INFLUENCE OF EARTHQUAKE INTENSITY MEASURE ON THE PROBABILISTIC EVALUATION OF RC BUILDINGS

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ABSTRACT:

The correct assessment of different performance levels of reinforced concrete (RC) structures still remains an unaccomplished task in the field of earthquake engineering. Starting from the expression for classical time-independent reliability formulation, and under a few established assumptions, the probability of exceeding a specified performance level can be written in a closed form via Probabilistic Seismic Demand Analysis. It has been recognized that the choice in terms of the intensity measure (IM) of a records plays a leading role in the performance assessment, because it is strongly related to the seismic hazard. The general framework for computing the probability of exceeding a specified limit state is here specialized to RC elements and implemented for different frame structures, showing a relatively computational efficiency for the range of buildings considered. The procedure is then applied using several alternative scalar ground motion IMs in order to observe the unavoidable variability of the results in terms of the computed total risk.

KEYWORDS: Seismic Reliability, Probability of Failure, Intensity Measure, RC Structures

1. INTRODUCTION

Particularly in seismic area, designing or assessing, respectively, new or existing buildings requires a full approach based on statistical fundamentals to assess the probability of exceeding a specified limit state. Performance-Based Earthquake Engineering (PBEE, Moehle and Deierlein, 2004) makes use of the classical definition of exceeding a limit state and introduces a set of cascading mutual-correlated random variables. Under the assumption of Poissonian occurrences, structural performance is expressed by the probability of exceeding a given achievement of those random variables. According to PBEE, Probabilistic Seismic Demand Analysis (PSDA, Bazzurro, 1998) combines the random variables ground motion intensity measure (IM) and engineering demand parameter (EDP) to express the seismic performance of structures by an integral on IM. PSDA represents a direct measure of structural performance because it is related to the probability of experiencing the event EDP is greater than a given value *edp* within the life-time of a structure. With structural capacity information, PSDA is used to compute the annual probability of exceeding a specified limit state (e.g., the collapse) through a further integral on EDP. This probability, which corresponds to the total risk, can be expressed as:

$$P_f = \int_{edp}\int_{im} P\left[\mathrm{LS}\middle|\mathrm{EDP}=edp\right]P\left[\mathrm{EDP}>edp\middle|\mathrm{IM}=im\right]\left|d\lambda_{\mathrm{IM}}(im)\right| \tag{1.1}$$

where $d\lambda_{\mathrm{IM}}$ is the differential of the ground motion hazard curve in terms of IM, and it is found by conventional Probabilistic Seismic Hazard Analysis (PSHA, McGuire, 2004).

The concept of a single limit-state function is not adequate to describe the state of most realistic structural systems. This is true even for simple elements, but it is not immediately clear how to deal with different failure modes simultaneously. It is obvious that the number of failure modes increases fast with the complexity of the structure, whose state depends on the states of its elements. In particular for RC structures, the compounding of the unavoidable scatter of the material properties and the uncertainty about the real structural configuration (i.e., geometric size of elements, amount and details of reinforcement, etc.) and the multiplicity of the possible failure modes (i.e., shear failure, joint failure, brittle fracture of concrete, etc.) represent a main topic. Anyway, the whole process of reliability assessment is affected by the choice in terms of IM. Primarily, this variable influences the hazard at the site; secondarily, it is usual to assume IM as scale factor (SF) of recorded ground motion when PSDA is performed through nonlinear incremental dynamic analysis (IDA, Vamvatsikos and Cornell, 2002). One of the most important properties of an IM is its "efficiency", which represents the relatively small variability of structural responses for a given IM level. A reduction in the variability reduces the number of records needed to achieve an accurate estimate of the mean of EDP given an IM level, and thereby reliable PSDA results.

Considering the limit state of collapse and evaluating the probability of failure by Eqn. 1.1, the aim of this work consists in testing the flexibility of the approach using different scalar elastic-based ground motion IMs and comparing the reliabilities in order to study the influence of IM on P_f. With regard to these aspects, a set of RC frame structures located in a city characterized by high seismicity is considered. The structures have been designed using the capacity design criteria and Eurocode 8 (CEN, 2003) provisions.

2. GROUND MOTION INTENSITY MEASURES

The topic concerning which ground motion IM is advisable to use in seismic reliability analysis via PSDA represents a main task for earthquake engineering community and currently it captures great attention. The concept of ground motion IM has been recognized also in Eurocode 8 (CEN, 2003), which defines the hazard at the site by the recorded peak ground acceleration (PGA) at stiff soil with a 10% probability of exceedance, i.e., with a return period equal to 475 years. In fact, peak ground values represent probably the easiest solution for the IM topic in PSHA and could be effective for those structures where the fundamental period lies in spectral regions sensible to the corresponding kinematic parameters. In fact, for spectral periods under 0.5 sec, structural responses are most directly related to ground acceleration; for spectral periods between 0.5 and 3.0 sec, pseudo-velocity may be considered as constant and structural responses are better related to ground velocity than to other ground motion parameters.

In PSDA, the most used IM is the pseudo-spectral acceleration at the fundamental period of structure $T^{(1)}$ with damping ratio ξ equal to 5%, $S_a(T^{(1)}, \xi)$, or briefly S_a. In the past, this parameter was widely used because national geological survey offices produced the hazard curves in terms of S_a for each earthquake-source. Particularly for structures dominated by first-mode, several studies have shown that S_a is more "efficient" than PGA, i.e., the variability of structural response given a S_a level is smaller than with PGA (Shome et al., 1998). Basically, the reason is that the single value of S_a does not account for the spectral shape, so that the structural response is strongly dependent on recorded ground motion characteristics (e.g., moment magnitude, site-to-source distance, ε parameter). It has been shown that S_a is strongly "insufficient" for structures with long fundamental period (e.g., bridges) or high-rise buildings.

In order to consider the spectral shape related to recorded ground motions, vector-valued ground motion intensity measures can be considered. For example, combining S_a with ε parameter, Baker and Cornell (2005) have shown that vector-valued IMs are better than scalar-valued IMs in improving structural response prediction. However, using a vector-valued IM in Eqn. 1.1 requires a vector-valued PSHA to obtain the joint hazard curve, which has not been commonly applied, or a conventional seismic disaggregation analysis. In proximity of fault lines or surfaces, recorded ground motions can present the so-called "pulse-like effect", i.e., the forward-directivity induces velocity discontinuities which may cause relatively severe elastic and inelastic responses in structures with certain periods. Since vector-valued IMs sometimes are inefficient and insufficient, inelastic spectral values were chosen as advanced IM. Tothong and Luco (2007) have used the inelastic spectral displacement S_{di} computed on nonlinear equivalent single-degree-of-freedom (SDF) systems with fundamental period $T^{(1)}$ and yielding displacement δ_y. For multi-degree-of-freedom (MDF) systems, pushover analysis can be used to define the SDF system with equivalent period and equivalent yielding displacement. As usual, in order to conduct PSDA using S_{di}, PSHA requires a specific attenuation law for inelastic spectral displacement.

Especially for practical applications, the difficulties in working with vector-valued or inelastic IMs can be a barrier which is hard to overcome. Structural response of MDF or inelastic systems is sensitive to multiple periods T_i, so an intensity measure which averages elastic spectral acceleration values over a certain range of periods might be a useful and convenient predictor of structural response of inelastic systems. This concept was already anticipated in federal provisions (ICC, 2006), although it is more of a rough guide based on design spectrum to choose records rather than to define predictors. Many codes states that the ordinates of the response spectra for the suite of motions should be not less than those of design response spectrum for periods ranging from $0.2T^{(1)}$ to $1.5T^{(1)}$ (ASCE, 2005). Bianchini (2008) showed the effectiveness of $S_{a,avg}(T_1,\ldots,T_n)$, or briefly, $S_{a,avg}$, as IM in PSDA. This new IM has been defined as the geometric mean of the spectral acceleration ordinates at a set of n periods, and it is applied to demand assessment of inelastic MDF systems. Furthermore, an attenuation law for $\ln[S_{a,avg}]$ can be easily developed by using existing ground motion prediction models which provide information for $\ln[S_a(T_i)]$ and by performing an average of the n regression coefficients in the range of periods $T_1,\ldots,T_n$.

3. ASSESSMENT OF PROBABILITY OF FAILURE FOR RC STRUCTURES

3.1. Basic procedure

Direct Probabilistic Seismic Analysis (DPSA, Jalayer 2003) is here assumed as direct procedure to assess the probability of failure, because it expresses in a closed form the classical integral time-independent formulation of reliability problems under a few established assumptions. Statistical basis of the method can be found in Cornell et al. (2002). Performance objective (i.e., the collapse) is quantified through the annual probability that the random variable "demand", D, exceeds the random variable "capacity", C, both indentified by selecting an appropriate EDP. D and C are distributed following a lognormal probability density function, as suggested by Shome et al. (1998). The first two moments of structural response random variables are called "median", D_m and C_m, and "dispersion", β_D and β_C, and they are respectively computed as the mean and standard deviation of natural logarithm of D and C. Failure occurs when the maximum of the demand exceeds the correspondent value of the capacity in the period of time matching the length of seismic event. In the range of values in the region of hazard levels in the proximity of the limit state probability, hazard curve λ_{IM} is approximated by a power law on im with exponent $-k_1$ times a factor k_0. Similarly, the predicted conditional median demand D_m is approximated by a power law on im with exponent b times a factor a. Factors and exponents are computed by a linear regression analysis on a log-log plot. In order to complete the probabilistic representation of the demand given an IM, it is necessary to assess also its dispersion, $\beta_{D|IM}$, which can be computed by regression analysis using IDA. From a general point of view and referring to the limit state of collapse, the time-independent reliability formulation shown in Eqn. 1.1 using DPSA assumes the following expression:

$$P_f = k_0 \left(\frac{C_m}{a}\right)^{-\frac{k_1}{b}} \exp\left[\frac{1}{2}\frac{k_1^2}{b^2}\left(\beta_{D|IM}^2 + \beta_C^2\right)\right] \tag{3.1}$$

3.2. Application to RC elements

In the original formulation, DPSA has been applied only for steel moment resisting frames, where demand and capacity are measured by a single EDP, i.e., the maximum inter-storey drift angle (i.e., the largest inter-storey drift over time over the structure). For RC structures, it is necessary monitoring a quite large number of possible critical elements and failure for each of them in order to obtain a reliable evaluation of total risk. A major simplifying assumption considers failure mechanisms as statistical independent events, providing an upper bound to the total risk. Under this assumption, if the system has no redundancy, failure of any of its components will imply failure of the systems itself. In this case, the structure will be considered as a "series system" of critical n_e elements, and the partial probability of failure related to the *j*-th mechanism is given by:

$$P_{f,j} = 1 - \prod_{i=1}^{n_e} \left[1 - P_{f,j}(i)\right] \tag{3.2}$$

where $P_{f,j}(i)$ is the basic probability of failure for the *j*-th generic mechanism and the *i*-th monitored element, which can be evaluated by Eqn. 3.1. If several mechanism are investigated simultaneously and the structure can be considered as a series system, the total probability of failure is evaluated for series elements as:

$$P_f = 1 - \prod_{j=1}^{n_m} \left[1 - P_{f,j}\right] = 1 - \prod_{j=1}^{n_m} \prod_{i=1}^{n_e} \left[1 - P_{f,j}(i)\right] \tag{3.3}$$

where n_m is the total number of considered mechanisms. Sometime, the bounds provided by Eqn. 3.3 can be quite conservative: in fact, further considerations could be done in terms of mutual correlation of different mechanisms or between different elements. However, the core method has been calibrated for a single mode of failure, and cannot be strictly extended to multiple-correlated modes without substantial changes in the approach and increase in complexity.

3.3. RC resisting mechanisms

In case of RC structures designed according to Eurocode 8 (CEN, 2003) and capacity design criteria, the limit state of collapse should be characterized by ductile mechanisms (e.g., flexural member) rather than brittle ones, as the shear mechanism. Anyway, two different mechanisms have been considered here: shear collapse, V, and failure associated to exceeding a certain level of chord rotation, θ, which is determined as the ratio between the inter-storey drift and the inter-storey height.

In a reliability-based procedure, structural capacities have to be expressed in statistical terms. As anticipated, the inelastic mechanisms capable of leading a RC structure to collapse are identified in terms of median and dispersion. About shear strength, as expressed by Priestly (1997), the scatter between experimental and predicted values is quite modest, thus the indicated value of the coefficient of variation for shear mechanism, δ_V, is about 0.13. Concerning chord rotation mechanism, Panagiotakos and Fardis (2001) state that the ratio of experimental values to the predictions of chord rotation mechanism has a coefficient of variation, δ_θ, of 0.28.

4. RC PLANE FRAME STRUCTURES

RC buildings are considered located in the city of Catania, in Southern Italy, which is a zone characterized by high seismicity. Sample frames are shown in Fig. 1, which shows three bays and one floor (3B1F), three bays and three floors (3B3F) and three bays and six floors (3B6F) RC frame buildings. The bay-width is set to 6.00 m and storey-height to 3.20 m. The structures have been designed according to Eurocode 8 (CEN, 2003) by considering first category seismic zone (PGA equal to 0.35 g) and soil type B, which is consistent with geology of area under study. The high ductility class has been selected for defining the behaviour factor (equal to 5.85) and the detailing rules. The considered material properties are: concrete C25/30 and reinforcing steel with characteristic yield strength equal to 430 MPa. The gravity and live loads are, respectively, 30 kN/m and 12 kN/m. All beams are characterized by the same rectangular cross-section, with depth equal to 600 mm and width equal to 350 mm, whereas cross-sections of columns are shown in Fig. 1.

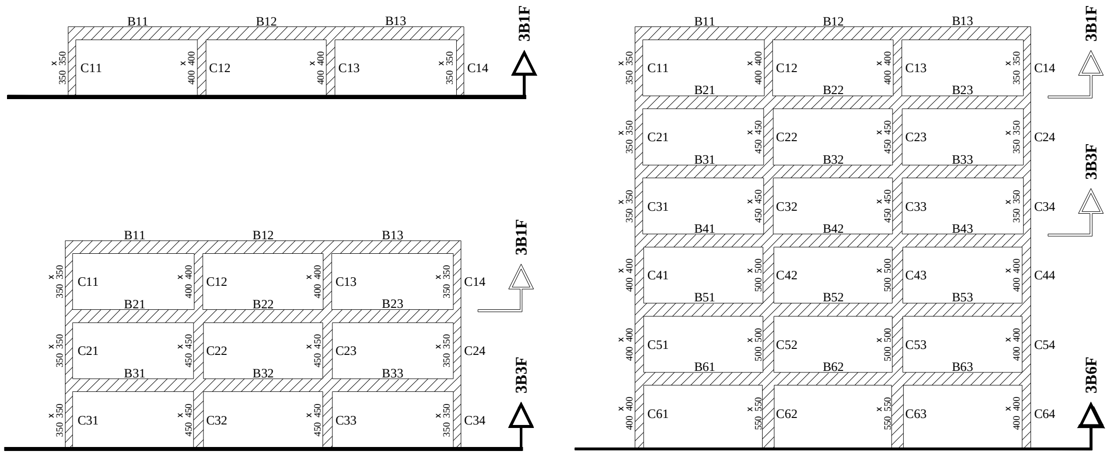

Figure 1 Front view of RC frame structures (cross sections are in mm)

Each element is identified by an abbreviation: letter "C" is referred to columns, letter "B" is referred to beams, first number is the level of floor from the top and the second one is the number of element from the left side. Both columns and beams have stirrup bars diameter equal to 8 mm and stirrup spacing equal to 80 mm. For structural elements, the secant stiffness at yield has been considered in the analytical model and modal dynamic analyses have been carried out to estimate the elastic fundamental period of structures: $T^{(1)}$ = 0.372 s for 3B1F; $T^{(1)}$ = 0.839 s for 3B3F; $T^{(1)}$ = 1.427 s for 3B6F.

PSHA for the city of Catania has been performed on the basis of the new Italian Parametric Catalogue. In order to link a generic IM at the site to the ground motion parameters (e.g., magnitude, distance, site geology, etc.), the law of Sabetta and Pugliese (1996) has been assumed here as empirical ground motion prediction model. Table 4.1 shows the coefficients of regression analysis of the hazard curves in terms of PGA, PGV, $S_a(T^{(1)})$ and $S_{a,avg}(0.2T^{(1)},\ldots, 1.5T^{(1)})$. Finally, it has been assumed that the earthquake source has a circular form with external and internal radius set to 35 and 15 km. Since buildings can be hypothetically located everywhere in the area, it has been supposed to treat an area surrounding the site adopting uniform probability of occurrence of the event.

Table 4.1 Coefficients of regression analyses for hazard curves in terms of the selected IMs

coefficients	PGA	PGV	$S_a(T^{(1)})$			$S_{a,avg}(0.2T^{(1)},\ldots, 1.5T^{(1)})$		
			3B1F	3B3F	3B6F	3B1F	3B3F	3B6F
k_0	0.000	0.234	0.148	0.018	0.006	0.281	0.041	0.011
k_1	1.956	1.491	0.868	1.169	1.158	0.641	0.137	1.157

5. PROBABILISTIC SEISMIC DEMAND ANALYSIS

In order to perform IDA, eight earthquake records have been selected from PEER database for soil type B. These records have been adopted as representative events in the Mediterranean Sea Basin. They are characterized by a moment magnitude varying from 5.8 to 7.5, and a variable site-to-source distance. Nonlinear dynamic analyses have been performed by assigning the median values of the material strength, which are computed from characteristic ones. Nonlinear dynamic analyses of RC frames have been performed by repeating the application of each earthquake record with increasing values of IM. In particular, for each record, fifteen values of spectral acceleration ranging from 0.1 g to 1.5 g have been considered; altogether, for each frame 120 nonlinear time-history analyses have been carried out. The nonlinear finite element code proposed by Diotallevi and Landi (2000) has been used to carry out the analyses. The code is based on a spread plasticity model and on a moment-curvature law that incorporates strength and stiffness deterioration under cyclic loading and the effects of changing axial forces. For each seismic analysis and for each element, the maximum value of demand in terms of ratio of shear demand versus shear supply and chord rotation has been evaluated.

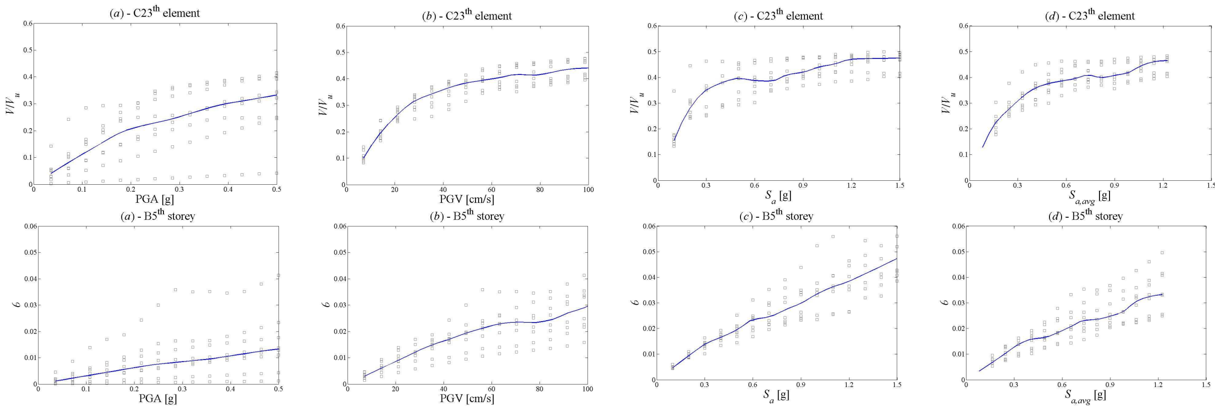

Figure 2 IDA curves results and median demand versus (*a*) PGA, (*b*) PGV, (*c*) S_a, and (*d*) $S_{a,avg}$ for shear (upper) and chord rotation mechanisms (lower) for 3B6F frame

As example, the results from IDA for C23-th element and B5-th storey in 3B6F frame are shown in Fig. 2 as a function of the four selected IMs, respectively, for shear and chord rotation mechanisms. Here, the points represent the numerical output and the bold line the interpolation of median demand. These two particular members for 3B6F frame are selected because, as probabilistic evaluation will display afterwards, they show the largest single probability of failure, hence the strongest influence on the partial performance evaluation of considered mechanism when IM coincides with S_a. In a similar way, for 3B1F frame the highest probability of collapse has been obtained for the C12-th element and B1-st storey, whereas for 3B3F frame it has been obtained for the C33-th element and B3-rd storey.

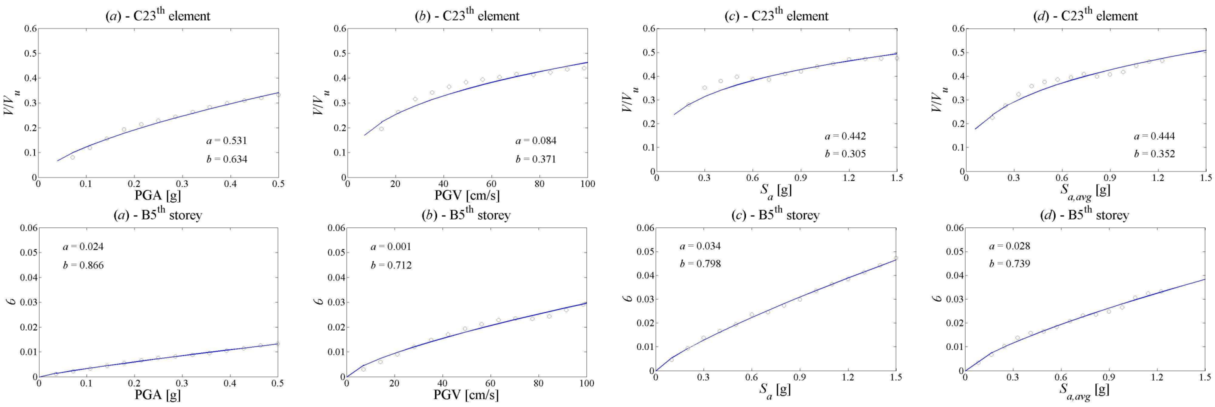

Figure 3 Predicted median demand versus (*a*) PGA, (*b*) PGV, (*c*) S_a, and (*d*) $S_{a,avg}$ for shear (upper) and chord rotation mechanisms (lower) for 3B6F frame

It is possible to observe that the selected IM can affect the dispersion in IDA curves. Using PGA as IM in 3B6F frame, results show a significant rise of the dispersion compared to other IMs. Instead, using S_a and $S_{a,avg}$, results show a relatively small and comparable level of dispersion. However, a general trend to larger level of dispersion for increasing values of IM can be observed for all IMs. Furthermore, the slope of IDA curves for shear mechanism decreases when IM-level increases, as well as the level of dispersion, because, corresponding to larger IM-level, the plasticity of structural members limits the shear value. On the other hand, IDA curves for chord rotation mechanism are characterized by increasing values for increasing IM levels, with no significant modifications in their slope. Fig. 3 shows the interpolation and the regression coefficients *a* and *b* respectively for shear and chord rotation mechanisms.

The tendency to increase the dispersion of demand with IM-level can be better emphasized in Fig. 4, which shows the dispersion of shear and chord rotation mechanisms as a function of IM for the C23-th element and B5-th storey of the 3B6F frame. These figures show also a linear fitting between dispersion and IM of the IDA curves using the least squares method. It should be noticed that dispersion levels associated to PGA as IM are larger than others. Furthermore, the dispersion associated to shear mechanism decreases when the IM-level increases. With regard to chord rotation mechanism, the level of PGA and PGV does not affect the dispersion. When structural dependent IM, and especially S_a, are used, this dispersion is lower but it is characterized by a tendency to increase.

The single probability of failure for the *i*-th monitored element and for the *j*-th considered mechanism, $P_{f,j}(i)$, has been evaluated by means of Eqn. 3.1 and subsequently combined through Eqn. 3.2 to obtain the partial probability of failure for the *j*-th mechanism, $P_{f,j}$. Finally, the values of $P_{f,j}$ of the two considered mechanisms have been combined according to Eqn. 3.3 in order to obtain the total probability of failure, P_f . The probabilities of collapse are illustrated in Fig. 5. It should be noticed that the failure associated to chord rotation mechanism has more influence on the total risk than the shear mechanism, as it can be expected for RC structures designed for high ductility. This result becomes more evident for structures characterized by high fundamental periods.

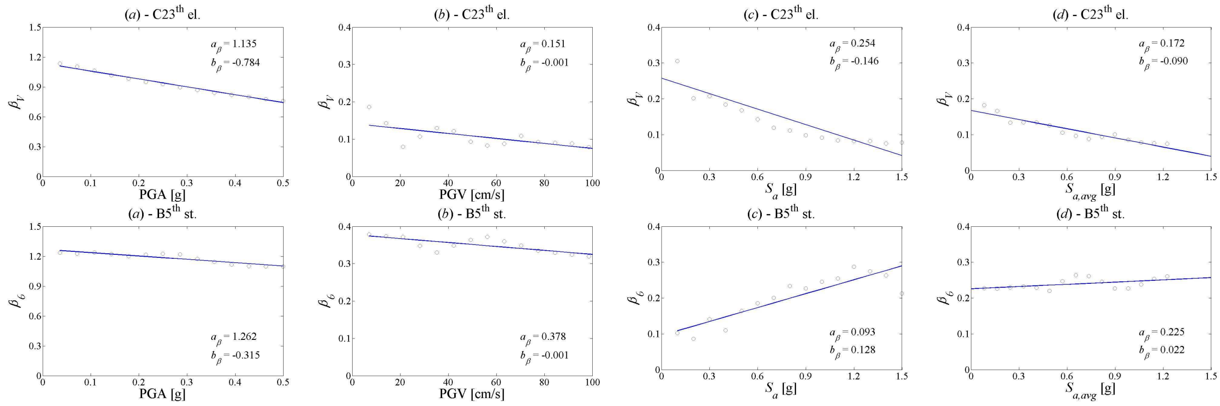

Figure 4 Dispersion of demand versus (*a*) PGA, (*b*) PGV, (*c*) S_a, and (*d*) $S_{a,avg}$ for shear (upper) and chord rotation mechanisms (lower) for 3B6F frame

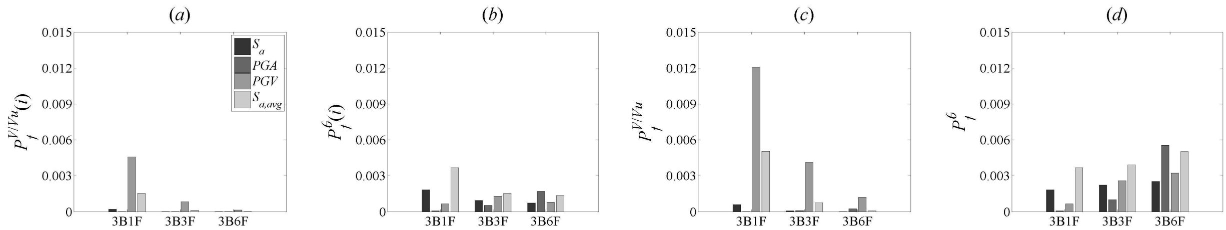

Figure 5 Probabilities of collapse in terms of each failure mechanism, frame structure and IM: maximum value for (*a*) shear mechanism and (*b*) chord rotation mechanism; partial value for (*c*) shear mechanism and (*d*) chord rotation mechanism

6. CONCLUSIONS

A full probabilistic methodology was tested for seismic design and evaluation of structures and applied to a set of RC buildings. This approach combined both probabilistic seismic hazard and demand analysis to obtain a measure of total risk in closed form for a given structure, at a certain site. Buildings were considered located in a region in southern Italy with high level of seismicity. The hazard curves were determined for this region and for each frame. Hazard analyses were carried out with regard to hazard aleatory variability, which is implicit in the choice of predictive model. Demand values were obtained through nonlinear IDA: statistical parameters in terms of median and dispersion were derived for different levels of IM. Shear and chord rotation mechanisms were chosen as EDP, considering adequate capacity models, by mechanical and statistical point of view.

The final result in terms of total risk matches expectations for new buildings designed according to Eurocode 8 (CEN, 2003), since brittle mechanism did not affect in relevant way the global value of probability of failure. Assuming failure mechanisms for RC elements as independent statistical series, the approach presented here showed a good statistical efficiency, even from computational point of view. In conclusion, the relative workability of the presented probabilistic approach makes it a useful tool for seismic assessment of RC structures in PBEE framework.

Concerning the effect of IM on total risk, it should be observed that the probabilities of collapse are strongly affected by the hazard curves, which in turn depends on IM, and the dispersion, which is conditioned to the choice of IM. Both hazard and IDA change in terms of IM. The probability of collapse in a closed form depends to hazard and dispersion, hence to IM. Using PGA, the probability of collapse increases from stiff-to-flexible systems, whereas using S_a it remains almost constant. In examined cases a larger variability of the results using different IMs was obtained for structures with lower number of storeys (or alternatively with shorter fundamental period). The variability of results seemed to be related also to some aspects of the procedure, as the properties of efficiency and sufficiency of the intensity measures and the assumptions related to the statistical correlation between mechanisms and elements.

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