EFFECT OF MODAL TRUNCATION IN MULTIPLE SUPPORT RESPONSE SPECTRUM ANALYSIS OF BRIDGES

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ABSTRACT:

The question of how many modes to include in the dynamic analyses of multiply-supported structures subjected to spatially varying ground motions is considered in this paper. The issue is investigated within the framework of the Multiple Support Response Spectrum (MSRS) method developed by Der Kiureghian and Neuenhofer (1992). The original MSRS rule is extended to approximately account for the contribution of the truncated high-frequency modes. A bridge structure designed by the California Department of Transportation (Caltrans) is used as an example application. The modal contributions to various response quantities are examined in conjunction with measures of the participating modal mass and the improvement achieved with the extended MSRS rule.

KEYWORDS: bridges; incoherence; modal truncation; MSRS rule; spatial variability; wave passage.

1. INTRODUCTION

Proper seismic design of extended structures, such as bridges, requires accounting for the spatial variability of seismic ground motions due to the loss of coherency of seismic waves, the wave passage effect and the difference in the local soil conditions. Differential support motions can enhance or decrease the seismic demand depending on the characteristics of the structure and the ground motion field. The MSRS rule, developed by Der Kiureghian and Neuenhofer [2], evaluates the structural response in terms of the response spectra of the support motions and a coherency function that characterizes the spatial variability of the ground motion random field. This method is based on modal analysis and the fundamental principles of random vibrations theory, and accounts for the cross-correlations between the modes of the structure as well as between the support motions.

An important practical problem that has not been properly addressed is the development of a reliable guideline to define the number of modes that should be included in the MSRS analysis. The criterion used in conventional seismic analysis is based on the percentage of structural mass represented by the modes. However, this measure does not account for differential support motions, nor does it account for the effect of closely spaced modes. Kahan [4] proposed a measure of the participating modal mass for the limiting case of totally incoherent ground motions. The aforementioned measures are examined in this paper considering the model of an existing bridge in California. Furthermore, an extended version of the MSRS rule is developed to account for the quasi-static contribution of the truncated modes. Numerical investigations show that the extended rule improves the response estimate when the contribution of higher modes is significant.

2. STRUCUTRAL RESPONSE TO DIFFERENTIAL SUPPORT MOTIONS

Consider a N-degree-of-freedom lumped-mass linear structural model subjected to m support motions. Let $\mathbf{x}$ be the N-vector of total displacements at the unconstrained degrees of freedom and $\mathbf{u}$ be the m-vector of prescribed support displacements. The total displacement vector is decomposed into pseudo-static and dynamic components, $\mathbf{x} = \mathbf{x}^s + \mathbf{x}^d$. The pseudo-static component, $\mathbf{x}^s$, is the response of the system when dynamic effects are neglected and is related to the support displacements through an influence matrix, $\mathbf{R}$, i.e. $\mathbf{x}^s = \mathbf{Ru}$. The k th column of the influence matrix, denoted $\mathbf{r}_k$, represents the displacements at the unconstrained degrees of freedom when the k th support degree of freedom is statically displaced by a unit amount while other support degrees of freedom remain fixed.

Let $\boldsymbol{\varphi}_i$, ω_i and ζ_i, $i = 1, \ldots, N$ denote, respectively the mode shapes, natural frequencies, and modal damping ratios of the fixed-base structure. Assuming classical damping and neglecting the damping forces associated with the constrained degrees of freedom, the total displacement vector can be decomposed as

$$\mathbf{x}(t) = \sum_{k=1}^{m} \mathbf{r}_k u_k(t) + \sum_{k=1}^{m} \sum_{i=1}^{N} \boldsymbol{\varphi}_i \beta_{ki} s_{ki}(t) \tag{1.1}$$

where $\beta_{ki} = \boldsymbol{\varphi}_i^{\mathrm{T}} \mathbf{M} \mathbf{r}_k / \boldsymbol{\varphi}_i^{\mathrm{T}} \mathbf{M} \boldsymbol{\varphi}_i$ is the modal participation factor in which $\mathbf{M}$ is the $N \times N$ mass matrix associated with the unconstrained degrees of freedom, and $s_{ki}(t)$ is the i th normalized modal response to base motion $u_k(t)$ obtained as the solution to

$$\ddot{s}_{ki} + 2\zeta_i \omega_i \dot{s}_{ki} + \omega_i^2 s_{ki} = -\ddot{u}_k(t) \tag{1.2}$$

A generic response quantity of interest, $z(t)$, such as a nodal displacement or an internal force component, can be written as a linear combination of the displacements at the unconstrained degrees of freedom and the support displacements, i.e. $z(t) = \mathbf{p}^{\mathrm{T}} \mathbf{u}(t) + \mathbf{q}^{\mathrm{T}} \mathbf{x}(t)$. Equivalently, one can write

$$z(t) = \sum_{k=1}^{m} a_k u_k + \sum_{k=1}^{m} \sum_{i=1}^{N} b_{ki} s_{ki}(t) \tag{1.3}$$

in which $a_k = p_k + \mathbf{q}^{\mathrm{T}} \mathbf{r}_k$ and $b_{ki} = \mathbf{q}^{\mathrm{T}} \boldsymbol{\varphi}_i \beta_{ki}$. The coefficients a_k and b_{ki} depend only on the properties of the structure (not the ground motion).

3. THE MSRS RULE

Using the decomposition of $z(t)$ in Eqn. (1.3) and assuming jointly stationary, zero-mean support motions, Der Kiureghian and Neuenhofer [2] developed the MSRS (Multiple Support Response Spectrum) combination rule:

$$\begin{aligned} \mathrm{E}[\max|z(t)|] = \Bigg[& \sum_{k=1}^{m} \sum_{l=1}^{m} a_k a_l \rho_{u_k u_l} u_{k,\max} u_{l,\max} + 2 \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{j=1}^{N} a_k b_{lj} \rho_{u_k s_{lj}} u_{k,\max} D_l(\omega_j, \zeta_j) \\ & + \sum_{k=1}^{m} \sum_{l=1}^{m} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ki} b_{lj} \rho_{s_{ki} s_{lj}} D_k(\omega_i, \zeta_i) D_l(\omega_j, \zeta_j) \Bigg]^{1/2} \end{aligned} \tag{1.4}$$

According to the above rule, the mean peak response is given in terms of the structural properties as reflected in the coefficients a_k and b_{ki}, the mean peak ground displacement, $u_{k,\max}$, and the ordinates of the mean displacement response spectrum, $D_k(\omega_i, \zeta_i)$, for each support motion, and three types of cross-correlation coefficients: The cross-correlation coefficients $\rho_{u_k u_l}$ describe the correlation between the support displacements; they only depend on the auto- and cross-power spectral densities (PSDs) of the support motions. The cross-correlation coefficients $\rho_{s_{ki} s_{lj}}$ describe correlations between the responses of two modes (i and j) to two support motions (k and l), while the cross-correlation coefficients $\rho_{u_k s_{lj}}$ describe correlations between the modal responses and the support motions. The latter coefficients are functions of the auto- and cross-PSDs of the support motions as well as the modal frequencies and damping ratios. The cross-PSDs of the support motions are given in terms of the auto-PSDs and a coherency function that characterizes the spatial variability of the ground motion field.

When the support motions are described through response spectra, consistent auto-PSDs are determined from relations given in [2]. Thus, the set of peak ground displacements and response spectra for the support degrees of freedom together with a coherency function provide complete specification of the ground motion random field for MSRS analysis.

4. EXTENDED MSRS RULE ACCOUNTING FOR TRUNCATED HIGH FREQUENCY MODES

For large ω_i, the last term in the left-hand side of Eqn. 1.2 dominates. Hence, for such cases, an approximation of the ith normalized modal response to base motion $u_k(t)$ is $s_{ki}(t) \approx -\omega_i^{-2}\ddot{u}_k$. Using this relation to approximate the ith modal response for $n < i \le N$, we can write

$$z(t) \approx \sum_{k=1}^{m} a_k u_k + \sum_{k=1}^{m}\sum_{i=1}^{n} b_{ki} s_{ki}(t) - \sum_{k=1}^{m} d_k \ddot{u}_k(t) \tag{1.5}$$

One can show that the coefficients d_k only depend on the dynamic properties of the first n modes and can be obtained as

$$d_k = q^{\mathrm{T}} \mathbf{K}^{-1} \mathbf{M} \mathbf{r}_k - \sum_{i=1}^{n} \frac{b_{ki}}{\omega_i^2} \tag{1.6}$$

where $\mathbf{K}$ is the stiffness matrix associated with the unconstrained degrees of freedom. Employing the approximation described in Eqn. (1.5), the extended MSRS rule that approximately accounts for the contributions of the truncated modes is

$$\begin{aligned} \mathrm{E}[\max|z(t)|] = \Bigg[& \sum_{k=1}^{m}\sum_{l=1}^{m} a_k a_l \rho_{u_k u_l} u_{k,\max} u_{l,\max} + 2\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{j=1}^{n} a_k b_{lj} \rho_{u_k s_{lj}} u_{k,\max} D_l(\omega_j,\zeta_j) \\ & + \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ki} b_{lj} \rho_{s_{ki} s_{lj}} D_k(\omega_i,\zeta_i) D_l(\omega_j,\zeta_j) \\ & + \sum_{k=1}^{m}\sum_{l=1}^{m} d_k d_l \rho_{\ddot{u}_k \ddot{u}_l} \ddot{u}_{k,\max} \ddot{u}_{l,\max} - 2\sum_{k=1}^{m}\sum_{l=1}^{m} a_k d_l \rho_{u_k \ddot{u}_l} u_{k,\max} \ddot{u}_{l,\max} \\ & - 2\sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{l=1}^{m} b_{ki} d_l \rho_{s_{ki} \ddot{u}_l} D_k(\omega_i,\zeta_i) \ddot{u}_{l,\max} \Bigg]^{1/2} \end{aligned} \tag{1.7}$$

In the above expression, $\ddot{u}_{k,\max}$ is the mean peak ground acceleration at the kth support degree of freedom and $\rho_{\ddot{u}_k\ddot{u}_l}$, $\rho_{u_k\ddot{u}_l}$ and $\rho_{s_{ki}\ddot{u}_k}$ are three new types of cross-correlation coefficients reflecting, respectively, the correlation between the ground accelerations at stations k and l, the correlation between the ground displacement at station k and the ground acceleration at station l, and the correlation between the ith modal response at station k and the ground acceleration at station l. Similar to the original MSRS rule, the cross-correlation coefficients in the extended rule are determined in terms of the modal properties of the structure and the ground excitation, the latter described in terms of the mean response spectra, the mean peak displacement and acceleration at each support degrees of freedom, and the coherency function characterizing the spatial variability.

5. MEASURES OF PARTICIPATING MODAL MASS

The measure commonly used in engineering practice to define the number of modes required in the dynamic analysis of ordinary structures is

$$r_n^U = \frac{\sum_{i=1}^{n} M_i \gamma_i^2}{M_T} \tag{1.8}$$

where $M_i = \boldsymbol{\varphi}_i^{\mathrm{T}} \mathbf{M} \boldsymbol{\varphi}_i$, $\gamma_i = (\boldsymbol{\varphi}_i^{\mathrm{T}} \mathbf{M1}) / M_i$, and M_T is the total structural mass associated with the unconstrained degrees of freedom. This ratio is a measure of the accuracy in the estimates of base shear forces for the case of uniform support motions with only n modes included in the analysis. Kahan [4] developed a measure of the accuracy in the base shear forces for the case of totally incoherent ground motions:

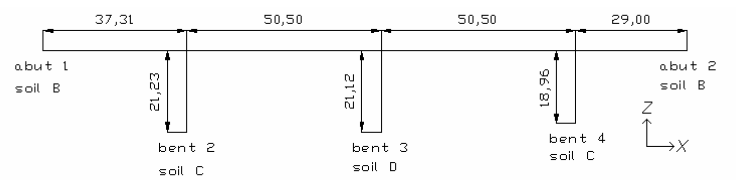
$$r_n^{TI} = \frac{\sum_{i=1}^{n} M_i \sum_{k=1}^{m} \beta_{ki}^2}{\sum_{k=1}^{m} \mathbf{r}_k^{\mathrm{T}} \mathbf{M} \mathbf{r}_k} \tag{1.9}$$

The derivation of this measure is based on the original MSRS formulation, where the cross-modal correlations as well as the cross-correlations between the modal responses and support displacements have been neglected.

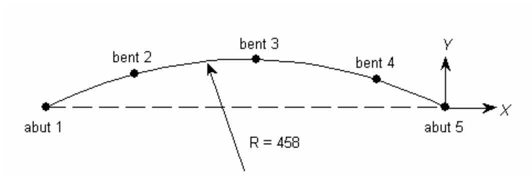
6. APPLICATION TO EXAMPLE BRIDGE

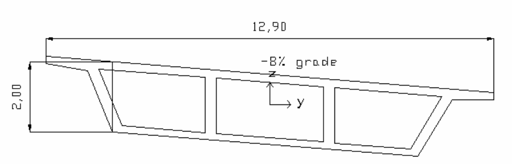
6.1 Description of the structure

As an example application, we consider the model of an existing bridge designed by Caltrans (California Department of Transportation). The elevation, plan and girder cross-section are shown in Figure 1. Each bent consists of a single column with circular cross section of diameter $D = 2.13\,\mathrm{m}$. The structure is made of concrete with a nominal compressive strength of $25\,\mathrm{MPa}$ for the columns and $28\,\mathrm{MPa}$ for the girder. Moment-curvature analysis indicated that the effective flexural stiffness of the columns is 25% of the uncracked flexural stiffness. The torsional moment of inertia of the columns is reduced to 20% of its uncracked value . No stiffness reduction is required for the prestressed concrete box girder. The columns are considered rigidly connected to the deck at the top and fixed at the bottom. The response of the seat abutments at the two ends of the bridge is modeled through two translational springs, one longitudinal and one transverse. The stiffness of the longitudinal spring is calculated by adjusting the initial embankment fill stiffness proportional to the backwall height. The stiffness of the nominal transverse spring is equal to 50% of the transverse stiffness of the adjacent bent. Vertical translations at the end supports are fully constrained, but free rotations are allowed in all directions.



Elevation





Plan View

Cross Section of the Girder

Figure 1 Bridge model (Dimensions are given in meters)

The finite element model of the bridge consists of 3 elements per bent and 6, 8, 8 and 4 elements in the 1st, 2nd, 3rd and 4th span from the left, respectively. The longitudinal axis of a girder element passes through the centroid of the girder. Three vertical rigid frame elements are used for the connection of the upper column elements with the girder elements. Condensing out the rotational degrees of freedom, the structure is modeled with 103 translational unconstrained degrees of freedom and 15 translational support degrees of freedom. The fundamental period of the structure is $T = 2.39\,\text{sec}$. All modes are assumed to have 5% modal damping.

6.2 Representation of the ground motion

The supports are subjected to translational ground motions in the longitudinal, X, transverse, Y and vertical, Z, directions (see Figure 1). The ground motions in these directions are considered to be statistically independent. The horizontal ground motions at each support are described by the Acceleration Response Spectrum (ARS) curves recommended by Caltrans for the corresponding soil types, assuming a peak rock acceleration of 0.3 g and a moment magnitude $M_w = 6.5$ for the maximum credible earthquake. The vertical ground motion is described by the model proposed by Bozorgnia and Campbell [1], which defines the vertical spectral acceleration in terms of the horizontal spectral acceleration, the source-to-site distance and the local site conditions. In this example we consider the bridge to be located $20\,\text{km}$ away from the source. For the spectral values at low frequencies, we assume a spectral shape varying quadratically with frequency. The PSDs of ground accelerations that are consistent with the specified ARS are obtained according to the procedure described in [2] for each soil type and are shown in Figure 2.

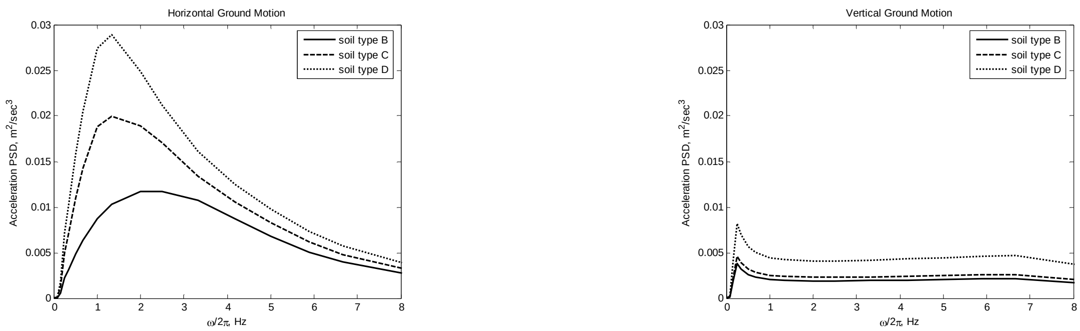


Figure 2 PSDs of Ground Acceleration for soil types B, C and D

The spatial variability of the ground motion is described by the coherency model developed in [3]

$$\gamma_{kl}(\omega) = |\gamma_{kl}(\omega)| \exp\{i[\theta_{kl}(\omega)^{wave\ passage} + \theta_{kl}(\omega)^{site\ response}]\} \tag{1.10}$$

in which $|\gamma_{kl}(\omega)|^{incoherence} = \exp[-(\alpha d_{kl}\omega / v_s)^2]$ describes the incoherence component, $\theta_{kl}(\omega)^{wave\ passage} = -\omega d_{kl}^L / v_{app}$ is the phase shift due to the wave-passage effect, and $\theta_{kl}(\omega)^{site\ response} = \tan^{-1}\{\text{Im}[h_k(\omega)h_l(-\omega)] / \text{Re}[h_k(\omega)h_l(-\omega)]\}$ is the phase sift due to the site-response effect. In these expressions, α is an incoherence parameter, d_{kl} is the distance between supports k and l, v_s is the shear wave velocity of the ground medium, d_{kl}^L is the projected horizontal distance in the longitudinal direction of wave propagation, v_{app} is the surface apparent wave velocity, and $h_s(\omega)$, $s = k, l$, is the frequency response function for the absolute acceleration response of the site associated with the s th support degree of freedom. In the current example we use the frequency response function

$$h_s(\omega) = \frac{\omega_s^2 + 2i\zeta_s\omega_s\omega}{\omega_s^2 - \omega^2 + 2i\zeta_s\omega_s\omega} \tag{1.11}$$

Table 2 Filter Parameters consistent with Acceleration Response Spectra

Response spectrum	ω_s (rad/sec)	ζ_s
Horizontal, soil type B (rock)	6.5 π	0.8
Horizontal, soil type C (soft rock)	5.0 π	0.8
Horizontal, soil type D (firm soil)	4.5 π	0.8
Vertical (soil types B, C, D)	13.0 π	0.8

which assumes that the soil column behaves as a single-degree-of-freedom oscillator with frequency ω_s and damping ratio ζ_s. In the subsequent analysis we use $\alpha / v_s = 1/600$ and $v_{app} = 400$ m/s. The values of ω_s and ζ_s consistent with the response spectra in Figure 4 are shown in Table 1. The waves are assumed to propagate in the direction of the *X* axis.

6.3 Results

Figure 3 shows the measures of the participating modal mass of the example bridge for the cases of uniform and totally incoherent support motions, r_n^U and r_n^{TI}, respectively, as a function of the number of modes considered. The figure suggests that the contribution of higher modes is more significant in the case of differential support motions. In the subsequent analysis, we assume that including the first 35 modes yields 'exact' results.

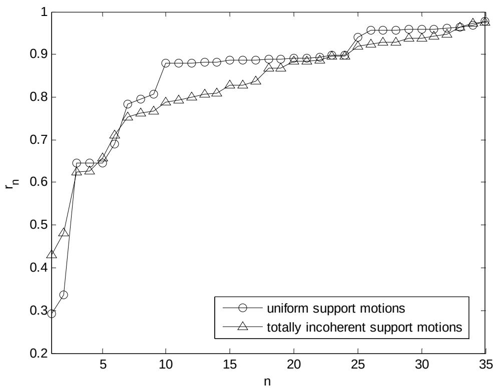


Figure 3 Participating modal mass for uniform and totally incoherent support motions

Figure 4 shows the ratios of the estimates of selected responses when the first $n = 1, \ldots, 10$ modes are included over the corresponding 'exact' estimates. The response quantities considered are the displacements, axial and shear forces, and torsional and bending moments at the middle of the 1st and 2nd spans. The displacements are given with respect to the global *X*, *Y* and *Z* axes, whereas the element forces and moments are given with respect to the local axes *x*, *y* and *z* of the girder cross section. Results based on the original MSRS rule as well as the extended MSRS rule are presented.

The results suggest that 2 or 3 modes are sufficient to accurately evaluate the displacement responses. High accuracy for almost all response quantities is achieved when the first 8 modes are considered. The corresponding measures of the participating modal mass are $r_8^U = 0.80$ and $r_8^{TI} = 0.76$. When fewer modes are included, the extended MSRS rule provides an improved approximation in most cases. However, in some cases, it significantly overestimates the response when very few modes are included. The peaks in the curves for F_y at $n = 7$ are due to the large values of the corresponding b_{ki} coefficients for the vertical support motions. We note that, for each vertical support motion, the coefficients b_{ki} of the closely spaced 7th and 8th modes have values of similar magnitude and opposite sign. Thus, the inclusion of the 8th mode balances the overestimation by the 7th mode. This observation suggests that pairs of closely spaced modes should not be separated in the analysis, especially when their corresponding coefficients b_{ki} have opposite signs.

The preceding analysis suggests that for structures subjected to non-uniform support motions satisfactory accuracy is achieved with a percentage of participating modal mass smaller than that required in the analysis of structures subjected to uniform support motions. This is because the response to non-uniform support motion includes a significant pseudo-static component, which is not affected by the modal truncation. Therefore, in order to obtain a better understanding of the modal contributions for non-uniform support motions, we need to examine the dynamic component of the total response, i.e. consider only the triple and fourfold sums in the MSRS formula, Eqn. (1.4). Figure 5 compares the modal contributions in the total response and the dynamic component of the response to non-uniform excitations with the modal contributions in the response to uniform excitations. We observe that although differential support motions can increase the contributions of higher modes, depending on the response considered, better accuracy may be achieved in the case of non-uniform excitations than in the case of uniform excitations, with the same number of modes included.

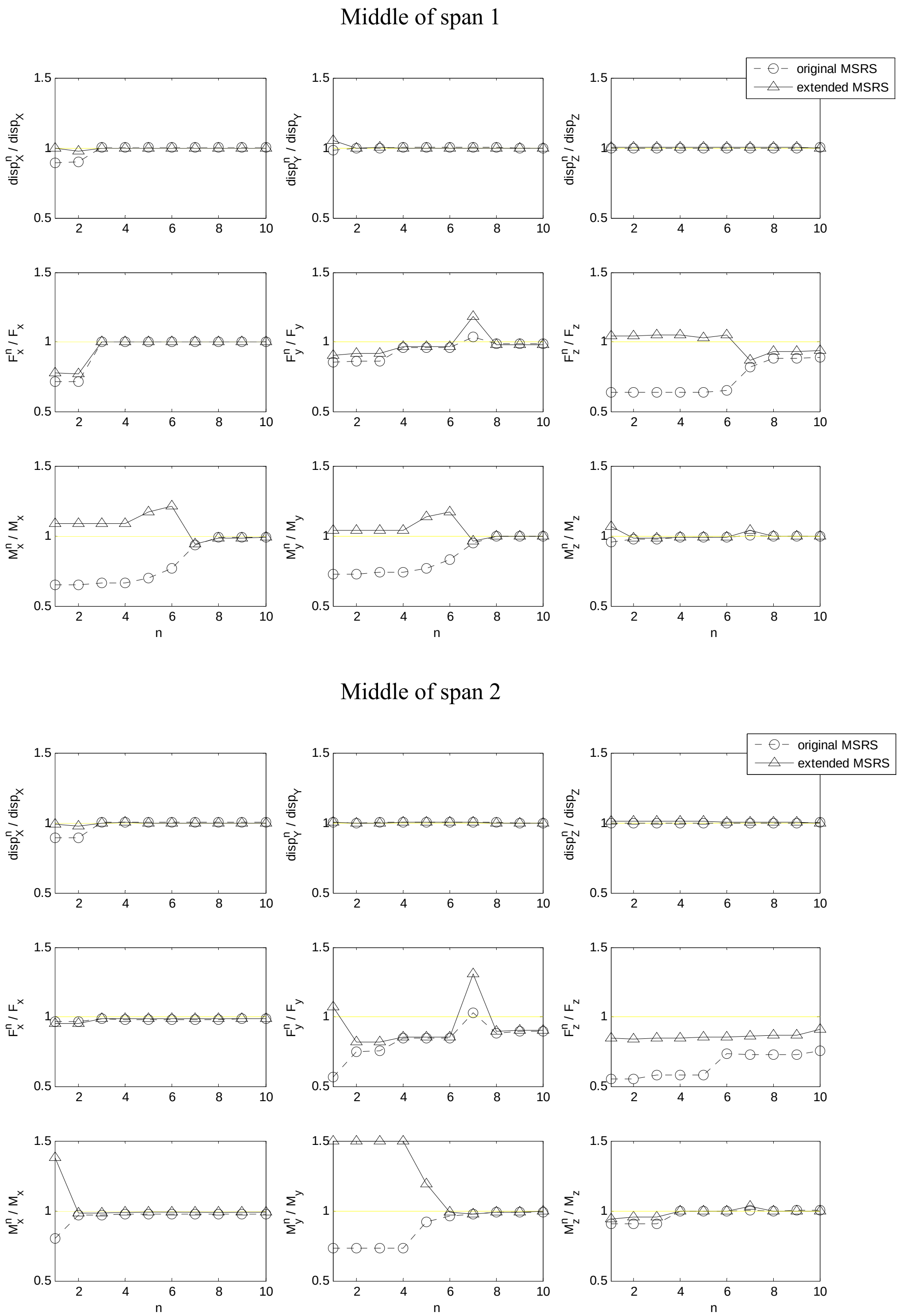


Figure 4 Response estimates based on the original and extended MSRS rules

Middle of span 2

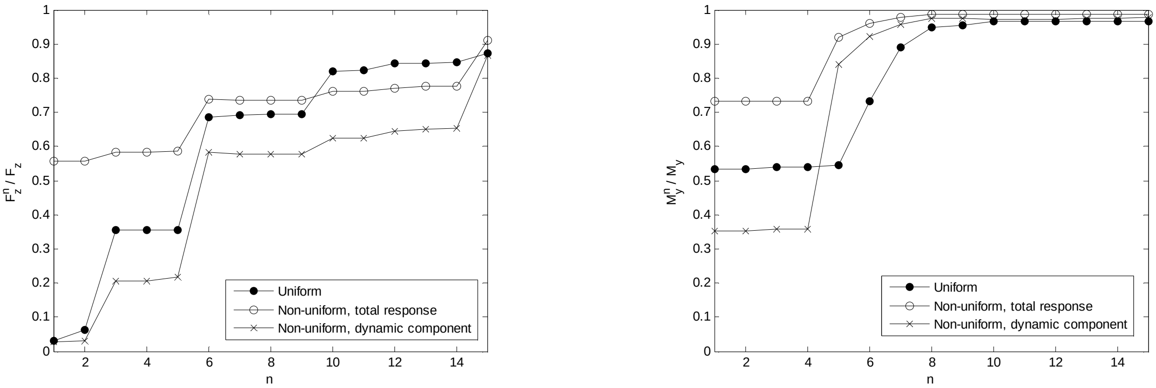


Figure 5 Comparison of modal contributions for uniform and non-uniform excitations

7 CONCLUSIONS

An evaluation of the modal contributions in the response of structures subjected to differential support motions is presented. An example application demonstrated that in order to achieve a desired level of accuracy, fewer modes are usually required in the case of non-uniform excitations than in the case of uniform excitations. This is because non-uniform support motions induce significant ‘pseudo-static’ contribution in the structural response, which is not affected by modal truncation. Improved results can be obtained with the extended MSRS rule that approximately accounts for the contributions of the truncated modes with small additional computational effort.

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