

# NONLINEAR STATIC PROCEDURE FOR REINFORCED CONCRETE ASYMMETRIC BUILDINGS WITH LINEAR VISCOUS DAMPERS

K. Fujii<sup>1</sup>

<sup>1</sup>Assistant Professor, Dept. of Architecture and Civil Engineering, Faculty of Engineering Chiba Institute of Technology, Chiba, Japan Email: kenji.fujii@it-chiba.ac.jp

## **ABSTRACT :**

In this paper, the Nonlinear Static Procedure (NSP) is extended for reinforced concrete single-story asymmetric building with linear viscous dampers. In this procedure, their responses are predicted through a nonlinear static analysis of MDOF model considering the contribution of linear viscous damper to the fundamental mode shape and an estimation of the nonlinear response of equivalent SDOF model using equivalent linearization technique. The peak drift of each frame predicted by the proposed procedure are compared with the results obtained by the time-history analysis. The results show that nonlinear response of single-story asymmetric buildings with viscous dampers can be satisfactory predicted by the procedure discussed in this paper.

**KEYWORDS:** asymmetric buildings, damper, nonlinear static procedure(NSP), pushover analysis

## 1. INTRODUCTION

It is well accepted that asymmetric buildings are vulnerable to earthquakes. This is because the excessive deformation may occur at the flexible and/or weak side frame due to the unfavorable torsional effect. That may lead to the premature failure of brittle members and finally to the collapse of whole buildings. In general, the excessive deformation at the flexible and/or weak side frame can be reduced by relocation of frames and/or members. However it may not be feasible for existing buildings because such relocation may be difficult due to architectural and functional constraints.

In recent years, the seismic rehabilitation of existing buildings using energy dissipative devices has been widely studied, and the seismic behavior of asymmetric buildings with viscous, viscoelastic dampers have been investigated by some researchers (Goel, 1998, Lin and Chopra, 2001). However few studies concerned about the Nonlinear Static Procedure (NSP) of asymmetric buildings with velocity-dependent dampers have been made.

In this paper, the extended NSP for single-story asymmetric buildings with linear viscous dampers is presented and its applicability is discussed. In this procedure, their responses are predicted through a pushover analysis of Multi-Degree-Of-Freedom (MDOF) model considering the contribution of linear viscous damper to the fundamental mode shape and an estimation of the nonlinear response of equivalent Single-Degree-Of-Freedom (SDOF) model using equivalent linearization technique (Otani, 2000). The results obtained by the proposed procedure are compared with the results obtained by the time-history analysis.

#### 2. DESCRIPTION OF NSP FOR ASYMMETRIC BUILDINGS WITH VISCOUS DAMPERS

# 2.1 Definition of First Mode Shape in Nonlinear Stage for Asymmetric Buildings with Linear Viscous Dampers

Buildings investigated in this paper is an idealized single-story reinforced concrete asymmetric building model (1-mass 3-DOF model) shown in Figure 2-1(a). In this paper, the same dampers are



installed in RC frame, and dampers are modeled as Maxwell model ( $C_D$ : damping coefficient of dashpot,  $K_D$ : elastic stiffness of spring) as shown in Figure 2-1(b).

The equivalent stiffness of RC frame  $K_{EQF}$  and damper  $K_{VD}(\omega)$  is defined as the secant stiffness at the peak drift  $d_{max}$  as shown in Figure 2-2;  $K_{EQF}$  and  $K_{VD}(\omega)$  are expressed as equation(2.1).

$$K_{EQF} = \frac{Q_{MAX}}{d_{MAX}}, K_{VD}(\omega) = \frac{F_{VD0}}{d_{MAX}} = \frac{K_D(C_D\omega)^2}{K_D^2 + (C_D\omega)^2} = \frac{\omega/\beta}{1 + (\omega/\beta)^2} \cdot C_D\omega$$
(2.1)

Where  $Q_{MAX}$  and  $F_{VD0}$  are the restoring force of RC frame and resistance force of damper corresponds to  $d_{max}$ , respectively,  $\beta = K_D / C_D$ , and  $\omega$  is circular frequency of harmonic excitation. Based on the equivalent stiffness defined above, the first mode shape in nonlinear stage of asymmetric building model is determined. Kasai has proposed an approximation method to determine the mode shape of elastic systems with viscoelastic damper (series viscoelastic damper-spring system), assuming that the equivalent stiffness of damper  $K_{VD}(\omega)$  can be determined based on the *i*-th natural circular frequency of system  $\omega_i$  (Kasai et al., 1999). Following this assumption, the first mode vector  $\varphi_1$  and equivalent circular frequency of the first mode  $\omega_{1eq}$  in nonlinear stage is determined from equation(2.2).

$$\omega_{leq}^{2} \mathbf{M} \boldsymbol{\varphi}_{1} = \left( \mathbf{K}_{EQF} + \mathbf{K}_{VD} \left( \omega_{leq} \right) \right) \boldsymbol{\varphi}_{1}$$
(2.2)

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, \mathbf{K}_{EQF} = \sum K_{EQFXi} \begin{bmatrix} 1 & 0 & l_{Yi} \\ 0 & 0 & 0 \\ l_{Yi} & 0 & l_{Yi}^2 \end{bmatrix} + \sum K_{EQFYi} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -l_{Xi} \\ 0 & -l_{Xi} & l_{Xi}^2 \end{bmatrix}$$
(2.3)

$$\mathbf{K}_{\mathbf{VD}}(\omega_{1eq}) = \mathbf{L}_{\mathbf{VD}} K_{VD}(\omega_{1eq}), \mathbf{L}_{\mathbf{VD}} = \sum n_{Xi} \begin{vmatrix} 1 & 0 & l_{Yi} \\ 0 & 0 & 0 \\ l_{Yi} & 0 & l_{Yi} \end{vmatrix} + \sum n_{Yi} \begin{vmatrix} 0 & 0 & 0 \\ 0 & 1 & -l_{Xi} \\ 0 & -l_{Yi} & l_{Yi} \end{vmatrix}$$
(2.4)

$$\boldsymbol{\phi}_{1} = \left\{ \phi_{X1} \quad \phi_{Y1} \quad \phi_{\Theta1} \right\}^{\mathrm{T}}, r = \sqrt{I/m}$$
(2.5)

Where **M** is mass matrix,  $\mathbf{K}_{EQF}$ ,  $\mathbf{K}_{VD}(\omega_{1eq})$  are equivalent stiffness matrix of RC frame and damper, respectively, *m* and *I* are mass and mass moment of inertia, respectively,  $K_{EQFXi}$  and  $K_{EQFYi}$  are equivalent stiffness of *i*-th RC frame in X- and Y-direction, respectively,  $l_{Xi}$  and  $l_{Yi}$  are location of *i*-th frame,  $n_{Xi}$  and  $n_{Yi}$  are the number of dampers installed in *i*-th frame, and *r* is the radius of gyration of floor mass. As shown in equations (2.1) and (2.2), the equivalent stiffness matrix of damper  $\mathbf{K}_{VD}(\omega_{1eq})$  is dependent of equivalent circular frequency of the first mode  $\omega_{1eq}$ ; therefore iteration process is needed to obtain inelastic first mode vector  $\varphi_1$  and  $\omega_{1eq}$  from equation(2.2). Figure 2-3 shows the flow of eigenvalue analysis shown in equation(2.2). Note that  $\omega_{1eqf}$  in Figure 2-3 is the equivalent circular frequency of the first modal mass. Considering a set of orthogonal U- and V- axes in X-Y plane, the *i*-th equivalent modal mass in U- and V-direction,  $M_{iU}^*$  and  $M_{iV}^*$ , respectively, are defined by equation(2.6).







Figure 2-3 Flow of eigenvalue analysis (equation (2.2))

$$M_{iU}^{*} = \Gamma_{iU} \boldsymbol{\varphi}_{i}^{T} \mathbf{M} \boldsymbol{\alpha}_{U}, M_{iV}^{*} = \Gamma_{iV} \boldsymbol{\varphi}_{i}^{T} \mathbf{M} \boldsymbol{\alpha}_{V}$$
(2.6)

$$\Gamma_{iU} = \frac{\boldsymbol{\phi}_{i}^{T} \mathbf{M} \boldsymbol{\alpha}_{U}}{\boldsymbol{\phi}_{i}^{T} \mathbf{M} \boldsymbol{\phi}_{i}}, \Gamma_{iV} = \frac{\boldsymbol{\phi}_{i}^{T} \mathbf{M} \boldsymbol{\alpha}_{V}}{\boldsymbol{\phi}_{i}^{T} \mathbf{M} \boldsymbol{\phi}_{i}}$$
(2.7)

$$\boldsymbol{\alpha}_{\mathrm{U}} = \left\{ \cos \psi_{i} - \sin \psi_{i} \quad 0 \right\}^{\mathrm{T}}, \boldsymbol{\alpha}_{\mathrm{V}} = \left\{ \sin \psi_{i} \quad \cos \psi_{i} \quad 0 \right\}^{\mathrm{T}}$$
(2.8)

By differentiating  $M_{iU}^{*}$  with respect to  $\psi_i$  and equating to zero, the principal direction of the *i*-th modal response is obtained and its tangent is given by equation(2.9).

$$\tan\psi_i = -\phi_{Y_i}/\phi_{X_i} \tag{2.9}$$

In this paper, the U-axis is taken as the principal axis of the first modal response of single-story asymmetric building model in elastic range as shown in Figure 2-4. Considering the first mode response and substituting  $\psi_1$  into equation(2.6),  $M_{1U}^*$  and  $M_{1V}^*$  are obtained as equation(2.10).

$$M_{1U}^{*} = \frac{\phi_{X1}^{2} + \phi_{Y1}^{2}}{\phi_{X1}^{2} + \phi_{Y1}^{2} + (r \cdot \phi_{\Theta 1})^{2}} \cdot m, M_{1V}^{*} = 0$$
(2.10)

It is interesting to note that equation(2.10) indicates that the ground motion in V-direction has no contributions in the first modal response.

# 2.2 Pushover Analysis for Asymmetric Building with Linear Viscous Dampers Representing the Nonlinear First Mode Response

First, the equation of motion of equivalent SDOF model representing the first mode response is formulated according to the previous study by author (Fujii, 2008). The equation of motion of asymmetric buildings with linear viscous dampers can be written as equation(2.11), considering the bidirectional excitation in U- and V-directions.

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}_{\mathbf{F}}\dot{\mathbf{d}} + \mathbf{f}_{\mathbf{R}} = -\mathbf{M}\left(\boldsymbol{\alpha}_{\mathbf{U}}\boldsymbol{a}_{gU} + \boldsymbol{\alpha}_{\mathbf{V}}\boldsymbol{a}_{gU}\right)$$
(2.11)

Where **d** is the displacement vector,  $C_F$  is the damping matrix of RC frame,  $f_R$  is the resistance force of whole building (=  $f_{RF} + f_{VD}$ ,  $f_{RF}$ : restoring force of RC frames,  $f_{VD}$ : resistance force of dampers) and  $a_{gV}$  and  $a_{gV}$  is the ground acceleration in U- and V-direction, respectively.

As is discussed in the previous study (Fujii, 2008), it is assumed that the building oscillates predominantly in the first mode under U-directional (unidirectional) excitation, and it oscillates predominantly in the second mode under V-directional excitation. Under bi-directional excitation, it is assumed that **d** and **f**<sub>R</sub> can be written in the form of equation(2.12), even if the building responses beyond the elastic range.









Figure 2-5 Equivalent SDOF model

$$\mathbf{d} = \Gamma_{1U} \boldsymbol{\phi}_1 D_{1U}^{*} + \Gamma_{2V} \boldsymbol{\phi}_2 D_{2V}^{*}, \mathbf{f}_{\mathbf{R}} = \mathbf{M} \left( \Gamma_{1U} \boldsymbol{\phi}_1 A_{1U}^{*} + \Gamma_{2V} \boldsymbol{\phi}_2 A_{2V}^{*} \right)$$
(2.12)

$$D_{1U}^{*} = \frac{\Gamma_{1U} \varphi_{1}^{T} \mathbf{M} \mathbf{d}}{M_{1U}^{*}}, D_{2V}^{*} = \frac{\Gamma_{2V} \varphi_{2}^{T} \mathbf{M} \mathbf{d}}{M_{2V}^{*}}, A_{1U}^{*} = \frac{\Gamma_{1U} \varphi_{1}^{T} \mathbf{f}_{\mathbf{R}}}{M_{1U}^{*}}, A_{2V}^{*} = \frac{\Gamma_{2V} \varphi_{2}^{T} \mathbf{f}_{\mathbf{R}}}{M_{2V}^{*}}$$
(2.13)

Where  $D_{1U}^*$  and  $D_{2V}^*$  are the first and second modal equivalent displacement,  $A_{1U}^*$  and  $A_{2V}^*$  are the first and second modal equivalent acceleration of whole building. It is also assumed that the change of the principal direction of the first modal response in nonlinear stage is negligibly small and equation(2.10) is still valid in nonlinear stage. By substituting equation (2.12) to equation (2.11) and by multiplying  $\Gamma_{1U} \varphi_1^T$  from the left side and considering equations (2.14) and (2.15), equation of motions of the equivalent SDOF model representing the first mode response can be obtained as equation(2.16).

$$\boldsymbol{\varphi}_1^{\mathrm{T}} \mathbf{M} \boldsymbol{\varphi}_2 = 0, \boldsymbol{\varphi}_1^{\mathrm{T}} \mathbf{C}_{\mathrm{F}} \boldsymbol{\varphi}_2 \approx 0 \tag{2.14}$$

$$\Gamma_{1V} / \Gamma_{1U} = \sqrt{M_{1V}^{*} / M_{1U}^{*}} = 0$$
(2.15)

$$\ddot{D}_{1U}^{*} + \frac{C_{1Uf}}{M_{W}^{*}} \dot{D}_{1U}^{*} + A_{1U}^{*} = -a_{gU}$$
(2.16)

$$C_{1Uf}^{*} = \Gamma_{1U}^{2} \left( \boldsymbol{\varphi}_{1}^{T} \mathbf{C}_{F} \boldsymbol{\varphi}_{1} \right)$$
(2.17)

Where  $C_{1Uf}^{*}$  is the first modal damping coefficient. Figure 2-5 shows the equivalent SDOF model for the asymmetric buildings.

The properties of the equivalent SDOF model representing the first mode response are determined based on the pushover analysis results described as follows. In the previous study, the pushover analysis for asymmetric buildings considering the change of the first mode shape in nonlinear stage has been proposed by author (Fujii et al., 2004). In this paper, this procedure is extended for buildings with linear viscous dampers. Figure 2-6 shows the flow of proposed pushover analysis. In the proposed pushover analysis, the following assumptions are made.

- 1) The equivalent stiffness of elements can be defined by their secant stiffness at peak drift previously experienced in the calculation (Figure 2-2).
- 2) The first mode shape at each loading stage can be determined from the equivalent stiffness.
- 3) The deformation shape imposed on a model is same as the first mode shape obtained in 2).

From the pushover analysis results, the equivalent acceleration of whole building  $A_{1U}^*$  corresponding to equivalent displacement  $D_{1U}^*$  at each pushover analysis steps can be determined by equation(2.18).

$$A_{1U}^{*} = \frac{\Gamma_{1U}^{2} \varphi_{1}^{T} \left\{ \mathbf{K}_{EQF} + \mathbf{K}_{VD} \left( \omega_{leq} \right) \right\} \varphi_{1}}{M_{1U}^{*}} \cdot D_{1U}^{*}$$
(2.18)





Figure 2-6 Flow of proposed pushover analysis

## 2.3 Outline of the NSP for Asymmetric Buildings with Linear Viscous Dampers

The proposed NSP for asymmetric buildings with linear viscous dampers consists of following 5 steps:

STEP 1: Pushover analysis of asymmetric building (first mode)

STEP 2: Prediction of seismic demand of equivalent SDOF model (first mode)

STEP 3: Pushover analysis of asymmetric building (second mode)

STEP 4: Prediction of seismic demand of equivalent SDOF model (second mode)

STEP 5: Prediction of peak drift in each frame of asymmetric building

The presented NSP is the extended version of the procedure presented in the previous study (Fujii et al., 2006). In this presented NSP, as well as that in previous NSP, two independent equivalent SDOF models are used to predict the responses of first and second mode (in STEP 2 and 4), and the peak drift of each frame in asymmetric building is predicted through the combination of four pushover analyses considering of the effect of bi-directional excitation (in STEP 5). Detail of the combination of four pushover analyses considering of the effect of bi-directional excitation can be found in (Fujii et al., 2006).

There are three modifications in the previous NSP (Fujii et al., 2006). The first modification is that pushover analysis in STEP 1 is carried out according to the procedure prescribed above (Section 2.2). The second modification is that the equivalent period  $T_{1eq}$  and equivalent damping  $h_{1eq}$  of equivalent SDOF model at each nonlinear stage is calculated by equations (2.19) through (2.21).

$$T_{1eq} = 2\pi / \omega_{1eq}, h_{1eq} = \left(\sum_{i} h_{eqfi} W_{efi} + \sum_{j} h_{eqvd} W_{evdj}\right) / \left(\sum_{i} W_{efi} + \sum_{j} W_{evdj}\right)$$
(2.19)

$$h_{eqfi} = \begin{cases} h_0 \sqrt{K_{EQFi}/K_{EFi}} & \mu_{fi} < 1\\ 0.2(1-1/\sqrt{\mu_{fi}}) + h_0 \sqrt{K_{EQFi}/K_{EFi}} & \mu_{fi} \ge 1 \end{cases}, \\ \mu_{fi} \ge 1, \\ \mu_{fi} \ge 1, \\ \mu_{fi} \ge 1 \end{cases}, \\ \mu_{fi} = \frac{d_{\max i}}{d_{yi}}, \\ W_{efi} = \frac{K_{EQFi}d_i^2}{2} \end{cases}$$
(2.20)

$$h_{eqvd} = 0.8h_{eqvd0}, h_{eqvd0} = \frac{1}{2} \frac{K_B}{C_D \omega_{leq}} = \frac{1}{2} \frac{\beta}{\omega_{leq}}, W_{evdj} = \frac{K_{VD} \left(\omega_{leq}\right) d_j^2}{2}$$
(2.21)



Where  $h_{eqfi}$  and  $h_{eqvd}$  are equivalent damping of RC frames and dampers, respectively,  $W_{efi}$  and  $W_{evdj}$  are potential energy of RC frames and dampers, respectively,  $h_0$  is the initial damping of RC frame and is assumed 0.03,  $K_{EFi}$  and  $K_{EQFi}$  are initial (elastic) and equivalent stiffness of *i*-th RC frame, respectively,  $\mu_{fi}$  is ductility of *i*-th RC frame,  $d_{maxi}$ ,  $d_{yi}$  and  $d_i$  are the peak, yield and current drift of *i*-th RC frame, respectively,  $h_{eqvd0}$  are equivalent damping dampers based on harmonic excitation. As shown in equation(2.21), the equivalent damping  $h_{eqvd}$  is reduced to 80% of  $h_{eqvd0}$ , which is based on the research by Kasai et al. (Kasai et al., 2004). Note that for the prediction of the seismic demand of second mode response (STEP 4), equivalent damping of dampers  $h_{eqvd}$  is determined based on  $\omega_{1eq}$ . The third modification is that for the pushover analysis using invariant force distribution (in STEP 3 and 5), equivalent stiffness of dampers  $K_{VD}(\omega)$  is assumed constant and is determined based on  $\omega_{1eq}$ .

#### **3** ANALYSIS EXAMPLES

#### 3.1 Building Data

Building investigated in this paper are idealized single-story asymmetric building models representing four-story building as shown in Figure 3-1. The model without damper is referred to as Model-O, while the model with dampers is referred to as Model-VD. Their height is assumed 11.16m and the total building mass m and moment of inertia I are 1524 ton, 1.075 x  $10^5$  ton-m<sup>2</sup>, respectively. Figure 3-2 shows the hysteresis model of RC frame, and Table 3-1 shows the properties of the each RC frame (elastic stiffness  $K_{EF}$ , yield strength  $Q_y$ , secant stiffness ratio at yield point  $\alpha_y$  and post-yielding stiffness degradation ratio  $\alpha_2$ ), which is determined based on the planer pushover analysis of each frame in original building model. The envelopes are assumed symmetric in both positive and negative loading directions. Torsional stiffness of member is neglected. No second order effect (ex. P- $\Delta$  effects) is considered. Muto hysteretic model (Muto et al., 1973) is employed for RC frame with one modification as shown Figure 3-2(b); the unloading stiffness after yielding stage is modified as it decreases with proportional to  $\mu_{f}^{-0.5}$ . In Model-VD, there are two linear viscous dampers which have the same properties;  $C_D = 1174$ kNs/m, and  $K_D = 21139$ kN ( $\beta = 18$  (1/s)). The properties of dampers are determined so that the maximum drift at Frame X6 is with 1% for the ground motion shown in section 3.2. The damping matrix is assumed proportional to the instant stiffness matrix of RC frame and 3% of the critical damping for the first mode. Figure 3-3 shows the mode shape and natural periods of (a) Model-O and (b) Model-VD. In this figure, the principal direction of modal response of







each mode obtained by equation (2.9) is also shown. As shown in this figure, the differences of the principal directions of all modes in two models are negligibly small.

#### 3.2 Ground Motion Data

In this study, the earthquake excitation is considered bi-directional in X-Y plane, and six sets of artificial ground motions are used. The first 60 seconds of two horizontal components (major and minor horizontal components) of the following records are used to determine phase angles of the ground motion: El Centro 1940(referred to as ELC), Taft 1952(TAF), Hachinohe 1968(HAC), Tohoku Univ. 1978(TOH), and JMA Kobe 1995(JKB) and Fukiai 1995(FKI). Target elastic spectrum of "major" components with 5% of critical damping  $S_A(T, 0.05)$  is determined by equation (3.1).

$$S_{A}(T, 0.05) = \begin{cases} 4.8 + 45T & \text{m/s}^{2} & T < 0.16s \\ 12.0 & 0.16s \le T < 0.576s \\ 12.0(0.576/T) & T \ge 0.576s \end{cases}$$
(3.1)

Where T is the natural period of the SDOF model. In this study, the target spectrum of "minor" components is reduced by 0.7 of "major" component determined by equation(3.1). Elastic response spectra of artificial ground motion with 5% of critical damping are shown in Figure 3-4, and Figure 3-5 shows the orbit of the set of artificial ground motion obtained from ELC. In this paper, the "major" components are applied in the principal direction of the first modal response of Model-O (U-direction), while the "minor" components are applied in V-direction.

#### 3.3 Analysis Results

Figure 3-6 shows the comparisons of the peak drift at each frame obtained from the NSP and the nonlinear time-history analysis. As shown in this figure, the procedure presented herein can predict the peak drift of Model-VD(with linear viscous dampers) as well as Model-O(without dampers).

## 4 CONCLUSION

In this paper, the extended NSP for single-story asymmetric buildings with linear viscous dampers is presented and its applicability is discussed. The procedure presented herein can predict the peak drift





Figure 3-6 Prediction of the peak drift at each frame

of asymmetric buildings with linear viscous dampers under bi-directional excitation as well as those without dampers. The presented NSP may also apply to asymmetric buildings with different linear velocity-dependent dampers, such as viscoelastic dampers. The extension of the proposed procedure to multi-story asymmetric frame building model is the next phase of this study.

## ACKNOWLEDGEMENT

The author acknowledges support by Grant-in-Aid for Young Scientist (Category (B), No. 18760426), Japan Ministry of Education, Culture, Sport, Science and Technology (MEXT).

## REFERENCES

Fujii, K., Nakano, Y. and Sanada, Y. (2004), Simplified nonlinear analysis procedure for asymmetric buildings, *Thirteenth World Conference on Earthquake Engineering*, Paper ID. 149.

Fujii, K., Nakano, Y. and Sakata, H. (2006), Nonlinear analysis of single-story unsymmetric buildings with elasto-plastic seismic control devices, *Eighth U.S. National Conference on Earthquake Engineering*, Paper ID. 217.

Fujii, K. (2008), Equivalent SDOF model for asymmetric buildings considering bi-directional excitation, submitted for Fifth European Workshop on the seismic behaviour of Irregular and Complex Structures

Goel, R. K. (1998). Effects of supplemental viscous damping on seismic response of asymmetric-plan systems, *Earthquake Engineering and Structural Dynamics*, **27:2**, 125-141.

Kasai, K. and Okuma K. (1999), Approximation analysis method for visco-elastically damped structures, *Summary of Technical Papers of Annual Meeting*, *Architectural Institute of Japan*, **B-2**, 1101-1104.(Japanese)

Kasai, K. and Okuma K. (2004), Evaluation rule and its accuracy for equivalent period and damping of frequency-dependent passive control systems, *Journal of Structural and Construction Engineering (Transaction of AIJ)*, **580**, 51-59.(Japanese)

Lin, W. and Chopra, A. K. (2001), Understanding and predicting effects of supplemental viscous damping on seismic response of asymmetric one-storey systems, *Earthquake Engineering and Structural Dynamics*, **30:10**, 1475-1474.

Otani, S. (2000), New Seismic Design Provision in Japan, *The Second U.S.-Japan Workshop on Performance-Based Earthquake Engineering Methodology for Reinforced Concrete Structures*, PEER Report 2000/10, 3-14.

Muto, K., Hisada, T., Tsugawa, T., and Bessho S. (1973) *Earthquake Resistant Design of a 20 Storey Reinforced Concrete Buildings, Fifth World Conference on Earthquake Engineering*, 1960-1969.