_R^{req} = \{1+\frac{d^*+a_N\xi l}{(1-2a_N\xi)\,l}\}(d^*+a_N\xi l)N_{\max} \tag{2.11.b}$$

In order to check Eq. (2.9), it is necessary to calculate stiffness of the lateral spring and the rotational spring. The lateral spring represents lateral displacement of the center of the girder, and is equal to the bending stiffness of the girder which bending moment acts on around the minor axis. And the rotational spring represents rotation of the center of the girder. Here, the angle of BRB is ϕ, and K_R is obtained from Eq. (2.12).

$$K_R = K_\psi / \sin^2\phi \tag{2.12}$$

Where, K_ψ means torsional stiffness of the girder and is given by the relationship between torsional moment and rotation angle at the center of the girder. If boundary conditions of the girder is assumed as shown in Fig. 7 (b), K_H and K_R can be obtained from next equations.

$$K_H = \frac{192(l_s+2\bar{l}_s)EI_Y}{(4l_s-\bar{l}_s)\bar{l}_s^3} \tag{2.13.a}$$

$$K_R = \frac{4GJ\alpha_t}{l_s(\alpha_t-\tanh\alpha_t-B_1^2/B_2)\sin^2\phi} \tag{2.13.b}$$

Where, EI_Y is minor axis rigidity, GJ is torsional rigidity, and EI_W is warping rigidity. And besides, α_t, B_1 and B_2 are given by next equations.

$$\alpha_t = \frac{l_s}{2}\sqrt{\frac{GJ}{EI_W}} \tag{2.14}$$

$$B_1 = \frac{l_s-\bar{l}_s}{l_s}\alpha_t - \frac{\sinh\frac{l_s-\bar{l}_s}{l_s}\alpha_t}{\cosh\alpha_t} \tag{2.15.a}$$

$$B_2 = \frac{l_s-\bar{l}_s}{l_s}\alpha_t - \frac{\cosh\frac{\bar{l}_s}{l_s}\alpha_t\cdot\sinh\frac{l_s-\bar{l}_s}{l_s}\alpha_t}{\cosh\alpha_t} \tag{2.15.b}$$

Fig. 8 shows the minimum value of stiffness of both the lateral spring and the rotational spring that just satisfy Eq. (2.9). Here, the length of the girder l_s is 8.0 meters, and the angle of BRB ϕ is $\pi/4$. Each plot in Fig. 8 can be obtained by varying both section of girder and ξ (ratio of the length of connection to the total length of BRB). From Fig. 8, in case of practical girders for BRBFs, it is considered that the lateral spring is rigid because stiffness of the lateral spring K_H is quite larger than the required stiffness of the lateral spring K_H^{req}.

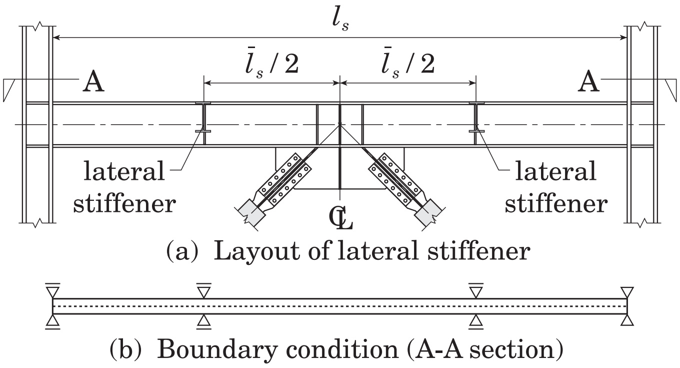


Figure 7 Boundary condition of girder

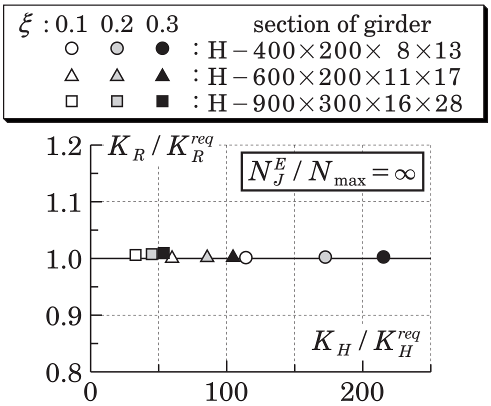


Figure 8 Relationship between K_H and K_R

Consequently, it is considered with K_H in Eq. (2.9) is infinity, and the design criterion of BRB to prevent out-of-plane buckling is given by Eq. (2.16).

$$K_R > K_R^{req} \tag{2.16}$$

3. LOADING TEST

3.1 Test specimen and setup

Loading test of one-story one-bay plane-frame specimens with BRBs which were installed in chevron bracing, as shown in Fig. 9, were conducted to verify the required stiffness of the rotational spring. Both columns and upper girder are wide-flange whose section is 'H−250×250×9×14,' and the lower girder is also wide-flange whose section is either 'H−300×150×6.5×9' or 'H−300×100×6.5×9.'

Fig. 10 shows detail of the BRB using specimen. The core member of the BRB is circular tube of LY225 and the restraining member is circular tube whose diameter is larger than the core member in order to ensure the clearance between inside of the restraining member and outside of the core member. Connections whose section is cruciform (width is 200 mm and thickness is 19 mm) were connected to the gusset plates by using high strength bolts. Ratio of the length of connection to the total length of BRB ξ is 0.244, as referring to Fig. 9. The mechanical properties of steel using specimen are shown in Table 1.

As shown in Fig. 9, BRBF specimen was supported by pin joints and was cyclically loaded through a horizontal force applied at the center of the upper girder. Firstly, lateral stiffeners of the girder were located at the center of the lower girder, and loading test was conducted within the range that the core member yielded and the frame kept elastic, herein the amplitude of story drift angle R was selected from 0.0067 to 0.01 radian. And next, the loading was repeated until out-of-plane deformation of the BRB became large with stiffened section of the girder (see triangle marks in Fig. 9) having been gradually separated from the center of the girder. Here, $\bar{l}_s$ in

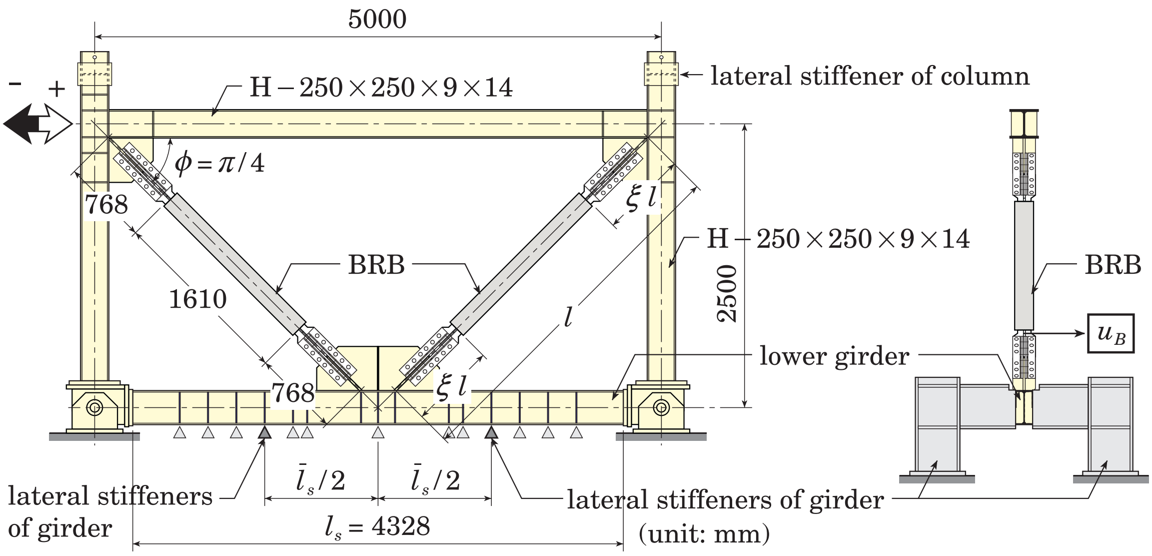


Figure 9 Test setup

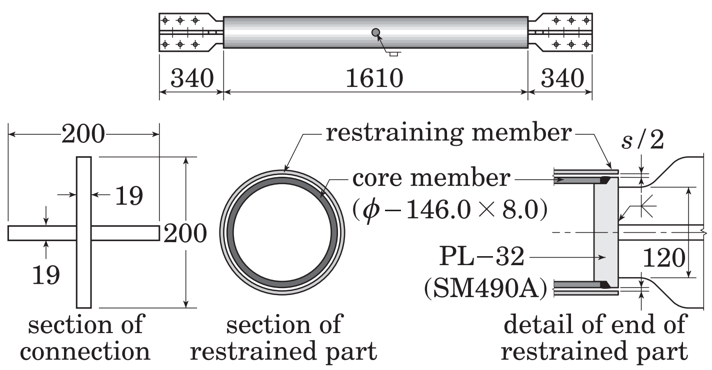


Figure 10 Detail of buckling-restrained brace

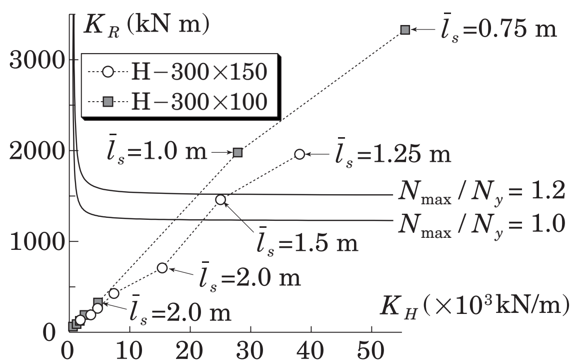


Figure 11 Relationship between K_H and K_R

Table 1 Mechanical properties of steel using specimen

Use part of specimen	Steel grade	Thickness (mm)	Yield strength (N/mm^2)	Tensile strength (N/mm^2)	Elongation (%)
Core member	LY225	7.77	207	305	55
Connection	SM490A	18.2	361	522	28
Gusset plate	SM490A	18.6	355	527	26
Splice plate	SM490A	15.6	371	550	26
Flange (Lower Girder)	SM490A	8.48	362	509	24
Flange (Column)	SM490A	13.7	360	527	26

Fig. 9 is the distance of lateral stiffeners.

Stiffness of both the lateral spring and the rotational spring were investigated by loading to the lateral direction at the top flange of the center of the lower girder when BRBs did not set in frame before loading test. Fig. 11 shows relationship between stiffness of the lateral spring K_H and stiffness of the rotational spring K_R of each lower girder. Solid lines in this figure mean the required stiffness given by Eq. (2.9), and it is predicted that out-of-plane buckling does not occur if plots are laid within the upper right area of solid lines.

3.2 Test results

Figs. 12, 13, and 14 show test results of left BRB in Fig. 9 in case of the distance of lateral stiffeners $\bar{l}_s$ varied 1.25 meters, 1.5 meters, and 2.0 meters respectively, when the section of lower girder is 'H−300×150×6.5×9'. Fig. (a) shows relationship between axial force N normalized by yield axial force N_y (= 699 kN) and strain of the core member ε. Here, strain of the core member is obtained by dividing axial deformation of the core member by the length of the core member. Fig. (b) shows relationship between normalized axial force N/N_y and out-of-plane deformation u_B at the boundary between the restrained part and the lower connection, as illustrated in Fig. 9. And Fig. (c) shows relationship between strain of the core member ε and out-of-plane deformation u_B. In each figure, the value of K_R/K_R^{req} which corresponds with the distance of lateral stiffeners $\bar{l}_s$ is represented, and K_R^{req} is obtained by substituting the maximum axial force to Eq. (2.11.b).

From Fig. 12 and Fig. 13, when K_R is larger than K_R^{req}, it is confirmed that out-of-plane deformation u_B increases slightly after the core member yields, then the increment of u_B becomes small step by step. On the

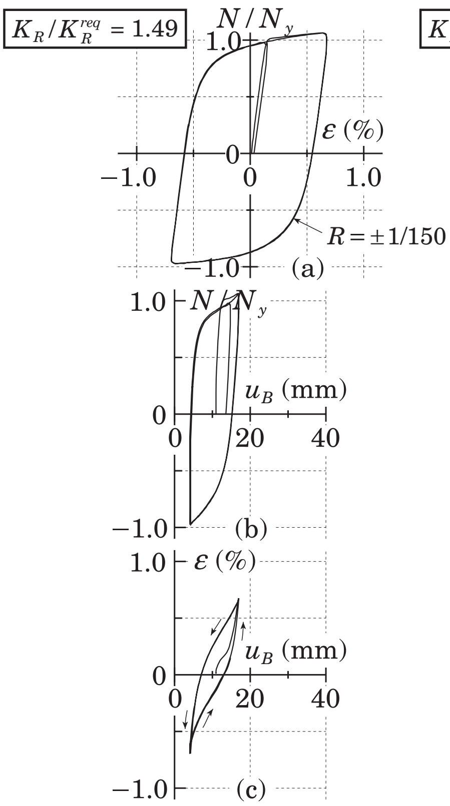


Figure 12 Test results ($\bar{l}_s$=1.25m)

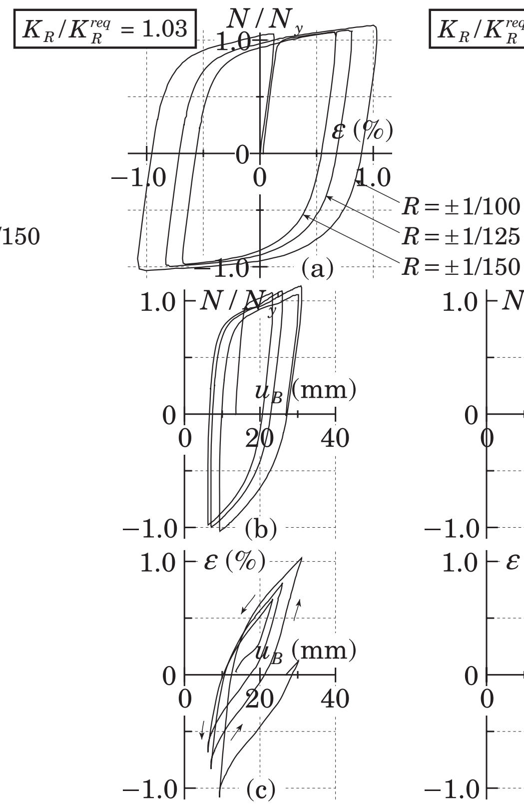


Figure 13 Test results ($\bar{l}_s$=1.5m)

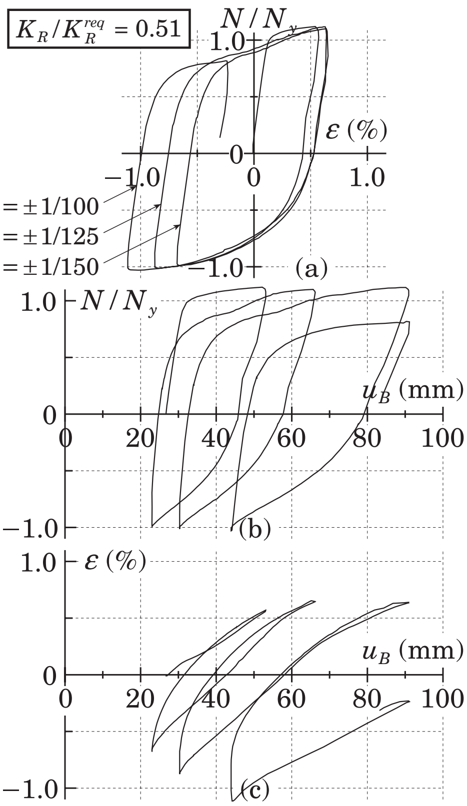


Figure 14 Test results ($\bar{l}_s$=2.0m)

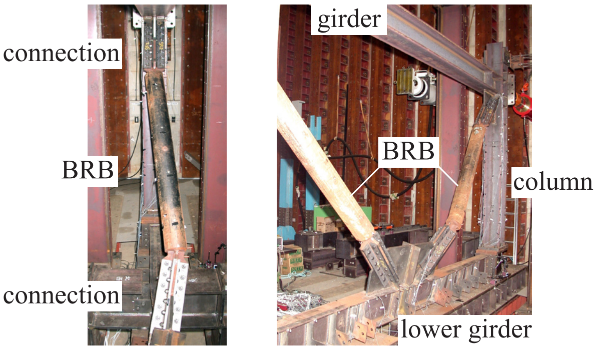


Photo 1 Specimen after occurring out-of-plane buckling

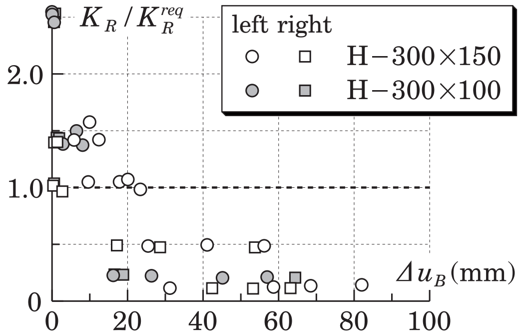


Figure 15 Increment of out-of-plane deformation

other hand, when K_R is smaller than K_R^{req} (referring to Fig. 14), out-of-plane deformation u_B increases rapidly after the core member yields. In the last, out-of-plane deformation of the BRB and torsional deformation of the lower girder become marked as shown in Photo 1. And distribution of out-of-plane deformation is identified with the predicted buckling mode, as illustrated in Fig. 6 (b).

K_R / K_R^{req} versus increment of out-of-plane deformation Δu_B, which are obtained by subtracting out-of-plane deformation of $N = 0$ from those of the maximum axial force, are represented in Fig. 15. These plots correspond with all test results of two kinds of section of girders and both side BRBs. It is clear that the increment of out-of-plane deformation increases as stiffness of the rotational spring K_R becomes less than the required stiffness K_R^{req}, and the validity of proposed design criteria is confirmed from Fig.15.

4. CONCLUSIONS

In this paper, design criteria of BRB to prevent out-of-plane buckling is proposed, and loading test was conducted to verify the criteria. As a result, we recommend the following criteria to prevent out-of-plane buckling; 1) if BRBs are installed in chevron bracing or inverted-V bracing and lateral stiffeners are not set at the center of the girder, the distance of lateral stiffeners must be as small as possible to make rotational stiffness (i.e. torsional stiffness) of the girder become lager than the required stiffness given by Eq. (2.16), and 2) bending stiffness of connections for all BRBs must be large enough to be satisfied with Eq. (2.8), that is the buckling load being lager than the maximum axial force of BRB.

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