FLEXURAL DUCTILITY ASSESSMENT AND CONCURRENT FLEXURAL STRENGTH AND DUCTILITY DESIGN OF REINFORCED CONCRETE BEAMS

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ABSTRACT :

Due to its higher strength-to-weight ratio, high-strength concrete is increasingly adopted in the construction of tall buildings and long span bridges. However, apart from better utilising its strength potential, there was only little attention paid to the ductility design of HSC members. Currently, the ductility design of HSC beams rely on some deemed-to-satisfy rules to provide a nominal level of ductility. The main drawback of this method is that it would not provide the same level of ductility, and most importantly much lower ductility, to HSC beams. This is dangerous because the lower ductility in beams would disallow the moment redistribution to occur during earthquake, and eventually lead to brittle collapse. In this regard, a new method that would enable the design of HSC beams with a minimum ductility not less than that provided in the past for normal-strength concrete beams is advocated in this paper. With this fixed minimum ductility set as a nominal requirement in the beams design, a maximum limit of tension steel ratio or neutral axis depth is imposed. These limits are subsequently evaluated using a new method of nonlinear moment-curvature analysis taking into account the stress-path dependence of steel reinforcement. The associated flexural strength that can be designed while achieving the recommended minimum ductility are also evaluated and presented in the form of design charts for practical application.

KEYWORDS: beams, ductility, flexural strength, high-strength concrete

1. INTRODUCTION

In recent years, because of the obvious advantages arising from its significantly higher strength to weight ratio, high-strength concrete (HSC) is increasingly being adopted in the design of tall buildings and long span bridges. However, the design guidelines developed for HSC focused only on how to utilise the strength of HSC, but did not paid particular attention to the flexural ductility provision. It should be noted that the flexural ductility of HSC beams could become very critical if the same design rules of NSC members are adopted without modification to cater for the use of HSC. For example, it has been demonstrated that HSC beams could fail in a very brittle manner even if they are under-reinforced with the tension steel provided close to the balanced steel area [1,2]. This obviously states that the method of ductility design in NSC and HSC members should be treated differently.

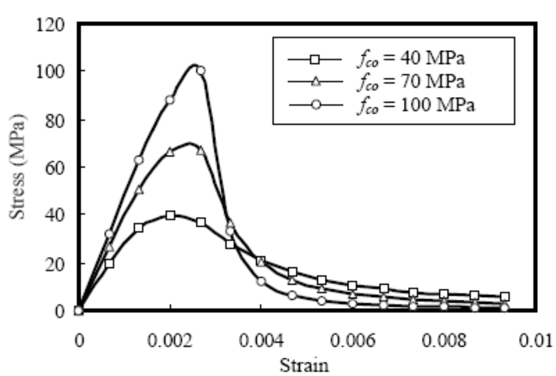
From structural safety point of view, ductility should be regarded as important as strength. Adequate ductility could allow moment redistribution to occur in the beams and prevent immediate collapse during earthquake attack [3] or accidental impact. Therefore, the design principle of strong columns and weak beams is not violated. However, although the flexural ductility is commonly accepted to be an important design parameter, the method of flexural ductility design has not been well specified in the existing design codes, particularly for the design of non-earthquake resistant structures. This is because the evaluation of ductility is not as straightforward as strength. To evaluate ductility, a nonlinear moment-curvature analysis extended into the post-peak range is required, which should also take into account the stress-path dependence of steel reinforcement [4] in addition to the stress-strain curves of the concrete and steel. Because of such complexity, the existing beams design relies on some empirical deemed-to-satisfy rules (by limiting the maximum tension steel area or neutral axis depth) for provision of a nominal level of ductility. However, since these rules are independent on concrete strength, it does not guarantee the provision of the same level of nominal ductility for beams constructed of HSC [5,6].

Since the flexural ductility of beams depends heavily on the concrete strength [1-6], the use of existing deemed-to-satisfy rules derived many years ago based on the behaviour of NSC would render a significantly lower level of flexural ductility provided to HSC beams. It would also cause the design of beams to have a wide range of available flexural ductility depending on the design concrete strength. Therefore, the existing deemed-to-satisfy rules are not conservative in the sense of ductility design. With a view to preserving the same level of flexural ductility in HSC beams, a new concept of providing an absolute minimum level of flexural ductility in the design of beams is advocated in this paper. This minimum level of flexural ductility should not be less than the nominal level of ductility that has been provided in the past to NSC beams, even for beams not subjected to earthquake attack. For the design of beams that would be subjected to earthquake attack, a larger value of the minimum flexural ductility could be assigned depending on the actual demand.

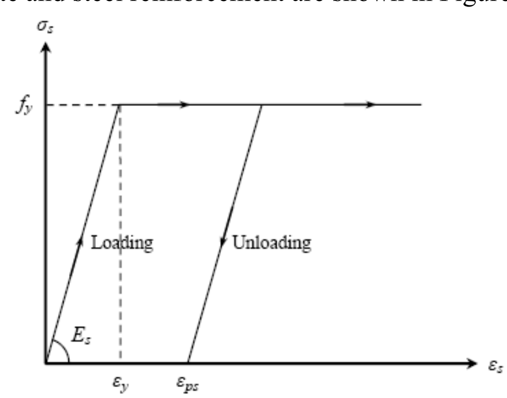
However, upon imposing the minimum ductility to be a nominal requirement of HSC beam design, it would automatically impose a maximum limit on the steel ratio or neutral axis depth. These maximum values would also limit the maximum flexural strength that could be achieved for each of the concrete strength. Therefore, under such circumstance, the design of HSC beams to achieve a pair of flexural strength and ductility requirement could only be done in an iterative approach. To enable a more rapid design that could simultaneously achieve a pair of specified strength and ductility requirements, some design charts plotting the relationship between ductility and strength for concurrent strength and ductility design in HSC beams are developed.

2. NONLINEAR MOMENT-CURVATURE ANALYSIS

A new method of nonlinear moment-curvature analysis has been developed for the flexural ductility analysis of HSC beams [4]. It takes into account the constitutive stress-strain curve of concrete and steel as well as the stress-path dependence of the longitudinal steel reinforcement. The unconfined and confined concrete stress-strain curves developed by Attard and Setunge [7], which are applicable to concrete strength from 20 to 130 MPa, are adopted. For longitudinal and confining reinforcement, the idealised linearly elastic – perfectly plastic stress-strain curve is adopted. The adopted stress-strain curves of concrete and steel reinforcement are shown in Figure 1.



(a) Stress-strain curves of concrete



(b) Stress-strain curve of steel with stress-path dependence considered

Figure 1 Stress-strain curves of concrete and steel reinforcement

The following assumptions have been adopted in the analysis: (1) plane sections before bending remain plane after bending; (2) the tensile strength of the concrete may be neglected; (3) there is no bond-slip between the concrete and steel reinforcement; and (4) the concrete core is confined while the concrete cover is unconfined. The moment-curvature behaviour of the beam section is analysed by applying prescribed curvatures to the section incrementally starting from zero. At a prescribed curvature, the stresses developed in the concrete and the steel are determined from the strain profile and their respective stress-strain curves. Then, the neutral axis depth and resisting moment are evaluated from the axial and moment equilibrium conditions respectively. The above procedure is repeated until the resisting moment has increased to the peak and then decreased to 50% of the peak moment.

3. FLEXURAL DUCTILITY ANALYSIS AND FAILURE MODES

3.1. Flexural Ductility Analysis

The flexural ductility of a beam section may be expressed in terms of the curvature ductility factor μ defined by Watson and Park [8] as:

$$\mu = \phi_u / \phi_y \tag{1}$$

where ϕ_u and ϕ_y are the ultimate and yield curvatures, respectively. The ultimate curvature ϕ_u is taken as the curvature when the resisting moment has dropped to 0.8 M_p after reaching M_p, where M_p is the peak moment. The yield curvature ϕ_y is taken as the curvature at which the peak moment M_p would be reached if the stiffness of the section is equal to the secant stiffness at 0.75 M_p.

Based on the above definition, a parametric study on the effects of various factors on the curvature ductility factor of beams has been conducted. The beam sections analysed in the parametric study are shown in Figure 2. The concrete strength f_{co} was varied from 40 to 100 MPa, the tension steel ratio ρ_t was varied from 0.4 to 2 times the balanced steel ratio, the compression steel ratio ρ_c was varied from 0 to 2%, the confining pressure f_r provided by the confinement was varied from 0 to 3 MPa, and the steel yield strengths are 250, 460 and 600 MPa.

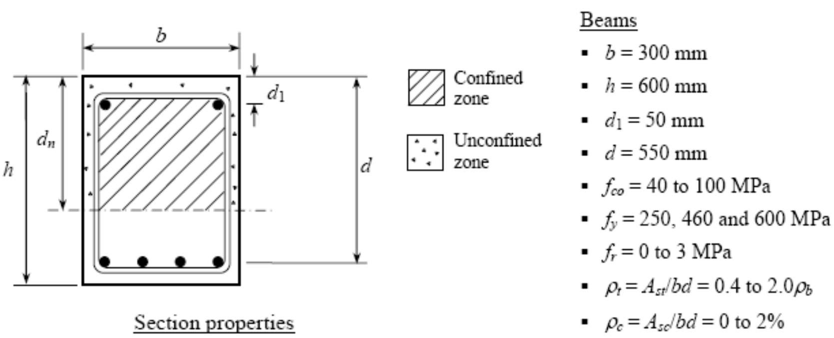


Figure 2 Beam sections analysed

3.2 Failure Modes

Three failure modes are observed: (1) tension failure, in which the tension steel yields during failure; (2) compression failure, in which none of the tension steel yields during failure; and (3) balanced failure, in which the tension steel just reaches the yield stress during failure. Tension failure occurs when the tension steel is less than the steel area that leads to balanced failure, which is defined as the balanced steel area A_{sb}; while compression failure occurs when the tension steel is larger than the balanced steel area. The balanced steel area is usually expressed as a ratio to the effective area of the beam section, which is defined as the balanced steel ratio, denoted hereinafter by $\rho_b = A_{sb}/bd$. For general beam section with or without compression steel, ρ_b is given by:

$$\rho_b = \rho_{bo} + (f_{yc} / f_{yt})\rho_c \tag{2}$$

where ρ_c is the compression steel ratio, f_{yc} and f_{yt} are the yield strengths of the compression and tension reinforcement respectively, and ρ_{bo} is the steel ratio that causes balanced failure in beam section without compression reinforcement. The values of ρ_{bo} are listed in Table 1.

Table 1 Balanced steel ratios

f_{co} (MPa)	Balanced steel ratio ρ_{bo}				
	f_r = 0 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	3.95	4.97	5.82	6.54	7.29
50	4.69	5.74	6.62	7.39	8.13
60	5.39	6.46	7.36	8.16	8.97
70	6.06	7.14	8.07	8.90	9.66
80	6.70	7.78	8.71	9.55	10.34
90	7.30	8.38	9.32	10.16	10.97
100	7.87	8.93	9.86	10.72	11.53

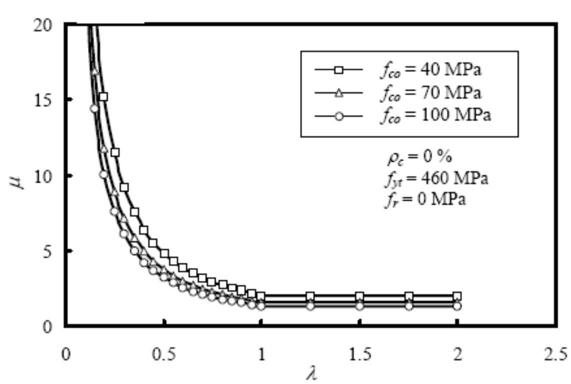
4. MINIMUM DUCTILITY DESIGN

4.1 Factors affecting ductility of beams

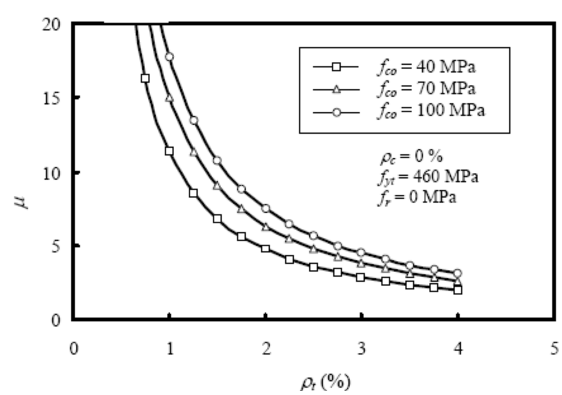
The major factors affecting the ductility of beams are the degree of reinforcement, concrete strength and confining pressure. The degree of reinforcement is denoted by λ, which can be expressed as:

$$\lambda = \frac{f_{yt}\rho_t - f_{yc}\rho_c}{f_{yt}\rho_{bo}} \tag{3}$$

where ρ_c and ρ_t are the ratios of compression and tension reinforcement respectively, f_{yc} and f_{yt} are the yield strength of compression and tension reinforcement respectively, ρ_{bo} is given by Equation (2). According to the above definition, λ is equal to 1.0 for balanced section, whereas λ is less than 1.0 for under-reinforced section, and larger than 1.0 for over-reinforced section. To investigate the effects of the degree of reinforcement, the value of μ is plotted against λ in Figure 3(a). It can be seen that the ductility decreases as λ increases until it reaches 1.0, after which the ductility remains constant. For the effects of concrete strength, it can be seen from the same figure that at a given λ, the ductility decreases as the concrete strength increases. However, if the value of μ is plotted against ρ_t as shown in Figure 3(b), it can be observed that the ductility increases at a given tension steel ratio as the concrete strength increases, albeit that HSC is more brittle *per se*. This is because as the concrete strength increases, the balanced steel ratio also increases and thus for a given tension steel, the degree of reinforcement is decreased and thereby leading to a higher ductility.



(a) Ductility plotted against degree of reinforcement



(b) Ductility plotted against tension steel ratio

Figure 3 Variation of ductility with concrete strength

For the effect of confining pressure f_r, the values of μ for are plotted against λ and ρ_t in Figures 4(a) and 4(b) respectively. It is evident that from Figure 4(a) that at a given λ, the flexural ductility increases significantly with the confining pressure. This is because the addition of confining pressure would increase the strength and ductility of the concrete core of the beam section. From Figure 4(b), it is also evident that at a given ρ_t, the provision of confining pressure could significantly increase the ductility of the beam section. This is because the balanced steel ratio increases as the confining pressure increases (see Table 1), and hence for a fixed tension steel ratio, the degree of reinforcement is reduced and thereby leading to a higher ductility. On the whole, it can be concluded that the addition of confining pressure is an effective means of improving the ductility of beams.

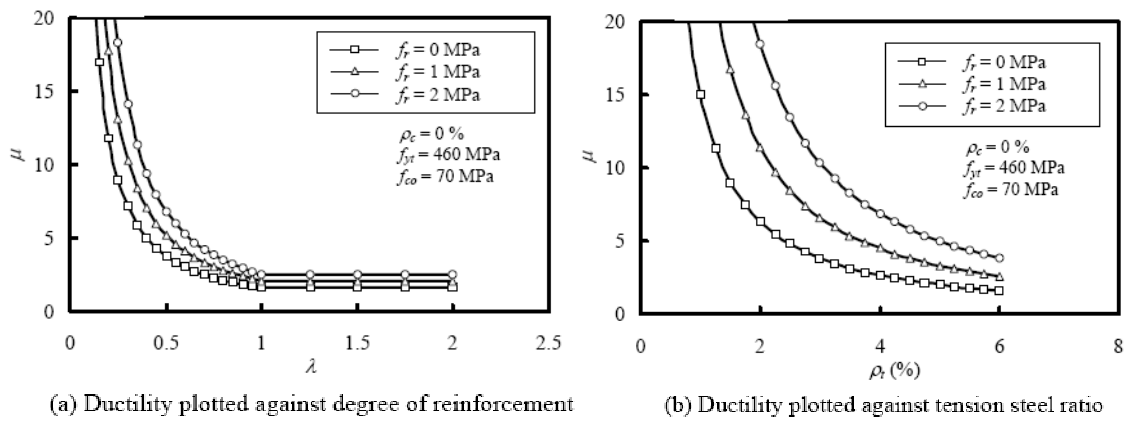


Figure 4 Variation of ductility with confining pressure

To enable a rapid evaluation of flexural ductility in beams, the values of μ plotted in Figures 3 and 4 are correlated to the studied parameters using regression analysis. The following formulas are thus obtained:

$$\mu = 10.7m\left(f_{co}\right)^{-0.45}\left(\lambda\right)^{-1.25n}\left(1+95.2\left(f_{co}\right)^{-1.1}\left(\frac{f_{yc}\rho_c}{f_{yt}\rho_t}\right)^3\right)\left(\frac{f_{yt}}{460}\right)^{-0.25} \quad (5a)$$

$$m = 1+2.5\sqrt{f_{co}}\,(f_r/f_{co}) \quad (5b)$$

$$n = 1+5(f_r/f_{co}) \quad (5c)$$

where f_{co} is in MPa and $\lambda \leq 1.0$, $250 \leq f_{yt}$ and $f_{yc} \leq 600$ MPa. Within the range of parameters studied, the above formula for μ is accurate to within 10% error.

4.2 Minimum ductility

In the existing design codes, there are no explicit design guidelines for the flexural ductility of beams. The design of beams needs only to comply with some empirical deemed-to-satisfy rules so that a nominal level of ductility could be provided. However, since the ductility of beams depends on the concrete strength, the resulting curvature ductility factors of beams designed according to the deemed-to-satisfy rules would vary significantly. It is of greater concern that the flexural ductility would drop to a very low level when ultra HSC is adopted. With a view to maintaining the flexural ductility even after using HSC, it is advocated that instead of following the deemed-to-satisfy rules, the beams should be designed with a minimum value of curvature ductility factor, denoted by $\mu_{\min}$. The value of $\mu_{\min}$ should not be less than the ductility provided in the past for NSC beams. From a series of study conducted by the authors [9,10], it was proposed to adopt $\mu_{\min}$ = 3.32 for the design of beams in non-earthquake resistant structures. For beams in earthquake resistant structures, a larger value of $\mu_{\min}$ could be assigned depending on the actual demand.

4.3 Maximum tension steel ratios and maximum neutral axis depth

Upon setting a fixed minimum curvature ductility factor of μ_{min} = 3.32 in the design of beams, it will automatically set up a limit on the maximum value of λ and maximum difference between tension and compression steel ratios denoted by $(\rho_t - \rho_c)_{max}$. The values of $(\rho_t - \rho_c)_{max}$ are evaluated using nonlinear moment-curvature analysis for each of the concrete strength and steel yield strength, which are listed together with the respective flexural strength that can be achieved in Table 2. From the table, it is clear that the use of HSC in beam design could allow a higher flexural strength to be achieved while maintaining μ_{min}, whereas the use of higher strength steel would only allow a smaller required steel area without increasing the maximum limit of flexural strength achievable at μ_{min}.

Table 2 Maximum steel ratios for minimum ductility design

f_{co} (MPa)	Maximum steel ratio $(\rho_t - \rho_c)$ (%)			Maximum M_p/bd^2 at ρ_c = 0 % (MPa)		
	$f_{yt} = f_{yc}$ = 250 MPa	$f_{yt} = f_{yc}$ = 460MPa	$f_{yt} = f_{yc}$ = 600 MPa	$f_{yt} = f_{yc}$ = 250 MPa	$f_{yt} = f_{yc}$ = 460MPa	$f_{yt} = f_{yc}$ = 600 MPa
40	6.63	2.67	1.76	12.86	10.24	9.05
50	7.32	2.93	1.92	14.63	11.49	10.07
60	7.92	3.15	2.05	16.18	12.55	10.90
70	8.46	3.35	2.18	17.57	13.51	11.71
80	8.95	3.53	2.29	18.83	14.37	12.40
90	9.39	3.69	2.38	19.97	15.14	12.98
100	9.79	3.89	2.47	21.01	15.82	13.55

The minimum ductility requirement would also limit the maximum neutral axis depth of the beams for each of the concrete strength and steel yield strength. The maximum values of neutral axis depth are evaluated using nonlinear moment-curvature analysis, which are listed in Table 3 as a ratio to the effective depth of beam, denoted by d_n/d. The flexural strength that can be achieved for each of the concrete strength and steel yield strength are also listed in Table 3. From the table, it is observed that the maximum neutral axis depth decreases substantially when the concrete or steel yield strength increases.

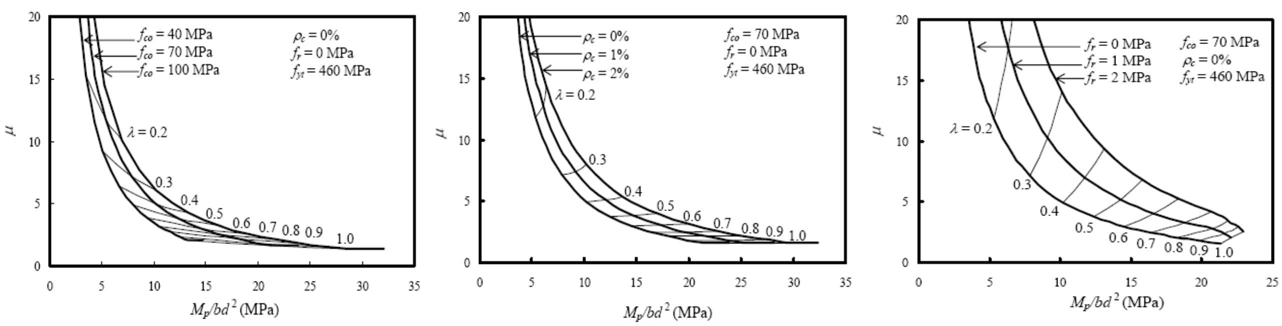
Table 3 Maximum neutral axis depth to effective depth ratio for minimum ductility design

f_{co} (MPa)	Maximum neutral axis depth to effective depth ratio (d_n/d)			Maximum M_p/bd^2 at ρ_c = 0 % (MPa)		
	$f_{yt} = f_{yc}$ = 250 MPa	$f_{yt} = f_{yc}$ = 460MPa	$f_{yt} = f_{yc}$ = 600 MPa	$f_{yt} = f_{yc}$ = 250 MPa	$f_{yt} = f_{yc}$ = 460MPa	$f_{yt} = f_{yc}$ = 600 MPa
40	0.570	0.428	0.371	12.86	10.24	9.05
50	0.523	0.391	0.338	14.63	11.49	10.07
60	0.489	0.364	0.314	16.18	12.55	10.90
70	0.462	0.343	0.295	17.57	13.51	11.71
80	0.441	0.327	0.281	18.83	14.37	12.40
90	0.423	0.313	0.268	19.97	15.14	12.98
100	0.409	0.302	0.258	21.01	15.82	13.55

5. CONCURRENT STRENGTH AND DUCTILITY DESIGN

The limits on either the maximum steel ratios or maximum neutral axis depth imposed by the requirement of satisfying the minimum ductility would also limit the flexural strength of the beam that can be achieved. Therefore, the design of beams satisfying both the minimum ductility and flexural strength requirement could only be carried out in an iterative manner. Normally speaking, structures should be designed with generous ductility but not strength. It is because an increase in the flexural ductility is always beneficial to the structures; however, an increase in the flexural strength would increase the shear demand and the flexural strength of the neighbouring columns. Therefore, it is important to establish a method that would allow the simultaneous design of flexural strength and ductility of beams without using an iterative approach.

Figure 5 shows a series of charts plotting the curvature ductility factor μ against strength M_p/bd^2, where M_p is the maximum resisting moment, for various concrete strengths, compression reinforcement ratio and confining pressure. From the figures, it is obvious that the use of HSC, addition of compression reinforcement and confining pressure would increase the maximum limit of flexural strength and ductility that could be achieved simultaneously. Therefore, these measures would increase the ductility of beam at a given strength, increase the strength at a given ductility, or increase both the strength and ductility. The charts shown in Figure 5 could also be employed as the design tools for designing beams to meet a pair of strength and ductility requirement.



(a) Various concrete strength (b) Various compression reinforcement (c) Various confining pressure

Figure 5 μ versus M_p/bd^2 for various concrete strength, compression reinforcement and confining pressure

A numerical example is given herein to illustrate how to obtain different design solutions using the charts as shown in Figure 5 to design a beam section satisfying the following pair of strength and ductility requirements: μ = 5.0 and M/bd^2 = 12.0. The first solution, being a beam section without compression reinforcement and confining pressure, can be obtained from Figure 5(a). By plotting (12.0, 5.0) on Figure 5(a), it is found that f_{co} = 100 MPa and λ = 0.38 and $\rho_t = 0.38\times7.87 = 3.0\%$. The second solution, being a beam section with compression reinforcement but without confinement, can be obtained from Figure 5(b). By plotting (12.0, 5.0) on Figure 5(b), it is found that ρ_c = 1%, f_{co} = 70 MPa, λ = 0.4 and $\rho_t = (0.4\times7.14)+1 = 3.9\%$. The third solution, being a beam section with confinement but without compression reinforcement, can be obtained from Figure 5(c). By plotting (12.0, 5.0) on Figure 5(c), it is found that f_r = 0.5 MPa, ρ_c = 0%, f_{co} = 70 MPa, λ = 0.45 and $\rho_t = 0.45\times6.6 = 3.0\%$. The choice of the final design from the above design options would depend on the engineering judgement taking into account the cost of construction and steel congestion problems at joint locations.

6. CONCLUSIONS

The flexural ductility of HSC beams has been studied by an extensive parametric study based on nonlinear moment-curvature analysis taking into account the stress-path dependence of steel reinforcement. From the study, it was evident that the major factors affecting the flexural ductility of beams are the degree of reinforcement λ, concrete strength and confining pressure. Generally, the ductility increases as λ decreases. However, the ductility decreases as concrete strength increases at a fixed λ, but increases at a fixed tension ratio as the concrete strength increases. On the other hand, the addition of confining pressure will improve the ductility at all values of λ and concrete strength. To enable rapid evaluation of ductility in beams, a formula was developed that correlates the curvature ductility factors to the above governing factors.

To better utilize the strength potential of HSC in beams without jeopardising the flexural ductility, a new concept of imposing a minimum flexural ductility in the beam design was advocated. For non-earthquake resistant structures, it was recommended that the minimum curvature ductility factor should not be less than 3.32. Based on this nominal requirement, it would impose limits on the maximum value of λ or neutral axis depth in beams. However, since these limits would also affect the maximum limit of strength that can be designed for, a set of design charts that enable simultaneous consideration of flexural strength and ductility were also developed. From the charts, it was evident that the use of HSC, addition of compression reinforcement and confining pressure, would increase the maximum limit of strength and ductility that could be achieved simultaneously.

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NOTATIONS

A_{sb}	Balanced steel area
A_{sc}	Area of compression reinforcement
A_{st}	Area of tension reinforcement
b	Breadth of beam section
d	Effective depth of beam section
d_n	Depth to neutral axis
E_s	Elastic modulus of steel reinforcement
f_{co}	Peak stress on stress-strain curve of unconfined concrete
f_r	Confining pressure produced by confining reinforcement
f_y	Yield strength of steel reinforcement
f_{yc}	Yield strength of compression reinforcement
f_{yt}	Yield strength of tension reinforcement
h	Total depth of the beam section
M_p	Peak moment
ε_{ps}	Residual plastic strain in steel reinforcement
ε_s	Strain in steel
ε_y	Yield strain of steel

λ	Degree of reinforcement
μ	Curvature ductility factor
$\mu_{\min}$	Minimum curvature ductility factor
ϕ_u	Ultimate curvature
ϕ_y	Yield curvature
ρ_b	Balanced steel ratio (= A_{sb}/bd)
ρ_{bo}	Balanced steel ratio for beam section with no compression reinforcement
ρ_c	Compression reinforcement ratio (= A_{sc}/bd)
ρ_t	Tension reinforcement ratio (= A_{st}/bd)
σ_s	Stress in steel reinforcement