MASONRY BARREL VAULTS: INFLUENCE OF THE PATTERN

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ABSTRACT :

Masonry barrel vaults were generally analyzed in the Centuries as sum of single arches without considering longitudinal effects. The finite element method (FEM) allows to assess curved structures in linear and nonlinear field. Nevertheless, particular attention must be addressed to the geometry and the material model. The aim of this paper is the assessment of the role of the pattern in the structural behaviour of masonry barrel vaults. For this scope, advanced numerical approaches (i.e. macro and micro models) were adopted. Linear and nonlinear analyses were performed with reference to some case studies characterized by different unit arrangements.

KEYWORDS: Masonry, Barrel Vaults, FEM, Macro-modelling, Micro-modelling

1. INTRODUCTION

Masonry vaults represent a typical structural part of several historic buildings. These elements influence the structural behaviour of the construction both toward vertical and horizontal loads. Indeed, several damages and failure modes were observed during the past earthquakes evidencing their role in the seismic vulnerability of buildings.
Various analytical and empirical methods for the analysis of masonry vaults are available in literature. Most of them are based on theoretical formulations used for arches (Breymann, 1905), and consequently they are not able to account the ‘three-dimensional’ behaviour of these structures. Recently Cattari et Al. (2008) modelled masonry vaults as equivalent plates taking into account also the units arrangement.
The adoption of punctual non-linear constitutive models, implemented in a Finite Element (FE) formulation, allows examining masonry structures in a deeper detail compared to the traditional empirical methods used in the engineering practice.
The paper here presented is part of a research activity devoted to investigate the structural behaviour of masonry vaults. In particular, different modelling approaches based on the use of the FEM are presented in order to assess the effect of the blocks arrangement on the global behaviour of the vaults. For this aim, some numerical analyses were performed using the commercial code DIANA 9.1 (2000) and some case studies characterized by different arrangements of the units were chosen.

2. MASONRY BARREL VAULTS: GENERAL ASPECTS

Masonry barrel vaults, generated by the translation of an arch (modulus) along an orthogonal directrix, are the simplest and oldest type of roofing structures. The construction process and the shape of the elements are an essential characteristic of these structures. Squared stones, due to the exact cut of the voussoirs, are very appropriate granting strength and steadiness to the construction. Uniform mortar joints, subsequently, contribute to link the stones flattening the contact surfaces and distributing stresses uniformly.
Along the Centuries, many kinds of pattern were used by master-builders for barrel masonry vaults (Protti, 1935; Astrua, 1996; Tomasoni, 2008). The most used types may be addressed in the following categories.
In Longitudinal Vaults the unit courses are parallel to the abutment development and mortar joints are orthogonal to the correspondent element of the arch. Two following mortar courses must not correspond on the face, on the

back and inside so that, at least two different courses have to alternate. This type is not suited for roofs with big spans or low heights. The construction starts from the abutments, symmetrically, until the key stone in order to avoid deformations in the centrings.
In the Transversal Vault the stones have the major side orthogonal to the abutments and mortar joint are in parallel planes. The vault consists of many independent arches, leaning one upon another. Each arch is linked to the adjacent ones only by mortar joints. To avoid lateral displacements and saving centrings, it may be built also along inclined planes. The construction starts on the fronts and go on along the axis with a sliding centring, sufficient if coupled to quick curing mortar.
The Bone Vault is generated by the mirroring of a simple diagonal vault which courses are placed at 45° respect the supporting walls. Mortar joints and cut stones in one or two sides, are linear in plan, sloped in their development and orthogonal to the diagonal section of the vault. The construction starts from the corners with elliptical cripple arches closed by a squared key stone. It has the advantage of single self-bearing courses and guide centrings but is more complicated than the previous ones and requires qualified workmanship. Transferring loads both on the longitudinal and transversal walls, it is generally used for great spans and small heights.
In the Inverted Bone Vault the construction process starts from the centre of the structure, placing on the centrings firstly four units inclined at 45° with the axis of the vault in right angles, then whole and cut units are set at the quarter until the boundary walls. The construction goes on with whole stones restarting with cut and entire units. Thanks to the better good-looking than the previous one, this vault is suitable for not-plastered structures.
The vaults so described are all illustred in Figure 1.
The aesthetical and construction reasons at the base of a particular pattern influence, naturally, the static and dynamic behaviour of these structures. Until the first half of the XX Century the spreadest method of studying barrel vaults was considering the structure as a series of independent arches put side by side. In this way, the interaction among the single strips along the generatrixes is neglected. Instead, it is well known that a barrel vault presents also a flexural regime in the longitudinal sense and each arch may be helped by contiguous arches, transmitting them a part of concentrated applied loads. It is evident that the construction process and, mainly, the units pattern may amplify this effect.

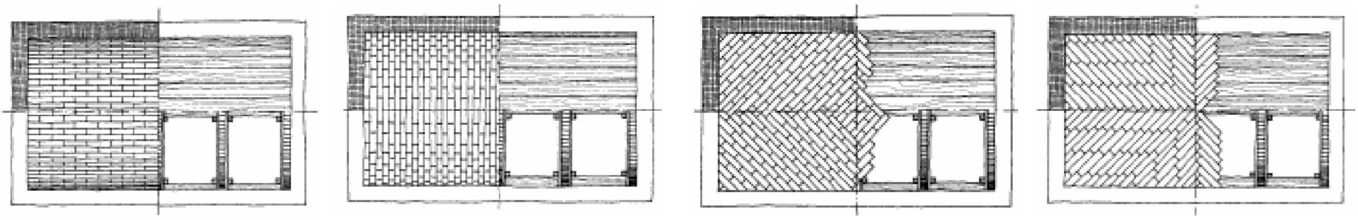
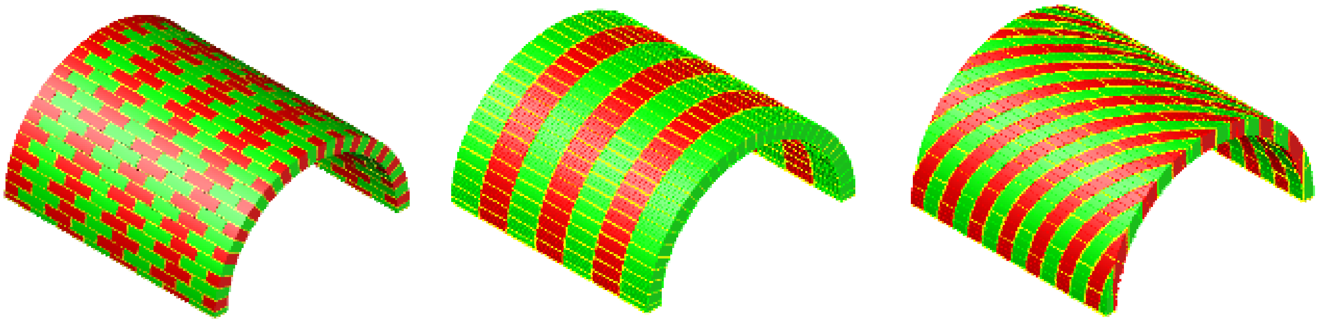


Figure 1 Different barrel vaults configurations (Levi, 1932).

3. STUDY CASES

In order to underline the role of the pattern, in this paper three different arrangements were examined (Figure 2). The first scheme corresponds to the Longitudinal Vault, the second one to the Transversal Vault and the third one to the Diagonal Vault.
All the examined structures are characterized by the same geometric dimensions (uniform thickness of 150 mm covering a squared plan 2 m long) and constituent materials (tuff units 300x150x100 mm^2 coupled to mortar lime joints 10 mm thick), but changing the pattern of the blocks.
Regarding the material properties used in the numerical analyses and reported in the following tables, some of the values were deduced from experimental tests performed by (Augenti and Romano, 2007), whilst some other ones were adopted by the authors in order to better underline the role of the parameters investigated in the paper.
Some load cases were applied on the vaults varying from vertical concentrated loads, to increasing vertical loads and increasing horizontal loads.

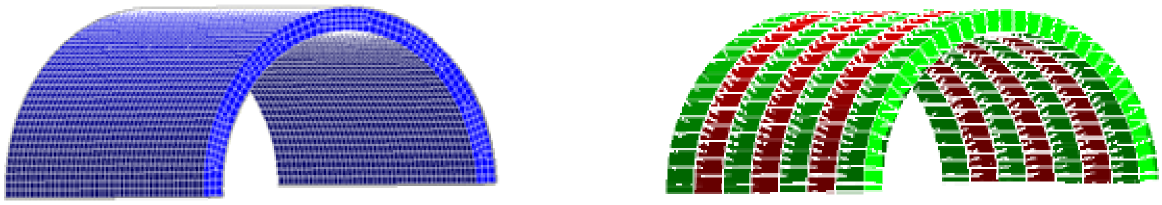


Longitudinal Vault Transversal Vault Diagonal Vault

Figure 2 Barrel vaults: study cases

4. MODELLING APPROACHES AND NUMERICAL ANALYSES

Some numerical analyses were performed through the commercial code DIANA 9.1 (2000) with reference to the study cases and choosing different modelling approaches (Figure 3), material models and load conditions.
The obtained results, presented in the next sections, underline the influence of the blocks arrangement on the global behaviour of the examined schemes and the capability of the selected modelling approaches to capture this effect.



a: Macro-model b: Micro-model

Figure 3 FEM Models

4.1. Macro-modelling

The macro-modelling approach is based on the homogenization process which involves the constituents of structures both in terms of geometry (arrangement of the elements) and material properties (Grande et Al., 2008). This modelling approach was used in the present paper in order to obtain a preliminary estimation of the influence of some parameters on the structural behaviour of masonry barrel vaults.
In particular, the modalities of the assemblage of the arches (or moduli) generating the vault may influence the global behaviour of the structure both in terms of load capacity and stiffness. Indeed, the interaction among the adjacent moduli generates a 'three-dimensional' effect in the vault which involves also a flexural behaviour. For this aim, different schemes were considered varying the length of the vault from 100 mm (scheme 2F) to 2000 mm (scheme 40F), as shown in Figure 4.
The material model selected is an orthotropic elastoplastic continuum model with the Hoffmann's yield condition (1967) and characterized by an elastic-perfectly plastic behaviour in tension and compression. The adopted yield condition is an extension of the Hill model (1947) describing different yield strength in tension and compression. An associate flow rule is adopted for this yield criterion. The reader is referred to Hoffmann (1967) and DIANA (2000) for further details. The mechanical parameters used for developing the numerical analyses are reported in Table 1. In particular, the mechanical properties along the x-direction were deduced from the experimental tests (Augenti and Romano, 2007), whilst the other properties were chosen by the authors in order to underline the different behaviour along the principal directions of the vault. Eight-node isoparametric solid brick elements (denoted 'HX24L' in the code) were used for the model discretization (Fig. 3.a).

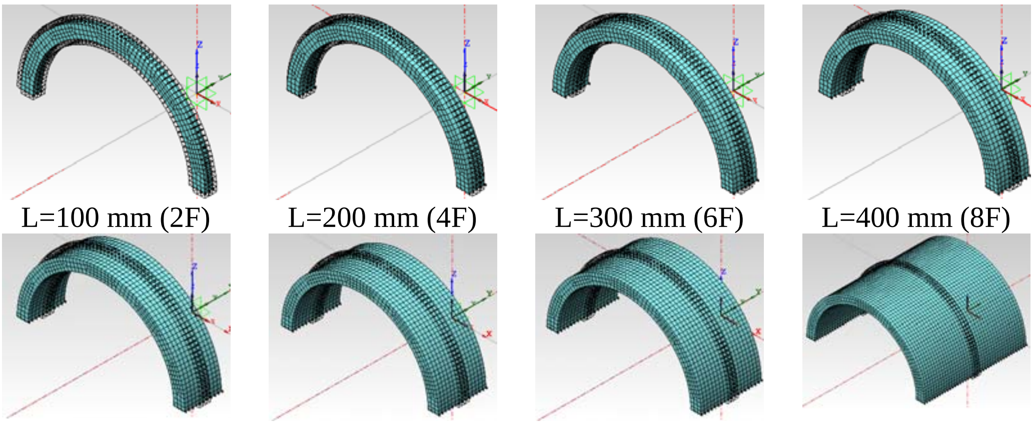


Figure 4 Macro-modelling approach: examined schemes

Table 1 mechanical properties for the macro-model

Direction	Compressive strength	Tensile strength	Tensile strength	Young's modulus	Shear modulus
X	4.0 MPa	0.4 MPa	0.1 MPa	1980 MPa	900 MPa
Y	2.0 MPa	0.2 MPa	0.1 MPa	990 MPa	450 MPa
Z	4.0 MPa	0.4 MPa	0.1 MPa	1980 MPa	900 MPa

The first examined results concern the influence of the longitudinal effects on the vertical deformability of the vault deduced through elastic analyses. In Figure 5 the displacement of the middle point of the vault is reported for the different examined schemes varying the value of the Young's modulus along the longitudinal direction (y-direction). From the plot it is clear the influence of the longitudinal effects on the vault deformability and the role of the material stiffness. In fact, starting from the case of the simple arch (scheme 2F), loaded on the entire surface, the vertical displacement of the middle point of the vault decreases with a nonlinear law evidencing the contribution of the unloaded parts. This effect is more evident for high values of the Young's modulus along the longitudinal direction, whilst, decreasing the value of the Young's modulus a minor contribution is observed.
In Figure 6 the maximum value of the axial stress along x and y-directions is reported for the scheme 40F and considering E_y in the reference value. From this figure it is interesting to notice that, while the axial stress along the x-direction decreases with the vault length, the axial stress along the y-direction decreases up to the scheme 8F and increase for the other schemes. The increase of the axial stress along the longitudinal direction may be considered as an indicator of the activation of the flexural behaviour of the vault.

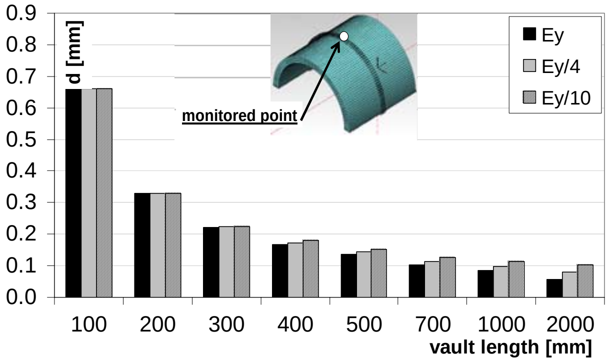


Figure 5 Variation of the vertical displacement

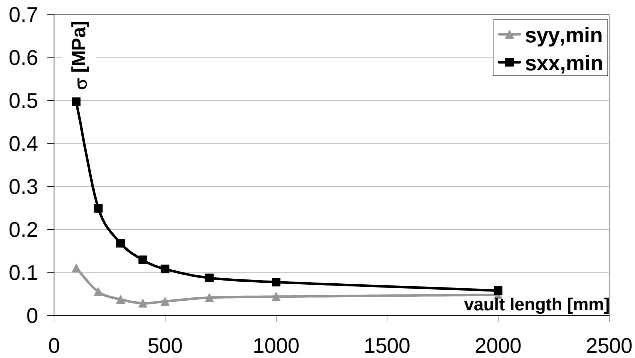


Figure 6 Variation of the axial stress

Clearly, the longitudinal effects also influence the nonlinear behaviour of the vault. For this aim, nonlinear static analyses were performed on the schemes used in the elastic analyses considering the self weight of the vault and an increasing concentrated vertical load.

The obtained results are reported in Figure 7 in terms of load factor (λ: applied force factorized self weight of the vault) vs vertical displacement (d) curves. From the plot it is clear the effect due to the increase of the vault length both in terms of initial stiffness and maximum sustained load. From the deformed shapes reported in the same figure, it is also evident the reduction of the vertical displacement of the schemes.

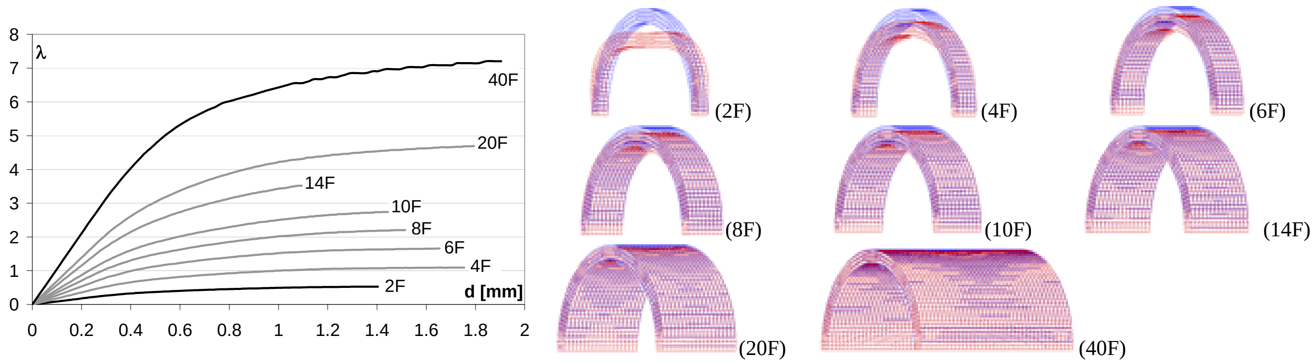


Figure 7 Push-over curves of the schemes and deformed configurations

On the basis of these results, it may be affirmed that a good interaction among the arches composing the vault improves the load capacity. As previously evidenced, this effect depends not only on the mechanical properties of the constituents materials but also on the moduli arrangement. It is clear that this effect may be taken into account considering the different assemblage configuration in the homogenization process. This involves the need to perform experimental tests on sub-assemblages (blocks + mortar) reproducing the effective blocks arrangement and the corresponding strength along different direction.
In order to underline this effect, in Figure 8 the pushover curves of the scheme 40F are reported varying the shear strength τ_{xz}. From the plot it is evident that for low values of the shear strength the behaviour of the vault tends to the behaviour of the single arch (scheme 2F) because of the not activation of unloaded parts.
A possible strategy for taking into account the arrangement of the blocks in the macro-modelling approach consists of providing different orientations of the material axes (option allowed in several commercial FE codes).
In Figure 9 the vertical displacement of the middle point of the vault is reported for the scheme 40F selecting different values of orientation of the material axes and considering the mechanical properties reported in Table 1 in the case of zero degrees (0°). In particular, 0° corresponds to the Transversal Vault, 90° to the Longitudinal one and 45° to the Diagonal one. From the plot it is possible to observe that the reference scheme (0°) shows the maximum elastic stiffness and an intermediate value of the peak load; on the contrary, the case with 90° material axes orientation shows the minimum value of the elastic stiffness and the maximum value of the peak load.

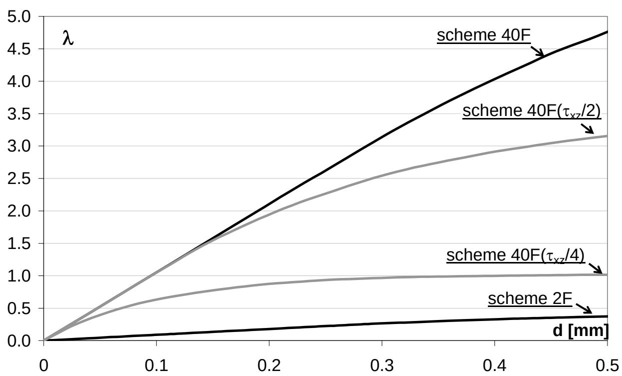


Figure 8 Effect of the strength in a vault

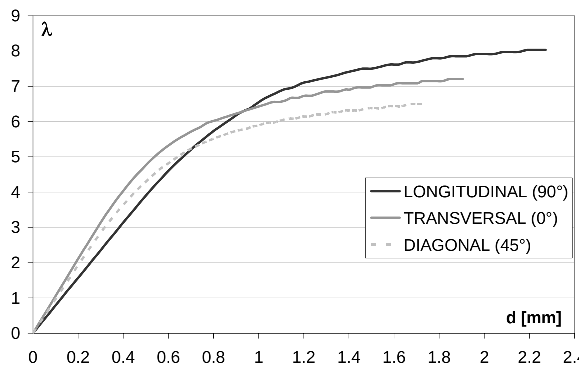


Figure 9 Vaults varying the material axes

It is clear that the reliability of the macro-modelling approach depends on the homogenization process which in some cases presents several drawbacks due to the particular configuration of the blocks arrangement. In this case, the use of a more detailed modelling approach, such as the micro-modelling, is therefore preferable.

4.2. Micro-modelling

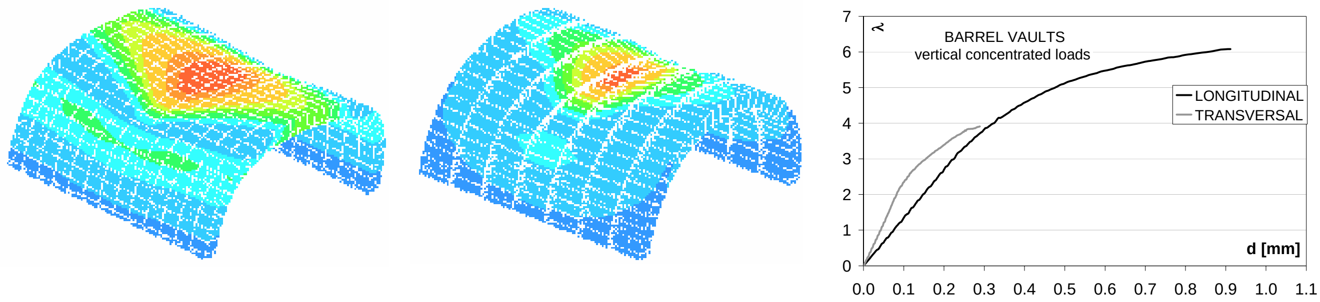
In the micro-modelling approach each single component of the masonry assemblage and their interaction is modelled in order to reproduce the behaviour of the structure. This method is particularly suited for studying structural details in which great stress concentration are present or when gradients inside the elementary volume are significant. Unfortunately, the required modelling phase and the computational efforts are rather burdensome.
In this paper, the micro-modelling approach was used in order to study the effect of the different blocks arrangements characterizing the first two study cases shown in Figure 2. Obviously, in this case the effective arrangement of the blocks was taken into account also considering the effective skew of the mortar joints homogenized in the macro-modelling (Fig. 3.b).
Regarding the material model, an orthotropic elastic-perfectly plastic continuum model with the Hill failure criterion (1947) was selected and eight-node solid brick elements ('HX24L') were used in the discretization. The parameters characterizing the material model, reported in Table 2, were deduced from experimental tests (Augenti and Romano, 2007). The choice of a simpler material model than the macro-modelling, depends only on the fact to reduce the computational process.

Table 2 mechanical characteristics of materials

Material	Weight density	Compressive strength	Shear strength	Young's modulus	Poisson modulus
Stone	16 kN/m^3	4 MPa	0.1 MPa	2500 MPa	0.15
Mortar	16 kN/m^3	2 MPa	0.1 MPa	800 MPa	0.16

Three different load conditions were used in the numerical analyses:
- a vertical concentred load applied only in a central portion of the vault (300 mm wide);
- a vertical distributed load proportionally to the self weight of the vault;
- a dead load plus an horizontal distributed load proportionally to the self weight of the vault.

The effect of the concentrated vertical load was studied performing nonlinear numerical analyses. The obtained results are reported in Figure 10 in terms of deformed shapes (also evidenced through contour levels. Figure 10.a, .b) and multiplier factor vs vertical displacement plots (Figure 10.c). Comparing the pushover curves, it is possible to notice that the longitudinal arrangement, although less stiff of the transversal one, is able to carry a bigger load due to the skew of each course with the adjacent ones. The transversal arrangement, made of single arches linked each other only by mortar, is a stiffer system but is characterized by a lower peak load.



a: Longitudinal Vault b: Transversal Vault c

Figure 10 Effect of a concentrated vertical load on a vault

Afterwards, the assessment of the influence of the blocks arrangement was made considering the masonry vaults subjected to distributed vertical and horizontal loads. Figure 11 shows the un-deformed and the deformed shape of the vault frontally (.a), the contour levels of the same vault deformed (.b) and a multiplicative load/vertical displacement graph of a control point at the extrados of the vault (.c). The initial stiffness of the Longitudinal and Transversal Vaults is likewise whilst the peak load of the transversal one is slightly bigger than the Longitudinal Vault. The same analysis was also conducted with the macro-modelling approach, evidencing a similar initial stiffness and a smaller value of the peak load, probably due to the different material model.

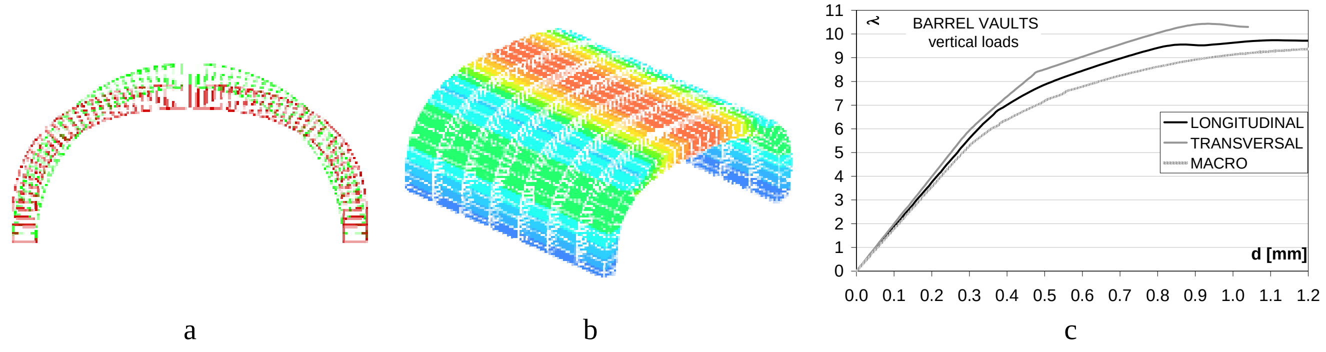


Figure 11 Barrel vaults: influence of the pattern under vertical loads

The results are slightly different when the structures are subjected to horizontal loads. Again the initial stiffness is similar for the two vaults but contrarily to the previous load case, the Longitudinal Vault is able to carry a greater peak load (Figure 12). This result underlines the good response of the Longitudinal Vault under seismic loads compared to the transversal one. Also in this case, the non linear analysis performed using the macro-model showed a curve which is in agreement in terms of elastic stiffness with micro-models. On the contrary, the macro-model presents a peak load value closer to the transversal vault maybe due to the low efficacy of this arrangement respect to the horizontal loads.

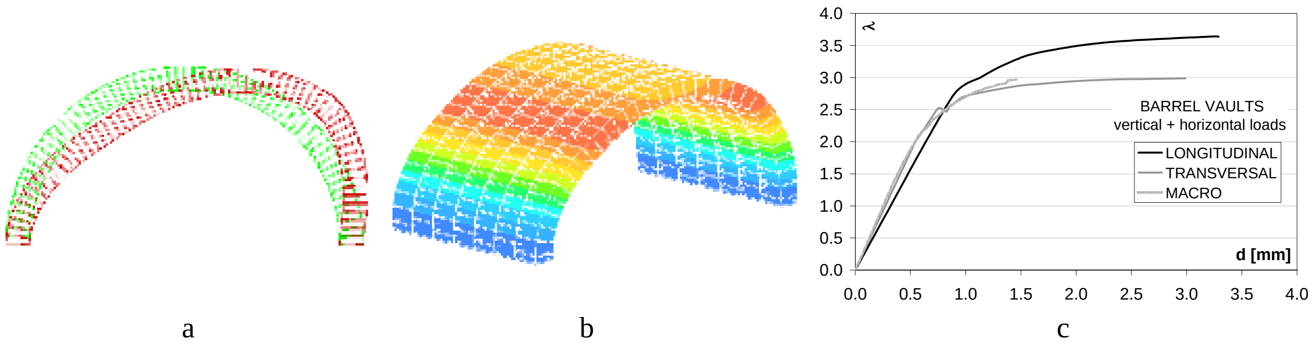


Figure 12 Barrel vaults: influence of the pattern under dead load + horizontal loads

6. CONCLUSIONS AND FUTURE DEVELOPMENTS

In this paper the influence of the pattern in masonry barrel vaults was assessed through the use of advanced FEM models. Some case studies were analyzed in detail considering different modelling approaches (micro and macro models), material models and loading conditions. The performed numerical analyses allowed to derive the following conclusions.

According to the macro-modelling approach:

- increasing the vault length, the variation of the stiffness shows a non linear law which underlines the contributions of the longitudinal effect and the activation of the flexural regime;
- the influence of the longitudinal effects particularly depends on the blocks arrangement which influences both the stiffness and the load capacity of the vault. The reduction of both the Young's Modulus along the longitudinal direction and the shear strengths decreases the longitudinal effects leading to a behaviour of the vault similar to the behaviour of the single modulus;
- the reliability of the macromodel depends on the homogenization process which in some cases requires experimental test on non-standard sub-assemblages reproducing the blocks arrangement of the vault. The use of the technique of rotating the material axes may furnish a good estimation of the influence of the blocks arrangement. The examined cases underlined some peculiarities both in terms of initial stiffness and peak load.

In particular it was observed that the scheme with the minimum stiffness value (Diagonal Vault) presents the maximum peak load whilst the scheme characterized by the maximum value of the stiffness (Transversal Vault) showed an intermediate value of the peak value.

According to the micro-modelling approach:

- for vertical concentrated loads, the behaviour in terms of strength of the Longitudinal Vault is better than the Transversal one. The latter, although stiffer, is influenced by the behaviour of the single arches which act almost independently. The other pattern, conversely, involves the collaboration of the courses in the nearby of the loaded part;
- for increasing vertical loads, no significant differences were found in the two vaults both in terms of stiffness and peak load with a slight prevalence of the Transversal Vault with an arch behaviour;
- for increasing horizontal loads, like for the previous load case, the Longitudinal Vault behaved better than the Transversal one, being able to bear a bigger peak load. This is due to the same reason given previously;
- the skew of the blocks furnishes a further contribution to the global capacity which also depends on the type of the load condition.

For these reasons it is reasonable to affirm that no difference are advisable in the examined masonry barrel vaults under uniform vertical loads, whatever is the units arrangement. On the contrary, in case of concentrated or horizontal loads, the skew of unit courses like the Longitudinal Vault could make the difference, increasing consistently the load capacity of these structures.

Future developments of this research will concern the use of more refined material models provided of softening, other case studies for barrel vaults developed in the micro-modelling approach like the bone and the inverted bone vaults and further shapes of combined vaults.

ACKNOWLEDGMENTS

The authors would like to thank sincerely P.E. Diego Renna for his indispensable help in the vault CAD models.

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