THEORETICAL SOLUTION OF BOND SPLITTING STRENGTH OF RC USING PARABOLIC MODEL OF BOND CONSTITUTION

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ABSTRACT :

This study presents the theoretical solution of bond stress distribution along the reinforcement in concrete structure solving fundamental differential equation on bond problem with modeled bond stress – slippage relationship by parabolic curve. The modeled relationship has almost the same shape with previously proposed by the authors considering the equilibrium conditions of bond stress and splitting stress of concrete. By the theoretical bond stress solution, the bond strength is solved by the integration of the bond stress distribution. The previously proposed calculation method using the equivalent bond stress block (EBSB) is verified by the theoretical solution, and the calculated values of bond strength show a good agreement with theoretical ones. The relationship between bond strength and bond fracture energy can be also led by the theoretical solution.

KEYWORDS: bond splitting strength, bond stress, slippage, differential equation, fracture energy

1. INTRODUCTION

Simple differential equation of concerning the bond problem of reinforcement bar and concrete is expressed as the following. [1]

$$\frac{d^2 s}{dx^2} = \frac{1+np}{E_b \cdot a_b} \phi_b \cdot \tau_b \tag{1.1}$$

s : slippage
ϕ_b: perimeter of reinforcement
a_b: cross-sectional area of reinforcement
E_b: elastic modulus of reinforcement
τ_b: bond stress
n : elastic modulus ratio of reinforcement to concrete
p : reinforcement ratio

In Eq. (1.1), if the constitutive law of bond is given, where bond stress is expressed as the function of slippage, solving Eq. (1.1) under the arbitrary boundary condition gives the distribution of slippage, stress distribution of reinforcement bar and distribution of bond stress. Integrating the distribution of bond stress through arbitrary region gives the average bond stress by calculating the difference of tensile force of reinforcement bar divided by the surface area of the reinforcement. In addition, the maximum value of average bond stress gives the bond strength. However, it is difficult to solve Eq. (1.1) mathematically when bond constitutive law is given in complicated function form. Until now, the complete theoretical solution can be obtained by the bond constitutive law of linear function (constant model or bi-linear model with proportional and softening expression). [2] The authors proposed the constitutive law for the case of bond splitting failure without lateral reinforcement considering hollow-thick cylinder model subjected to inner pressure. However, it is difficult to solve Eq. (1.1) mathematically using this constitutive law with complex functional form. The numerical analysis is the only way to obtain average bond stress or bond strength. The authors have proposed an easier way to obtain bond strength by introducing Equivalent Bond Stress Block (EBSB) which expresses the equivalent area of bond stress distribution at the maximum tensile force. However, bond strength obtained by EBSB is not equal to the area which is given by mathematical solution.

On the other hand, bond strength is given by Eq. (1.2) using bond fracture energy G_{fb}. Eq. (1.2) can be adopted for the case of long enough bond length and high stiffness of concrete compared with the stiffness of the reinforcement material (steel plate). [3]

$$P_{max} = b_s \cdot \sqrt{2 \cdot G_{fb} \cdot E_s \cdot t_s} \tag{1.2}$$

P_{max}: bond strength
b_s : width of plate （bond area per unit length）
G_{fb}: bond fracture energy
E_s: elastic modulus
t_s : thickness of plate

The following equation can be obtained by replacing bond area per unit length to perimeter of reinforcement bar and cross sectional area of reinforcement material to cross sectional area of reinforcement bar.

$$P_{max} = \sqrt{2 \cdot G_{fb} \cdot E_b \cdot a_b \cdot \phi_b} \tag{1.3}$$

From the simple consideration, it can be shown that bond strength led by EBSB is equal to Eq (1.3) when bond length is long enough,. However, generally bond length in ordinary reinforced concrete member is finite. There are unclear points on the accuracy of bond strength by EBSB.

In this paper, a bond constitutive law with parabola which has similar shape with previously proposed constitutive law is proposed, then the theoretical solution is led by solving differential equation. The bond splitting strength calculated by EBSB method is confirmed by the theoretical solution. In addition, Eq. (1.3) is also verified theoretically.

2. MODELING OF LOCAL BOND STRESS– SLIPPAGE RELATIONSHIP BY PARABOLA

2.1. Proposed Local Bond Stress – Slippage Relationship

The authors had proposed the local bond stress – slippage relationship as shown in Eq. (2.1). [4]

$$\tau_b = 2 \cdot \sigma_t \cdot \beta \cdot s \cdot \frac{(r_u/d_b)^2 - (\beta \cdot s)^2}{(r_u/d_b)^2 + (\beta \cdot s)^2} \cdot \cot\alpha \tag{2.1}$$

τ_b: bond stress
σ_t: splitting strength of concrete
β : relationship between inner crack width and slippage = 10.2 (1/mm)
s : slippage
d_b: diameter of reinforcement bar
r_u: $C + d_b/2$ （C : concrete cover thickness）
α : angle between splitting force to axial direction = 34 degree

Figure 1 shows local bond stress – slippage curve by Eq. (2.1). Bond stress and slippage is standardized by maximum bond stress ($\tau_{b,max}$) and slippage at bond stress equals to 0 (s_u), respectively. $\tau_{b,max}$ and s_u are given by Eq. (2.2) and (2.3), respectively.. Slippage at the maximum bond stress (s_{max}) is given by Eq. (2.4).

$$\tau_{b,max} = \left(\sqrt{5}-1\right)\sqrt{\sqrt{5}-2}\cdot\sigma_t\cdot\frac{r_u}{d_b}\cdot\cot\alpha \tag{2.2}$$

$$s_u = \frac{r_u}{d_b\cdot\beta} \tag{2.3}$$

$$s_{max} = \sqrt{\sqrt{5}-2}\cdot s_u = 0.486\cdot s_u \tag{2.4}$$

2.2. Parabola Model

The local bond stress – slippage relationship expressed by Eq. (2.1) is modeled by parabola expressed by Eq. (2.5). The parabola model is also shown in Figure 1. The curve has a similar shape as Eq. (2.1).

$$\tau_b = -a\cdot s\cdot(s-s_u) \tag{2.5}$$

where, a is defined as follows.

$$a = \frac{4\cdot\tau_{b,max}}{s_u^2} \tag{2.6}$$

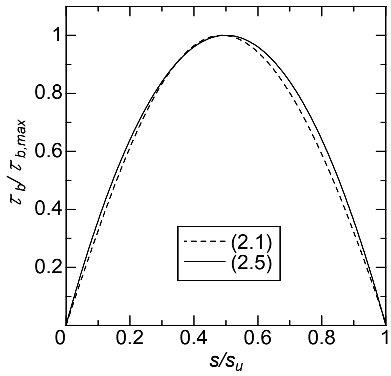

Figure.1 Local bond stress – slippage

3. THEORETICAL SOLUTION

3.1. Solution

Ordinary differential equation of second order expressed by Eq. (1.1) and (2.5) is solved. Firstly, ν_b is defined as follows.

$$\nu_b = \frac{1+np}{E_b\cdot a_b}\phi_b \tag{3.1}$$

Eq. (1.1) is substituted by Eq. (3.2).

$$\frac{d^2s}{dx^2} = \nu_b\cdot\tau_b \tag{3.2}$$

$$\therefore 2\frac{ds}{dx}\cdot\frac{d^2s}{dx^2} = 2\nu_b\frac{ds}{dx}\cdot\tau_b \tag{3.3}$$

$$\therefore \frac{d}{dx}\cdot\left(\frac{ds}{dx}\right)^2 = 2\nu_b\frac{ds}{dx}\cdot\tau_b \tag{3.4}$$

$$\therefore \left(\frac{ds}{dx}\right)^2 = 2\nu_b \int \tau_b ds + C_1 \quad (3.5)$$

$$\therefore \frac{ds}{dx} = \sqrt{2\nu_b \int \tau_b ds + C_1} \quad (3.6)$$

$$\therefore \int dx = \int \frac{ds}{\sqrt{2\nu_b \int \tau_b ds + C_1}} \quad (3.7)$$

C_1 is a constant of integration. C_1 should be 0 in order to simple calculation. C_1 will be included in the integral constant C_2 mentioned later. Calculation of the right side of Eq. (3.7) is as follows.

$$(right\ \ side) = \int \frac{ds}{\sqrt{2\nu_b \int \tau_b ds}} \quad (3.8)$$

Substituting Eq. (3.8) into Eq. (2.5) gives

$$(right\ side) = \int \frac{ds}{\sqrt{2\nu_b\left(-\frac{a}{3}s^3 + \frac{a}{2}s_u s^2\right)}} = \sqrt{\frac{3}{2\nu_b a}} \cdot \int \frac{ds}{\sqrt{s\left(\frac{9}{16}s_u^2 - \left(s - \frac{3}{4}s_u\right)^2\right)}} \quad (3.9)$$

Substitution integration using Eq. (3.10) is as follows.

$$s - \frac{3}{4}s_u = \frac{3}{4}s_u \cdot \cos\theta \quad (3.10)$$

$$(right\ side) = -\frac{1}{\sqrt{\nu_b a s_u}} \int \frac{d\theta}{\cos\frac{\theta}{2}} = -\frac{1}{\sqrt{\nu_b a s_u}} \ln\left|\frac{1+\sin\frac{\theta}{2}}{1-\sin\frac{\theta}{2}}\right| \quad (3.11)$$

The left side of Eq. (3.7) is given by following equation.

$$(left\ side) = x + C_2 \quad (3.12)$$

C_2 is the constant of integration. Eq. (3.11) and (3.12) give Eq. (3.13).

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$$s = \frac{3}{2} \cdot s_u \cdot \left\{1 - \left(\frac{e^{-(x+C_2)\sqrt{\nu_b a s_u}} - 1}{e^{-(x+C_2)\sqrt{\nu_b a s_u}} + 1}\right)^2\right\} \quad (3.13)$$

Bond stress is given by Eq. (3.14) by substituting Eq. (3.13) into Eq. (2.5).

$$\tau_b = 6 \cdot a \cdot s_u^2 \cdot \frac{e^{-(x+C_2)\sqrt{\nu_b a s_u}}\left(e^{-2(x+C_2)\sqrt{\nu_b a s_u}} - 4e^{-(x+C_2)\sqrt{\nu_b a s_u}} + 1\right)}{\left(e^{-(x+C_2)\sqrt{\nu_b a s_u}} + 1\right)^4} \quad (3.14)$$

C_2 expresses the position of bond stress distribution along the axial direction depending on the boundary condition. When bond length is long enough and the force is the maximum at the loaded end, C_2 is given by Eq. (3.15).

$$C_2 = \frac{1}{\sqrt{\nu_b a s_u}} \ln\left(2+\sqrt{3}\right) \tag{3.15}$$

3.2. Theoretical Solution of Bond Strength

Figure 2 shows theoretical distributions of bond stress, tensile force of reinforcement and slippage for several bond lengths. These distributions are cases for maximum tensile force described later. The hatched rectangles indicate EBSB. The shape of distributions do not vary even if the bond length changes. The positions of the distributions move along the axial direction. This phenomenon can be represented the value of integration constant C_2.

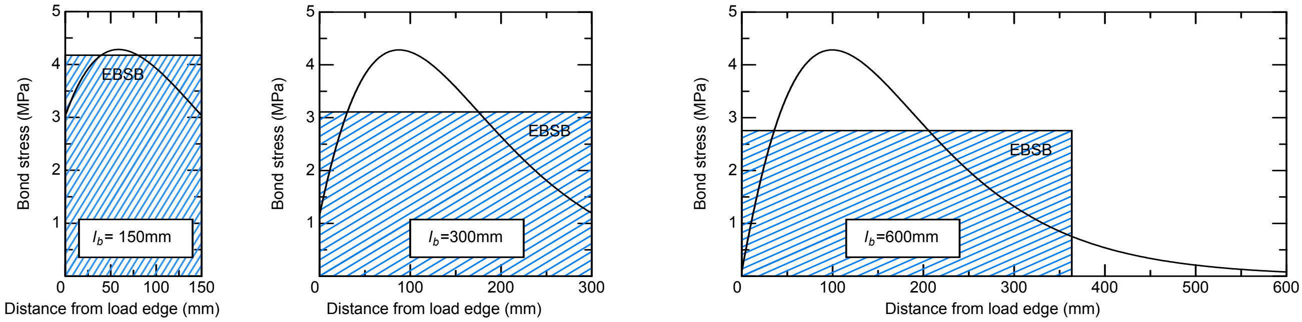
The tensile force of reinforcement can be estimated by integration of Eq. (3.14). The average bond stress at this time can be determined by the tensile force divided by surface area of reinforcement. So, the bond strength can be evaluated by solving maximum tensile force. Namely, the bond strength can be obtained by solving Eq. (3.16). Where, l_b expresses the bond length that is equal to the integration bounds. The variable x_1 represents the position of bond stress distribution along the axial direction including C_2. So, the problem is to determine x_1 when tensile force becomes maximum value.

$$\frac{P}{\phi_b} = \int_{x_1}^{x_1+l_b} \tau_b dx = \frac{6as_u^2}{\sqrt{\nu_b a s_u}} \left\{ \frac{1}{Z_2}\left(\frac{1}{Z_2}-1\right)\left(\frac{2}{Z_2}-1\right) - \frac{1}{Z_1}\left(\frac{1}{Z_1}-1\right)\left(\frac{2}{Z_1}-1\right) \right\} \tag{3.16}$$

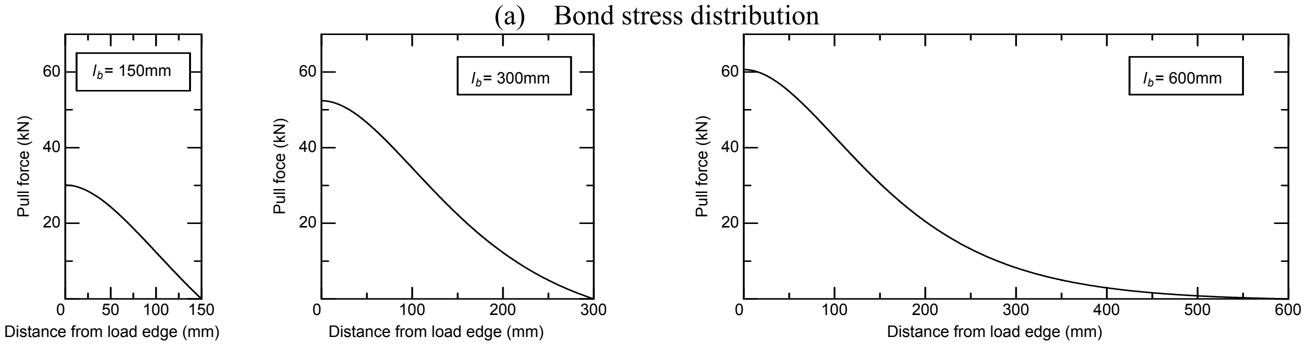
l_b : bond length
P : tensile force of reinforcement
ϕ_b: perimeter of reinforcement

$$Z_1 = 1 + e^{-x_1 \cdot \sqrt{\nu_b a s_u}} \tag{3.17}$$

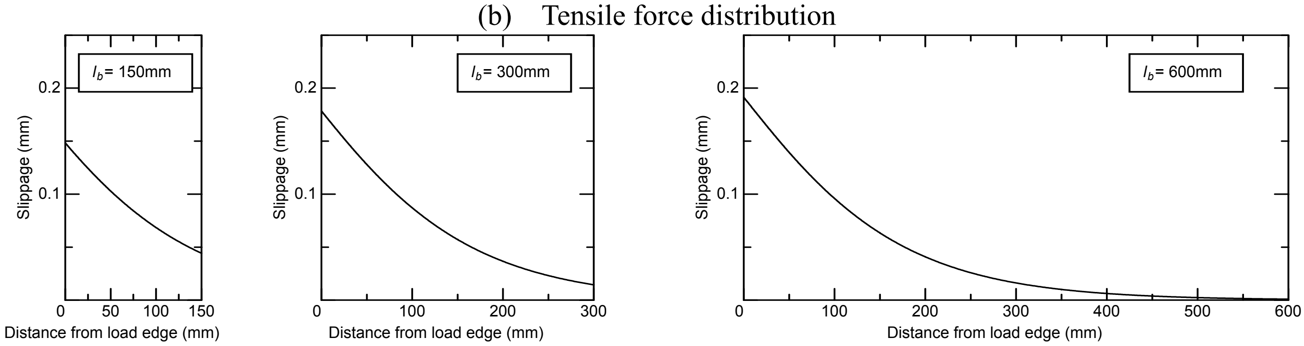
$$Z_2 = 1 + e^{-(x_1 + l_b) \cdot \sqrt{\nu_b a s_u}} \tag{3.18}$$



(a) Bond stress distribution



(b) Tensile force distribution



(c) Slippage distribution

Figure.2 Theoretical distributions of bond stress, tensile force and Slippage

Differentiate equation of Eq. (3.16) for x_1 is given by Eq. (3.19).

$$\frac{d}{dx_1}\left[\left\{\frac{1}{Z_2}\left(\frac{1}{Z_2}-1\right)\left(\frac{2}{Z_2}-1\right)-\frac{1}{Z_1}\left(\frac{1}{Z_1}-1\right)\left(\frac{2}{Z_1}-1\right)\right\}\right]=0 \tag{3.19}$$

Therefore, x_1 is given by following equations.

$$x_1=-\frac{1}{\sqrt{\nu_b a s_u}}\ln\left\{\frac{A-\sqrt{A^2-4e^{-l_b\sqrt{\nu_b a s_u}}}}{2e^{-l_b\sqrt{\nu_b a s_u}}}\right\} \tag{3.20}$$

$$A=2(e^{-l_b\sqrt{\nu_b a s_u}}+1)+\sqrt{3(e^{-2l_b\sqrt{\nu_b a s_u}}+10e^{-l_b\sqrt{\nu_b a s_u}}+1)} \tag{3.21}$$

The theoretical bond strength is given by Eq. (3.22).

$$\tau_{co}=\frac{P}{\phi_b\cdot l_b}=\frac{6as_u^2}{l_b\sqrt{\nu_b a s_u}}\left\{\frac{1}{Z_2}\left(\frac{1}{Z_2}-1\right)\left(\frac{2}{Z_2}-1\right)-\frac{1}{Z_1}\left(\frac{1}{Z_1}-1\right)\left(\frac{2}{Z_1}-1\right)\right\} \tag{3.22}$$

τ_{co}: bond strength

4. CONFIRMATION OF EBSB METHOD BY THEORETICAL BOND STRENGTH

The bond strength calculated by EBSB method is compared with theoretical one using experimental data reported in previous studies [5]. The number of experimental data is 56. All data are obtained from the specimen failed by side splitting mode. In calculating bond strength in reference [5], deformation of concrete is omitted because that is relatively smaller than reinforcement deformation. So, theoretical bond strength is also determined without considering concrete deformation, i.e., the value of numerator is to be 1 in Eq. (3.1).

The comparisons of bond strength are shown in Figure 3. The black marks indicate the case of the specimen which bond length (lb) is smaller than effective bond length (le). The average ratio of experimental data to determined bond strength is 1.22 for theoretical strength and 1.21 for calculated strength by EBSB method. The right figure shows the comparison between theretical strength and calculated strength by EBSB method. The average ratio and coefficient of variation is 1.01 and 2%, respectively. It is observed that the calculated value by EBSB is a little smaller than theoretical one for the black marks. In EBSB method, the estimated average bond stress is determined by the simple formula by regression analysis of numerical solution. It is considered that the difference for black marks is due to the error of regression analysis.

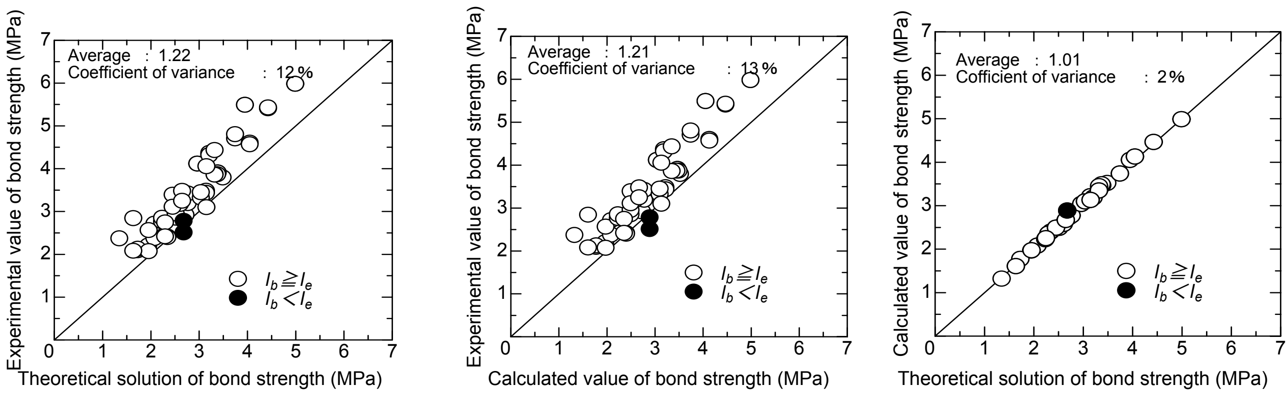


Figure.3 Comparison of bond strength

5. Relationship between Bond Strength and Bond Facture Energy

The area of local bond stress – slippage curve expresses the facture energy. There is a relation between bond facture energy and bond strength. In this chapter, this relation is theoretically led under arbitrary bond constitutive law. This lead is only for the case that bond length is long enough (mathematically infinity). Integration of Eq. (1.1) gives Eq. (4.1).

$$\frac{ds}{dx} = \frac{1+np}{E_b \cdot a_b} \cdot P = \frac{\nu_b}{\phi_b} \cdot P \tag{4.1}$$

Substituting this to Eq. (3.6) gives Eq. (4.2).

$$\frac{\nu_b}{\phi_b} \cdot P = \sqrt{2\nu_b \int \tau_b ds + C_1} \tag{4.2}$$

The maximum tensile force P_{max} is given by the next equation assuming that the bond length is long enough.

$$P_{max} = \int_0^\infty \tau_b dx \tag{4.3}$$

Bond fracture energy G_{fb} is;

$$G_{fb} = \int_0^{s_u} \tau_b ds \tag{4.4}$$

Considering the corresponding integral range, Eq. (4.2) can be expressed as;

$$\frac{\nu_b}{\phi_b} \cdot P_{max} = \sqrt{2\nu_b \cdot G_{fb} + C_1} \tag{4.5}$$

In the case of pullout loading with long bond length, tensile force is equal to 0 when the slippage is 0. This leads C_1 is equal to 0. So, Eq. (4.5) is given as;

$$P_{max} = \sqrt{\frac{2 \cdot G_{fb} \cdot \phi_b^2}{\nu_b}} \tag{4.6}$$

Substituting Eq. (3.1), P_{max} can be expressed as the following equation by G_{fb}.

$$P_{max} = \sqrt{\frac{2 \cdot G_{fb} \cdot E_b \cdot a_b \cdot \phi_b}{1 + np}} \tag{4.7}$$

Eq. (4.7) gives bond strength for the case of arbitrary constitutive law. When the stiffness of concrete is large enough, Eq. (4.7) shows same expression as Eq. (1.3).

6. CONCLUSION

The purpose of this paper is to derive the theoretical solution of bond strength by solving the simple differential equation about bond problem mathematically using a parabolic constitutive law which has a similar shape with experimental proposal reported previously. The simple calculation method using EBSB, which had already proposed by authors, gives almost same bond strength with the theoretical solution.

REFERENCE

[1] H. Muguruma, S. Morita, Fundamental Study on Bond Between Steel and Concrete (Part 1, Basic Laws of Bond Stress Distribution I), Journal of Structural Construction.Engng., Architectural Institute of Japan, No.131, pp.1-8, 1967.1
[2] H. Muguruma, S. Morita, Fundamental Study on Bond Between Steel and Concrete (Part 1, Basic Laws of Bond Stress Distribution II), Journal of Structural Construction Engng., Architectural Institute of Japan, No.132, pp.1-6, 1967.2
[3] B. Täljsten, Strengthening of Concrete Prisms using the Plate- Debonding Technique, International Journal of Fracture, Vol.82, pp.253-266, 1996
[4] T. Kanakubo, K. Yonemaru and H. Fukuyama, Study on Bond Splitting Behavior of Reinforced Concrete Members, Part 1 Local bond stress and slippage without lateral reinforcement, Transactions of Architectural Institute of Japan, No.492, pp.99-106, 1997.2
[5] A. Yasojima and T. Kanakubo, Study on Bond Splitting Behavior of Reinforced Concrete Members, Part 3 Predicting equation for bond splitting strength without lateral reinforcement, Transactions of Architectural Institute of Japan, No.567, pp.117-123, 2003.5