

# Non-linear Dynamic Response Analysis of Bridge crossing Earthquake Fault Rupture Plane

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# **ABSTRACT :**

On condition that fault displacement affects bridge, we formulated the equation of motion for bridge structure under inertial force and the relative displacements of support points of the bridge and developed a special algorithm to solve this equation. Using this algorithm, we calculated the dynamic response of the RC arch bridge subjected to the inertial force, and the relative displacements induced by fault movement. According to our analytical results, we evaluated the damage to the RC arch bridge by Damage Index. As a result of these analyses, we can conclude that damage to bridges considering both the relative displacements, and the inertial force by acceleration, will become more serious than that considering only the relative displacements by static analysis.

**KEYWORDS:** Equation of Motion, EPS Method, Fault Movement, Damage Index

# **1. INTRODUCTION**

A number of devastating earthquakes tragically took place successively in 1999. Especially, the August 17, 1999 Kocaeli earthquake in Turkey (Ulusay et al., 2001) and September 21, 1999 Chi-Chi earthquake in Taiwan (Kosa et al., 2001) were extraordinary. One of the special features of these earthquakes was the damage to structures directly inflicted not only by ground acceleration but also by fault movements. These two earthquakes caused ground surface failures across a broad area. These failures included lateral displacements, and rise with vertical displacements. Furthermore, the fault movement occurred right beneath various bridge structures. There is no doubt that these failures are of great concern to earthquake engineering because this phenomenon may be observed in any country having activated faults.

At present, the seismic design of bridge structures is generally achieved by considering only the possible shaking characteristics of ground acceleration. However, the seismic design of bridge structures under earthquake loading with fault displacements must deal with both internal force, caused by ground acceleration, and relative displacements between support points of bridge.

The objectives of this paper are to describe an algorithm to calculate non-linear dynamic response analysis of a bridge structure subjected to both inertial forces by acceleration and relative displacement between support points, and to evaluate the damage level of an RC arch bridge under ground surface faulting between its abutments.

First, we carried out the equation of motion including relative movement of support points and the algorithm to solve the equation of motion. Secondly, we show the transformations of acceleration record into displacement data by EPS (Erratic Pattern Screening) method. Lastly, we evaluate dynamic response and damage level of RC arch bridge being encountering fault-induced attacks quantitatively in comparison with static analysis.

# **2.THE EQUATION OF MOTION CONSIDERING INERTIAL FORCE AND RELATIVE DISPLACEMENTS**

### 2.1. Formulation

Generally, an equation of motion, neglecting damping effect, for full structure in the absolute coordinates can



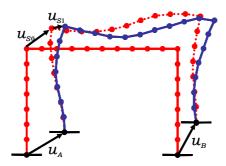


Figure 1 Division of displacement vector

be described as :

$$M\ddot{u} + Ku = F \tag{2.1}$$

where M and K are the mass and stiffness matrices.  $\ddot{u}$  and u are the absolute acceleration and absolute displacement vectors respectively, F is the external force vector. First, we divide these matrices and vectors as follows.

$$M = \begin{bmatrix} M_{s} & 0 & 0 \\ 0 & M_{A} & 0 \\ 0 & 0 & M_{B} \end{bmatrix}, \quad K = \begin{bmatrix} K_{ss} & K_{sA} & K_{sB} \\ K_{As} & K_{AA} & K_{AB} \\ K_{Bs} & K_{BA} & K_{BB} \end{bmatrix}, \quad u = \begin{cases} u_{s} \\ u_{a} \\ u_{B} \end{cases}, \quad F = \begin{cases} F_{s} \\ F_{A} \\ F_{B} \end{cases}$$
(2.2)

In these matrices and vectors, each element has indices: the DOF index about support point A, A, that about support point B, B, and load control points S, S, as shown in Figure 1. The stiffness equation is written as

$$\begin{bmatrix} K_{SS} & K_{SA} & K_{SB} \\ K_{AS} & K_{AA} & K_{AB} \\ K_{BS} & K_{BA} & K_{BB} \end{bmatrix} \begin{bmatrix} u_{S0} \\ u_{A} \\ u_{B} \end{bmatrix} = \begin{cases} F_{S0} \\ F_{A} \\ F_{B} \end{bmatrix}$$
(2.3)

with  $F_A$ ,  $F_B$  and  $F_{s0}$  being reaction forces and external forces of load control points for  $u_{s0}$ ,  $u_A$  and  $u_B$  being the location of the A, B, and load control points. Therefore

$$K_{SS}u_{S0} + K_{SA}u_{A} + K_{SB}u_{B} = F_{S0}$$
(2.4)

Secondly, we denote by  $u_{s_1}$  the displacement vector of load control points by dynamic effect. The total displacement vector are

$$u_{\rm s} = u_{\rm s0} + u_{\rm s1} \tag{2.5}$$

Substituting Eqs. (2.2) and (2.5) into Eq. (2.1), we arrive at

$$M_{s}(\ddot{u}_{s0}+\ddot{u}_{s1})+K_{ss}(u_{s0}+u_{s1})+K_{sA}u_{A}+K_{sB}u_{B}=F_{s0}$$
(2.6)

By use of Eqs. (2.4) and (2.6), we obtain

$$M_{s}\ddot{u}_{s1} + K_{ss}u_{s1} = -M_{s}\ddot{u}_{s0} \tag{2.7}$$



**TIME INCREMENT LOOP :** n  

$$\Delta u_{a} = u_{a}^{(n)} - u_{a}^{(n-1)}, \quad \Delta u_{a} = u_{a}^{(n)} - u_{a}^{(n-1)}$$
**STATE DETERMINATION BY STATIC ANALYSIS**  
Enter Newton-Raphson iteration loop :  $k = 1, 2, 3, \cdots$ , until convergence  

$$\begin{bmatrix} \Delta F_{so} \\ \Delta F_{so} \\ \Delta F_{so} \end{bmatrix} = \begin{bmatrix} K \end{bmatrix}^{t} \begin{bmatrix} \Delta u_{so} \\ \Delta u_{a} \\ \Delta u_{b} \end{bmatrix}$$

$$u_{so} = u_{so} + \Delta u_{so}$$
Assemble structure resisting force vector  $F_{ao}$   
Compute unbalanced force vector  $F_{u} = F_{o} - F_{zo}$   
Goto next Newton-Raphson iteration  
**CALCULATE**  $\ddot{u}_{so}, \quad \Delta \ddot{u}_{so}$  **AND**  $P = -M_{s}\ddot{u}_{so}$   
**STATE DETERMINATION BY DYNAMIC ANALYSIS**  
Enter Newton-Raphson iteration loop :  $k = 1, 2, 3, \cdots$ , until convergence  
 $\overline{K} = K_{ss}^{t} + \frac{1}{2\beta\Delta t}C_{ss} + \frac{1}{\beta\Delta t^{2}}M_{s}$   
 $\Delta \overline{P} = -M_{s}\Delta \ddot{u}_{so} + M_{s}\left(\frac{1}{\beta\Delta t}\dot{u}_{so}^{(n-1)} + \frac{1}{2\beta}\dot{u}_{so}^{(n-1)}\right) + C_{ss}\left(\frac{1}{2\beta}\dot{u}_{so}^{(n-1)} + \left(\frac{1}{4\beta} - 1\right)\ddot{u}_{so}^{(n-1)}\Delta t\right)$   
 $\Delta u_{s1} = \overline{K}^{-1} \cdot \Delta \overline{P}$   
 $u_{s1} = u_{s1} + \Delta u_{s1}$   
Solve  $\Delta \dot{u}_{s1}$  and  $\Delta \ddot{u}_{s1}$  by Newmark method and update  $\dot{u}_{s1}$  and  $\ddot{u}_{s1}$   
Assemble structure resisting force  $P_{a}$   
 $P_{a}^{p} = M_{s}\dot{u}_{s1} + C_{s}\dot{u}_{s1} + P_{a}$   
Compute unbalanced force vector  $P_{v} = P - P_{a}^{p}$   
Goto next Newton-Raphson iteration

GOTO NEXT TIME STEI

Figure 2 Integration Procedure of dynamic analysis subjected to both inertial force and relative displacement between abutments

Furthermore, Eq. (2.4) can be represented as :

$$u_{\rm S0} = K_{\rm SS}^{-1} \left( F_{\rm S0} - K_{\rm SA} u_{\rm A} - K_{\rm SB} u_{\rm B} \right)$$
(2.8)

Therefore, Eq.(2.7) can be rewritten as :

$$M_{s}\ddot{u}_{s1} + K_{ss}u_{s1} = M_{s}K_{ss}^{-1} (K_{sA}\ddot{u}_{A} + K_{sB}\ddot{u}_{B})$$
(2.9)

Note that  $u_{s1}$  is the displacement vector from  $u_{s0}$ . Hence  $u_{s1}$  is corresponding to relative displacement in normal equation of motion. Therefore absolute acceleration and absolute displacement can be derived as :



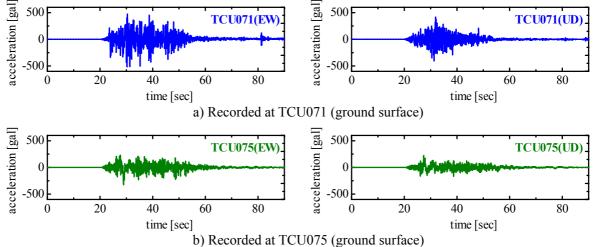


Figure 3 Ground accelerations of Chi-Chi Earthquake in Taiwan (21 September 1999)

$$u_{\rm s} = u_{\rm so} + u_{\rm si} \tag{2.10}$$

$$\ddot{u}_{\rm s} = \ddot{u}_{\rm s0} + \ddot{u}_{\rm s1} \tag{2.11}$$

Eq.(2.9) cannot calculate non-linear response of structures because this equation is for elastic body. The incremental expression of Eq.(2.9) is

$$\boldsymbol{M}_{S}\boldsymbol{\Delta}\boldsymbol{\ddot{u}}_{S1} + \boldsymbol{K}_{SS}^{t}\boldsymbol{\Delta}\boldsymbol{u}_{S1} = \boldsymbol{M}_{S} \left[\boldsymbol{K}_{SS}^{t}\right]^{-1} \left(\boldsymbol{K}_{SA}^{t}\boldsymbol{\Delta}\boldsymbol{\ddot{u}}_{A} + \boldsymbol{K}_{SB}^{t}\boldsymbol{\Delta}\boldsymbol{\ddot{u}}_{B}\right)$$
(2.12)

where  $K_{SS}^{t}$ ,  $K_{SA}^{t}$  and  $K_{SB}^{t}$  are the tangent stiffness matrices.

However, It is impossible to define constant external force from right-hand-side of this equation, because  $K_{SS}^{t}$ ,  $K_{SA}^{t}$  and  $K_{SB}^{t}$  are changing among one iteration step. In addition to this, inverse matrix of tangential stiffness matrix cannot define when softening of RC occurs. Hence, we apply Eq. (2.7) in place of Eq. (2.12) to solve non-linear response as :

$$M_{s} \Delta \ddot{u}_{s1} + K_{ss}^{t} \Delta u_{s1} = -M_{s} \Delta \ddot{u}_{s0}$$

$$\tag{2.13}$$

In Eq. (2.13),  $\ddot{u}_{s_0}$  is the second derivative of  $u_{s_0}$ : absolute displacement vector of load controll points, which can be obtained by the algorithm as shown in 2.2.

#### 2.2. Non-linear Dynamic Analysis Procedures

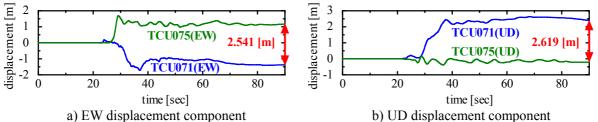
To investigate the non-linear dynamic response of the structure, the classical Newmark method (Newmark 1959) that is one the most popular for dynamic analysis, is adopted and combined with the non-linear static analysis.(Nakano et al. 2008).

The integration procedure of dynamic analysis formulated in Eq. (2.13) can be described in Fig. 2. The dynamic displacement solution at a given time is determinated as a summation of  $u_{s_0}$  computed by static analyses and  $u_{s_1}$  by Newmark method. This algorithm was developed by the authors for the purpose of simulating of the dynamic response of bridge under both internal force and relative displacement between abutments.

#### **3.THE EPS(ERRASTIC PATTERN SCREENING) METHOD**

In the algorithm mentioned above, it is necessary to use ground displacement responses. However, in general,







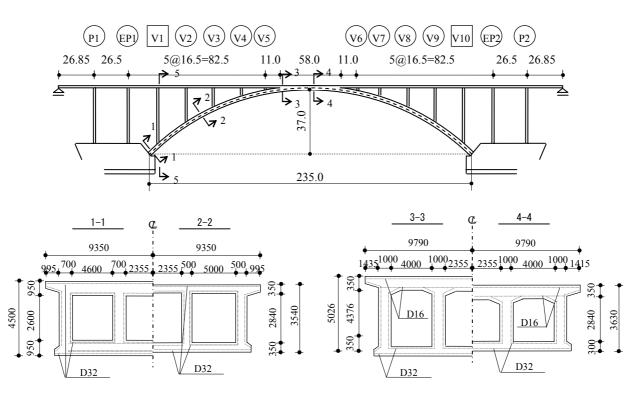


Figure 5 RC arch bridge system

integrating acceleration directly, the velocity and displacement responses will diverge because of background noise being caused by electrical acceleration records. The reason of this phenomenon, Ohta and Aydan (2007) considers integral constants influenced by background noise. Therefore, they suggested the EPS method which is the displacement calculation method. In this method, we screen effects of erratic pattern from electrical acceleration records and hence, we can control divergence phenomena of integration of acceleration.

Figure 3 shows the acceleration observed in Taiwan Chi-Chi earthquake, 1999 (Taiwan Central Weather Bureau, TCU071 and TCU075). Integrating these acceleration records by EPS method, we obtain displacement responses as shown in Figure 4. Our parametric study mentioned later will use these displacement waves.

### 4.NUMERICAL SIMULATION ANALYSIS

#### 4.1. Target RC Arch Bridge

In this study, an arch bridge designed based on Japan Design Specifications for Highway Bridges, Part V Seismic Design(2002) is analyzed to evaluate damage due to fault movements as shown in Figure 5. This bridge is idealized as a 177 degrees of freedom lumped-mass system. The member cross section is divided into cells. In this model (fiber model) material models are applied to concrete and reinforcing bars constituting



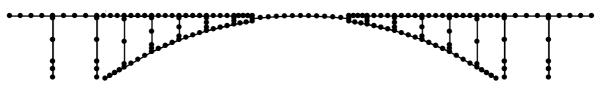


Figure 6 Beam element model

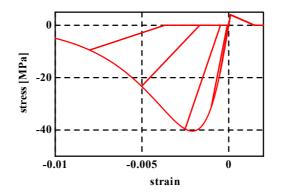


Figure 7 Stress-strain relation of concrete

each cell. We used constitutive relation proposed by Maekawa and Tsuchiya (2002) for concrete (Figure 7) and bi-linear model for reinforcing bars.

#### 4.2. Damage Index

It is difficult to evaluate the damage level of an arch bridge quantitatively because an RC arch bridge has various section shapes. For that reason, we employ a cross sectional damage index proposed by Maekawa and Tsuchiya(2002). According to them, damage index is defined as :

$$F = 1 - \overline{K} \equiv \frac{1}{A_c} \int_{A_c} (1 - K) dA \approx 1 - \frac{\Sigma K \cdot \Delta A}{A_c}$$
(4.1)

where F :cross sectional damage index,  $\overline{K}$  :average fracture parameter, K :local fracture parameter in each concrete cell and  $A_c$  :concrete cross-sectional area. Fracture parameter K can be computed as:

$$K = \exp\left[-0.73\varepsilon'_{\max}\left\{1 - \exp\left(-1.25\varepsilon'_{\max}\right)\right\}\right]$$
(4.2)

$$\varepsilon'_{p} = \varepsilon'_{\max} - 20\{1 - \exp(-0.35\varepsilon'_{\max})\}/7$$
(4.3)

where  $\varepsilon'$ :normalized axial strain divided by  $\varepsilon'_{peak}$  that is strain corresponding to compressive strength,  $\varepsilon'_{p}$ :normalized plastic strain and  $\varepsilon'_{max}$ : the experimental maximum value of the normalized strain.

Damage index F represents no damage when F = 0 and shear collapse completely when F = 1. Maekawa and Tsuchiya describe that the point in the softening range after the peak capacity approximately corresponding to the F = 0.5. Therefore, we apply this standard to damage level evaluation of RC arch bridge.



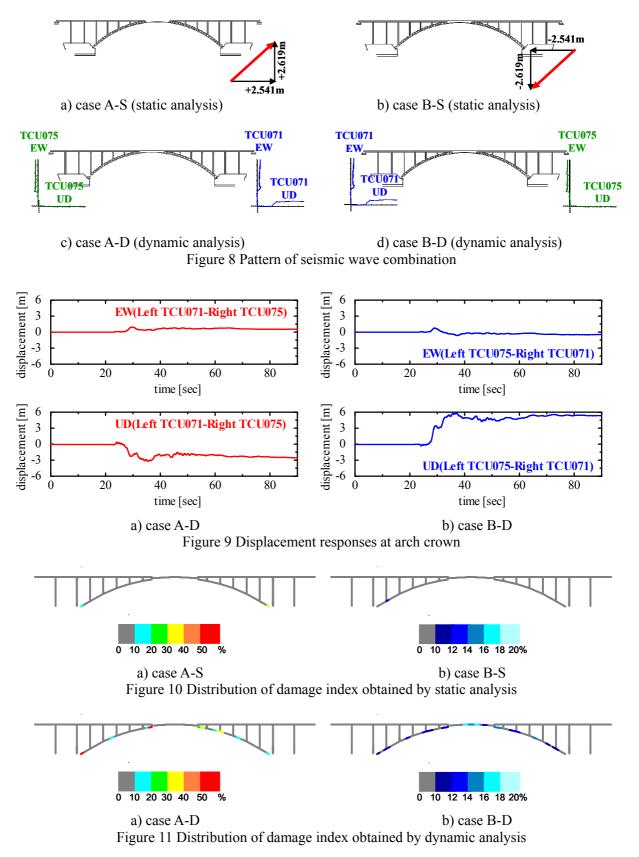




Figure 8 shows both the static and dynamic analysis conditions. In static analysis (Figure 8 a) and b)), we let



right support points be acted relative displacement step by step statically. In dynamic analysis (Figure 8 c) and d)), we employ the algorithm mentioned above.

The displacement responses of arch crown and damage index distribution are shown in Figure 9 to 11. These results imply that the damage considering dynamic effect will be more severe than the damage of arch bridge under only relative displacement of support points. Especially, in case that arch ring opens, the effect of inertial force produce ultimate damage i.e. collapse of section (compare case A-S with case A-D). Hence, it is necessary to take account of the effect of inertial force when we predict the damage of an RC arch bridge under fault movements.

#### **5.CONCLUSION**

In this paper, a numerical simulation method for bridges subjected to both inertial force and relative displacements of support points was introduced and we predicted seismic performance and damage level of an RC arch bridge under fault displacements. According to analytical results, we can conclude that the effect of inertial force will influence the damage level of an RC arch bridge enormously; hence we have to consider not only relative displacement but also the consequence of a fault-generated acceleration.

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