SEISMIC INELASTIC DESIGN OF STEEL STRUCTURES BY SPECTRUM ANALYSIS AND EQUIVALENT DAMPING

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ABSTRACT :

This paper presents a new method for the seismic design of steel structures. The method involves the use of equivalent modal damping for predefined deformation limits and spectrum analysis and modal synthesis for the calculation of the design base shear of the structure. All the work of dissipation due to material and geometric nonlinearities is converted into work of dissipation due to linear damping. Design curves providing equivalent damping ratios as functions of period for the first few modes as well as design accelerations as functions of period for given equivalent damping are constructed using numerical data coming from a large number of plane steel moment resisting frames excited by various long duration seismic motions. The proposed design procedure is illustrated by means of the seismic design of a steel moment resisting framed structure and is checked by means of non – linear inelastic dynamic analysis. It is concluded that the presented design method can be viewed as a more rational and effective alternative of the force based method of current seismic codes with equivalent modal damping values playing the role of the behaviour or strength reduction factor.

KEYWORDS: Equivalent modal damping, seismic design, steel structures, spectrum analysis

1. INTRODUCTION

Dynamic non – linear (with respect to geometry and material) analysis of structures by the finite element method in conjunction with a stepwise time integration of the equations of motion is the only direct and reliable way for obtaining their response to seismic excitations. However, when a structure is to be seismically designed, this approach is usually not practical since the structure has to be modeled very carefully and excited by a number of seismic excitations. To avoid the need of performing several non – linear dynamic analyses, earthquake engineering has been focused over the past 40 years or so on the computation of seismic demands by various indirect (simplified) procedures of varying degrees of simplicity and accuracy. Most of these procedures at the end make use of the most versatile tool in earthquake engineering, the elastic response spectrum. In general, these simplified approaches can be separated in four major categories: a) those based on equivalent linearization, b) those based on inelastic response spectra – reduction factor, c) those based on non – linear modal superposition and d) those based on transform techniques. A brief review of the most important methods of each category is given in Papagiannopoulos and Beskos (2008).

In this work, an equivalent linear, with respect to geometry and material, multi-degree-of-freedom (MDOF) structure that can substitute the original geometrically and materially non – linear MDOF structure for seismic response applications is constructed. The goal is the determination of the maximum structural response through a linear elastic modal analysis using the equivalent modal damping values instead of the crude strength reduction factor used by modern codes. The proposed equivalent linear structure: a) retains the mass and initial stiffness of the original non – linear structure and takes into account non – linearities in the form of equivalent time – invariant modal damping values; b) is obtained through an iterative formation of a frequency response transfer function until this function satisfies certain smoothness criteria. Equivalent damping values for the first few modes are then numerically computed by solving a set of non – linear algebraic equations. Thus, the proposed method for seismically analyzing non – linear MDOF structures by simplified linear methods belongs to the category of equivalent linearization methods. However, the equivalent linear structure remains here a MDOF structure and it is not reduced to a single-degree-of-freedom (SDOF) one, as it is usually the case in the literature, with obvious gains in modeling accuracy. Furthermore, this equivalent structure here is characterized only by equivalent damping ratios and not by equivalent stiffness and damping as it is usually the case in the literature.

A design - oriented scheme is developed in order to apply the equivalent structure with modal damping to the seismic response calculation of MDOF building structures. This scheme involves a) the quantification of equivalent modal damping of a structure for predefined deformation limits and b) the use of spectrum analysis and modal synthesis for the calculation of the design base shear of the structure. For illustration purposes, curves providing equivalent damping as function of period for the first few significant modes as well as design acceleration versus period for given equivalent damping are constructed using a number of steel moment resisting frames excited by various seismic motions (Papagiannopoulos and Beskos 2008). The whole design procedure is illustrated by means of a steel moment resisting framed structure. It is concluded that the proposed design scheme can be viewed as an alternative to the force based method of current seismic codes with equivalent modal damping values playing the role of the strength reduction factor.

2. THE EQUIVALENT STRUCTURE WITH DAMPING

In order to reproduce the seismic response of a non – linear structure, the aim is to construct a MDOF linear structure with the effects of non – linearities (both geometric and material) being taken care of by appropriately quantified modal damping ratios. Since damping is the only parameter that controls the proposed linearization procedure it is very important to realize that if a structure with such a value of viscous damping is to remain in the linear region, non – linear work should not be produced. Therefore, viscous damping is fed into the original non – linear MDOF structure in order to prevent members from being stressed in the non – linear region. Consequently, a critical condition will be reached at which the non – linear MDOF structure will start behaving as linear. This means that the non – linear MDOF structure can be theoretically substituted by an equivalent linear MDOF structure with the same mass and stiffness properties as the original non – linear one. Thus, only one parameter of the equivalent structure needs to be defined, that of the equivalent modal damping with such values so as to maintain the behavior of the structure linear.

The abovementioned thoughts can be interpreted as an effort to balance the work done by non – linearities and viscous damping. This balance is practically realized as follows. It is assumed that the original non – linear MDOF structure possesses well separated classical normal modes and zero initial viscous damping. Damping is added to the structure, by being assigned to each of its normal modes following Rayleigh formula (Papagiannopoulos and Beskos 2008), and its non – linear seismic response is then computed via direct stepwise time integration of the equations of motion. The roof to basement frequency response transfer function $R_r(\omega)$ of a linear MDOF plane structure subjected to a horizontal seismic excitation is defined in the frequency domain as the ratio of the absolute roof acceleration response $\bar{\ddot{y}}_r$ of the structure over the seismic excitation $\bar{\ddot{u}}_g$ (Papagiannopoulos and Beskos 2006)

$$R_r(\omega) = \bar{\ddot{y}}_r(\omega)/\bar{\ddot{u}}_g(\omega) = 1 + \sum_{k=1}^{N}\left[\omega^2 \cdot \bar{\phi}_{rk} \cdot \Gamma_{rk} /(\omega_k^2 - \omega^2 + 2\cdot\xi_k\cdot\omega_k\cdot\omega\cdot i)\right] \tag{2.1}$$

where $\bar{\phi}_{rk}$ denotes the roof component of mode k, while ξ_k, Γ_{rk}, ω_k are the damping ratio, the participation factor and the modal frequency in mode k, respectively and N is the total number of modes of the structure. If it is assumed that both participation factors and mode shapes of the structure are a priori known, then the calculation of modal damping can be obtained on the basis of the modulus of the transfer function $|R_r(\omega)|$, which reads as

$$|R_r(\omega)|^2 = 1 + 2\cdot\sum_{k=1}^{N}\mathrm{Re}(z_k) + \sum_{k=1}^{N}|z_k|^2 + 2\cdot\sum_{k\neq j,\, j>k}^{N}\mathrm{Re}(z_k\cdot z_j^*) \tag{2.2}$$

where

$$z_k = \left[\bar{\phi}_{rk}\cdot\Gamma_{rk}\cdot\omega^2\cdot\left(\omega_k^2-\omega^2\right) - 2\cdot\xi_k\cdot\omega_k\cdot\omega\cdot i\right] / \left[\left(\omega_k^2-\omega^2\right)^2 + (2\cdot\xi_k\cdot\omega_k\cdot\omega)^2\right] \tag{2.3}$$

with the asterisk (*) denoting the conjugate of the corresponding complex number. Equation (2.2) for known $|R_r(\omega)|$, $\bar{\phi}_{rk}$, Γ_{rk} and ω_k, represents a set of non – linear algebraic equations, the solution of which leads to the calculation of modal damping ratios of all modes that appear in the transfer function. On the basis of the results presented in Papagiannopoulos and Beskos (2006), a linear structure exhibits a smooth transfer function with well defined visible peaks. These peaks correspond to the resonant frequencies of the structure. Modal damping ratios can then be calculated from Eqn.(2.2) by using the modulus of the transfer function as well as the resonant frequencies and the participation factors of the structure as obtained by modal analysis. When the structure ceases to behave linearly, non – linearities begin to take place and the abovementioned transfer function looses its smoothness. This lack of smoothness in the transfer function is depicted by multiple peaks or a jagged (distorted) shape.

The distortion (jaggedness) of the transfer function is exhibited as spurious peaks to the right or left side of the resonant peaks. The number and the position of the spurious peaks as well as the visibility of the resonant ones depend on the magnitude of non – linear deflections of the structure. Since the goal here is to construct a linear structure equivalent to the original non – linear one, it is important to realize that when the shape of the transfer function of the equivalent linear structure follows a smooth (undistorted) pattern, the corresponding calculated equivalent modal damping ratios will reflect the work done due to geometrical and material non – linearities.

Assuming that the non – linear absolute acceleration time history of a structure under a known seismic excitation can be obtained, the construction of its transfer function using Eqn.(2.1) is straightforward. This transfer function can be very jagged (unsmoothed) or it can depict only light jaggedness. The existence or lack of jaggedness depends on the inherent damping of the structure. In case that inherent damping of the structure balances or surpasses the work done due to non – linearities, the transfer function will be smooth. On the other hand, if the inherent damping of the structure cannot balance this work, the transfer function will be jagged. By adjusting (increasing) the inherent damping of the structure until it balances exactly the non – linear work done, one obtains a linear structure that exhibits a smooth transfer function and possesses such modal damping values that take care of all non – linear effects. These are the equivalent modal damping values.

On the basis of the developments in Papagiannopoulos and Beskos (2008), the smooth pattern of the transfer function of a MDOF structure is ensured when certain monotonicity criteria between all its peaks (maximum points) are established. This is the case depicted in Fig.1. To obtain the increasing and the decreasing branches of a transfer function initial damping is assigned in all modes of the structure according to Rayleigh damping formula. The transfer function of a MDOF structure has more than one resonant peaks corresponding to those structural modes excited by the seismic input used. The satisfaction of the smoothness (monotonicity) criteria does not occur simultaneously for all modes, except in some cases. Consequently, when the satisfaction of the monotonicity criteria is accomplished for one or more modes, the values of the transfer function $R_r(\omega)$ and the corresponding frequencies at its peaks (modes) are not allowed to alter. If additional damping is assigned to the modes that had already satisfied the criteria, their participation in the total response will be overestimated. Therefore, using Rayleigh's formula, initial damping is increased as long as the rest of the modes manage to satisfy the criteria. The damping increase for the mode(s) that have already satisfied the criteria is taken to be zero as long as the iterative procedure continues towards its final goal, i.e., the satisfaction of the monotonicity criteria of for all modes. In other words, these monotonicity criteria for various parts of the transfer function can be satisfied in different cycles of its iterative formation. However, there are cases where one should also check the derivative of $R_r(\omega)$ to ensure accurate calculation of the equivalent modal damping values (Papagiannopoulos and Beskos 2008).

When all modes manage to satisfy the criteria, the transfer function will attain a very smooth shape corresponding to a linear elastic MDOF structure. Then, Eqn.(2.2) is numerically solved in order to obtain the modal damping values of the equivalent linear elastic MDOF structure. For the calculation of the modal damping ratios one uses: a) the modulus of the transfer function at the peaks, b) the frequencies that correspond to these peaks and c) the participation factors as calculated from the classical eigenvalue problem involving the mass and initial stiffness of the given structure.

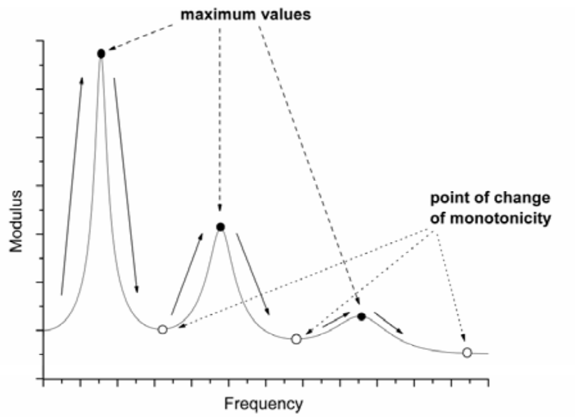

Figure 1 Transfer function for a MDOF structure and its peaks at resonant frequencies (↑ strictly increasing; ↓ strictly decreasing)

3. APPLICATIONS OF EQUIVALENT DAMPING IN SEISMIC DESIGN

The proposed technique by means of the iterative formation of the frequency response transfer function leads to equivalent modal damping values that take into account geometrical non – linearities, cyclic inelastic deformations, number of yield excursions and yield reversals because the non – linear seismic response of the structure is successively used till it becomes linear. Therefore, the equivalent modal damping values keep all deformations as well as the base shear of the structure to values that correspond to those just before its first time yielding. Consequently, equivalent modal damping ratios can be viewed as playing the role of the strength reduction factor in seismic design because the non – linear seismic demands posed by a given seismic motion against a given structure are quantified. In contrast to the strength reduction factor, the determination of equivalent modal damping values is more rational. Thus, by using the equivalent modal damping ratios in conjunction with spectrum analysis and modal synthesis, one can easily compute the maximum base shear of a building structure. However, there are two important things that demand further investigation. The first is related to the determination of deformation dependent equivalent modal damping ratios and the second concerns the effect of higher modes.

According to the developments of the previous section, the equivalent modal damping values are derived numerically but no reference is made regarding loss of stability of the structure. Thus, the equivalent modal damping values obtained by using Eqn.(2.2) are found under no restriction concerning the deformability state of the structure. Therefore, it should be checked if these damping values are realistic by posing a limit to the deformation of the structure under consideration.

Deformation limits are expressed herein in terms of interstorey drift ratios that should not surpass predefined limit values, even though other deformation measures could have been used as well. The calculation of the equivalent modal damping values follows the procedure described in Papagiannopoulos and Beskos (2008) but this time the roof response and the earthquake excitation time signals used in the formation of the roof to base transfer function are modified. More specifically, the structure is assumed to have very light damping, e.g., 0.1% and the earthquake time signal is considered just up to the time step that the violation of the predefined interstorey drift limit occurs. A band of zeros then replaces the values of the rest of the earthquake signal that follows (Papagiannopoulos and Beskos 2008). The non – linear response of the roof is obtained by using the part of the earthquake time signal that corresponds up to the violation of the interstorey drift with the rest of the signal having zero values. Therefore, equivalent modal damping values calculated by using part of the earthquake time signal will be different from the ones found by considering the whole earthquake time signal. This procedure enables the calculation of deformation dependent equivalent modal damping ratios and is justified by current efforts in earthquake engineering to control the displacements of the structure.

On the other hand, equivalent modal damping value can be assigned only to these modes that appear in the transfer function. Modes that do not appear in the transfer function cannot be considered in the solution of Eqn.(2.2) used for the quantification of the equivalent modal ratios but as it has been mentioned in Papagiannopoulos and Beskos (2006) they have to be taken into account for accurate response purposes. These higher modes that essentially correspond to high frequencies are not excited, behave statically and thus, a high damping value has to be taken into account for them. Regarding the proposed design scheme, a simple yet accurate solution is adopted for treating this issue of modal truncation. The number of modes taken into account in order to apply the modal combination rule is derived by considering the first modes that ensure 95% of modal mass participation in the response. Equivalent damping is considered for the modes that appear in the transfer function, while for the rest modes needed to satisfy the aforementioned modal mass participation requirement a very high value of damping is used.

4. DESIGN CURVES

The proposed method to evaluate equivalent damping is applied to twenty MDOF steel moment resisting framed structures with periods varying between 0.5 and 3.0sec. The time domain non – linear response of the MDOF structures for the long duration seismic excitations of Table 1 (Papagiannopoulos and Beskos 2008). Both geometrical and material non – linearities have been taken into account while the limit of allowable interstorey drift ratio (IDR) for the structures under study has been considered not to exceed the values of 0.6, 1.5 and 2.0% of their storey height for the limit states of fully operational, operational and life safety, respectively and the damage of structural elements cannot exceed acceptable values of plastic hinge rotations (Papagiannopoulos and Beskos 2008). Initial assumed damping was computed by Rayleigh's formula, using the first mode and the last mode needed to obtain participation of modal mass greater than 95%.

Figures 2-3 illustrate the variation of equivalent damping with period for the case of 1.5% (IDR) and the first two modes, respectively. The dashed lines in these figures indicate the design values for $IDR = 1.5\%$ whereas the solid and dash-dot lines shown correspond to the limits for plastic hinge rotations to further satisfy the damage performance levels mentioned in Papagiannopoulos and Beskos (2008). More specifically, points shown with solid circles (●), bounded by a dash-dot line constitute the lower bound where the damage limit of the beams occurs whereas, points that correspond to the same period and lie beyond the points indicated by (●) do not satisfy the damage demand at beams. In Figs.2-3 there is a period range where no damage at beams was detected and an arbitrarily bound indicated by a solid line is considered for design purposes. Design equations for equivalent modal damping for the cases of $IDR = 1.5\%$ and $IDR = 1.5\%$ & damage are tabulated in Table 2 for the first five modes.

Table 1 Long duration seismic excitations

Earthquake, Country	Station	Date
Tokachi Oki, Japan	HAC1,HAC2	16/05/1968
Valparaiso, Chile	LLO, LLA, ISI, VDM	03/03/1985
Michoachan, Mexico	SCT	19/09/1985
Manjil, Iran	AT2	20/06/1990
El Salvador, El Salvador	OB, ST	13/01/2001
Tokachi Oki, Japan	HKD 092, HKD 100	25/09/2003
Ica Pisca, Peru	ICA2	15/08/2007

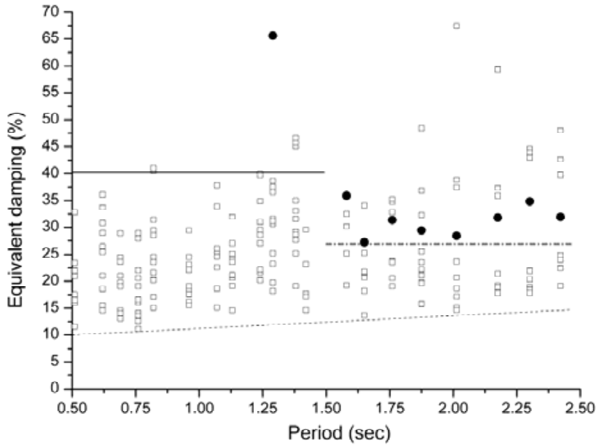

Figure 2 Design values for equivalent damping for 1.5% IDR (1st mode)

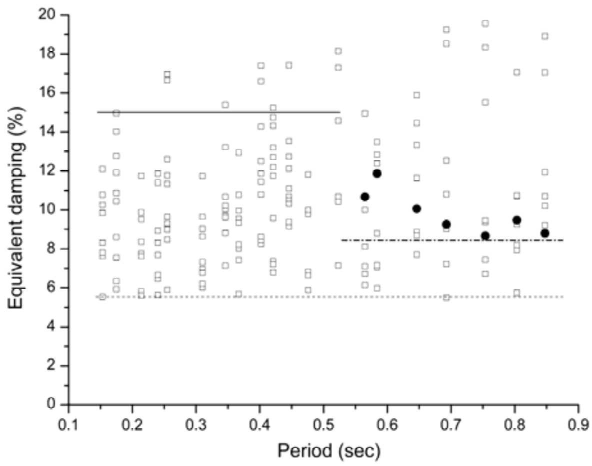

Figure 3 Design values for equivalent damping for 1.5% IDR (2nd mode)

Table 2 Modal damping ξ as function of period T, IDR and damage.

Mode	$IDR = 1.5\%$	$IDR = 1.5\%$ & damage
1	$\xi = 0.025 \cdot (T - 0.5) + 0.1$ for $0.5 \le T \le 2.5$ sec	$\xi = 0.40$ for $0.5 \le T \le 1.5$ sec $\xi = 0.27$ for $1.5 \le T \le 2.5$ sec
2	$\xi = 0.055$ for $0.15 \le T \le 0.85$ sec	$\xi = 0.15$ for $0.15 \le T \le 0.5$ sec & $\xi = 0.085$ for $0.5 \le T \le 0.85$ sec
3	$\xi = 0.035$ for $0.11 \le T \le 0.48$ sec	$\xi = 0.07$ for $0.11 \le T \le 0.48$ sec
4	$\xi = 0.035$ for $0.11 \le T \le 0.27$ sec & $\xi = 0.8 \cdot (T - 0.27) + 0.035$ for $0.27 \le T \le 0.32$ sec	$\xi = 0.05$ for $0.11 \le T \le 0.27$ sec & $\xi = (T - 0.27) + 0.05$ for $0.27 \le T \le 0.32$ sec
5	$\xi = 0.929 \cdot (T - 0.17) + 0.035$ for $0.17 \le T \le 0.24$ sec	$\xi = 1.036 \cdot (T - 0.17) + 0.0375$ for $0.17 \le T \le 0.24$ sec

5. NUMERICAL EXAMPLE

A moment resting framed structure having twelve stories and four bays is used to illustrate how Figs.2-3 or Table 2 can be used for seismic design. HEB profiles are used for columns and IPE for beams. Each bay of the steel frame has 4.0m span and each storey 3.0m height. The dead plus live load on beams is equal to 27.5KN/m. Interstorey drift is not allowed to surpass 1.5% of storey height. The structure has been designed according to EC3 (1992) and seismic hazard is defined by means of the mean plus one standard deviation damped elastic design spectrum of Fig.4, which has been constructed using the seismic motions of Table 1 for several damping ratios (Papagiannopoulos and Beskos 2008).

Performing eigenvalue analysis one first obtains the mode shapes and the natural periods of the given structure. To ensure modal participation mass of 95%, five modes need to be taken into account. Equivalent damping for modes 1 to 5 comes from Table 2, while design acceleration from Fig.4. The initial value for viscous modal damping of 2% in the elastic region was subtracted from the equivalent modal damping values for reasons of consistency (Papagiannopoulos and Beskos 2008). Equivalent modal damping ratios read as: $\xi_1 = 0.25$, $\xi_2 = 0.065$, $\xi_3 = 0.05$, $\xi_4 = 0.03$ and $\xi_5 = 0.035$. One finally has the following sections: 360/400/400/400/360-360 (1-5) & 320/360/360/360/320-300 (6-9) & 300/320/320/320/300-300 (10-12) which means that for the first five stories there is a variation of column sections in each bay, i.e., sections HEB360 and HEB400 for the first bay, sections HEB400 and HEB400 for the next two bays and sections HEB400 and HEB360 for the last bay and all beams have IPE360 section, for the next four stories there is a variation of column sections in each bay (sections HEB320 and HEB360) and the beams have IPE300 section and for the last three stories there is a variation of column sections in each bay (sections HEB300 and HEB320) and the beams have IPE300.

Non – linear dynamic analyses are executed using the accelerograms of Table 1, in order to check if the designed frame satisfies the target performance criteria. The results from non – linear dynamic analyses (assuming an initial value for viscous modal damping of 2%) regarding minimum, median and maximum value for interstorey drift and plastic hinge rotation have as follows: $IDR_{\min} = 0.92\%$, $IDR_{med} = 1.27\%$, $IDR_{\max} = 1.42\%$, $\theta_{pl,\min} = 0.79\theta_y$, $\theta_{pl,med} = 0.91\theta_y$ and $\theta_{pl,\max} = 0.96\theta_y$, where θ_y stands for the yield rotation. On the basis of these results, it is concluded that the proposed method of employing modal damping ratios results in good accuracy regarding median values of deformation and damage levels for the case examined.

The frame is now designed according to EC3 (1992) by employing the 2% damped acceleration spectrum of Fig.4 and a common value for all modes for the reduction factor, i.e., $q = 3$. On the basis of the equal displacement rule for target $IDR = 1.5\%$ and by using the aforementioned type of expression for the steel sections, one finally finds them to be: 360/400/400/400/360-360 (1-5) & 320/360/360/360/320-330 (6-9) & 320/320/320/320/320-330 (10-12). Non – linear dynamic analyses are executed using the accelerograms of

Table 1 in order to check if the designed frame satisfies the target performance criteria. The results from non – linear dynamic analyses (assuming an initial value for viscous modal damping of 2%) regarding minimum, median and maximum value for interstorey drift and plastic hinge rotation lead to $IDR_{min} = 0.92\%$, $IDR_{med} = 1.27\%$, $IDR_{max} = 1.42\%$, $\theta_{pl,min} = 0.81\theta_y$, $\theta_{pl,med} = 0.89\theta_y$ and $\theta_{pl,max} = 0.94\theta_y$. It is concluded that the conventional seismic design approach using a common value for all modes for the strength reduction factor results in overestimated seismic deformation and damage levels for the considered seismic motions.

On the basis of the above results and similar results provided elsewhere (Papagiannopoulos and Beskos 2008), it can be said that the conventional method of employing a single modal value for the strength reduction factor leads to overestimation or underestimation of seismic deformation and damage levels, depending on the frame and the seismic motion expected, because it does not recognize the fact that each mode contributes in a different way to the final seismic response and design results. The proposed approach of using equivalent modal damping ratios leads to more accurate seismic response and design results in a more rational way.

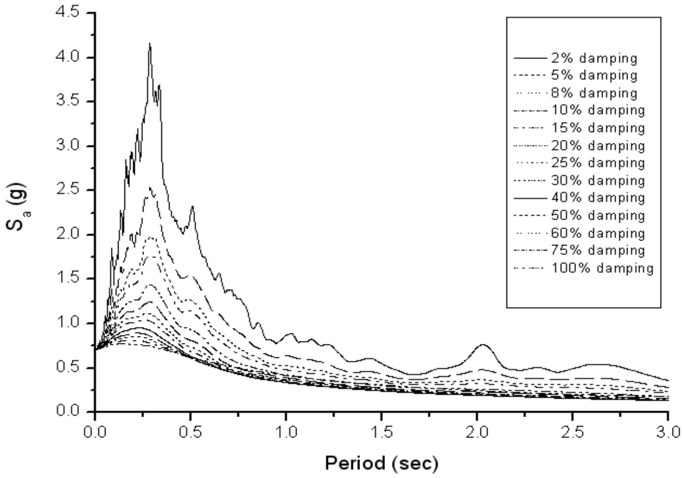

Figure 4 Elastic design spectrum for several damping values

6. CONCLUSIONS

On the basis of the preceding developments, the following conclusions can be stated: i) an equivalent linear MDOF structure has been proposed in order to determine the earthquake response of the original non – linear MDOF structure in an approximate yet of satisfactory accuracy way; ii) this equivalent linear MDOF structure is constructed by retaining the mass and initial stiffness of the original non – linear MDOF structure and expressing material and geometrical non – linearities in the form of time – invariant modal damping values. These values can be viewed as playing the role of the strength reduction factor in code – based seismic design; iii) curves providing equivalent damping versus period for the first few modes as well as design acceleration for given equivalent damping are constructed. Thus, one can establish a seismic design method on the basis of modal synthesis and spectrum analysis; iv) the proposed seismic design scheme can be viewed as an alternative to the force based method of current codes with equivalent modal damping values playing the role of the strength reduction factor

REFERENCES

Papagiannopoulos, G.A. & Beskos, D.E. (2008). Use of equivalent modal damping for seismic design of steel structures, Parts I & II. *Earthquake Engineering and Structural Dynamics (under preparation)*.
Papagiannopoulos, G.A. & Beskos, D.E. (2006). On a modal damping identification model for building structures. *Archive of Applied Mechanics* 76:6, 443-463.
EC3 (1992). *Eurocode 3,* Design of steel structures, Part 1.1: General rules for buildings, European Prestandard ENV 1993-1-1/1992. *European Committee for Standardization (CEN): Brussels.*