

# Effects of vertical ground motions on the seismic response of isolated structures with XY-Friction Pendulum system

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# **ABSTRACT:**

The effects of vertical ground motions on the response of isolated structures with XY-Friction Pendulum (XY-FP) systems are investigated. The structure is idealized as a three-dimensional singlestory building resting on the XY-FP system. The response of this idealized system subjected to three components (including vertical component) and two components (excluding vertical component) of Tabas 1978 earthquake excitations is investigated. The variation of the base shear and bearing displacement under variation of important structure parameters such as superstructure period, isolation period and sliding coefficient of friction is studied. It is demonstrated that error caused by neglecting the vertical component of earthquake in determining the peak bearing displacement and base shear of the structure are 4 and 31 percent, respectively.

**KEYWORDS:** XY-friction pendulum system, vertical ground motion, base shear, bearing displacement

# **1. INTRODUCTION**

In the recent years, the concept of base isolation has attracted considerable attention in the seismic design of buildings. The main idea is to isolate the structure from the ground, instead of the conventional techniques of strengthening the structural members. This procedure appears to have considerable potential in preventing earthquake damage to structures and their internal sensitive equipment. The devices, which isolate the structure at its base, have two important characteristics: horizontal flexibility and energy absorbing capacity. The flexibility of the isolation system increases the fundamental period of the structure, shifting it out of the region of dominant earthquake energy. The energy absorbing capacity increases damping, and therefore, reduces excessive displacements due to the lateral flexibility of the isolation system. A variety of isolation devices including elastomeric bearings (with and without lead core), frictional/sliding bearings and roller bearings have been developed and used practically for aseismic design of buildings during past years (Naeim et al., 1999 & Skinner et al., 1993).

Among the base-isolation devices, the FPS isolator proposed by Zayas et al. (1987) has been proven as an effective tool for isolating seismic transmitted energy in comprehensive experimental and numerical studies. Although FP system has a lot of advantages, but, like other system, it has its own defects. One of these weaknesses is the lack of an uplift restrainer device, which can leads to significant increase in the responses of the isolated structure with FP bearings (Almazan et al., 1998). The recent investigations have focused on mitigation of this defect, and one of the innovative proposed solutions is the usage of XY-FP bearings instead of conventional FP isolators. The XY-FP bearing is a modified Friction Pendulum bearing that consists of two perpendicular steel beams (rails) with opposing concave surfaces and a mechanical unit that connects the rails (the connector)(Fig. 1). The connector resists tensile forces, slides to accommodate translation along the rails and provides rotation capacity about a vertical axis. The idealized connection allows independent sliding in the two orthogonal directions when the XY-FP bearing is subjected to bi-directional (horizontal) excitation.





Figure 1 3D drawing of XY-FP bearing (Roussis, 2004)

The XY-FP bearing can be modeled as two uncoupled unidirectional FP bearings oriented along the two orthogonal directions (rails) of the XY-FP bearing (Roussis, 2004).

Roussis (2004) showed the effectiveness of the XY-FP bearings as an uplift-prevention isolation system in a 1/4- length-scale five-story isolated frame that was subjected to earthquake shaking applied in the vertical and one horizontal direction of the frame. Marin carried out a series of Numerical and experimental studies on an isolated truss-bridge model to study both the behavior of an XY-FP isolated system under three-directional excitation and the potential uses of XY-FP bearings for the seismic isolation of bridges (Marin, 2006). He indicated the effectiveness of this new generation of bearings as an uplift-prevention isolation system, it means that the XY-FP bearings can simultaneously resisted significant tensile loads and functioned as seismic isolators.

Herein, the response of an idealized three-dimensional structure isolated by XY-FP system subjected to three-component (including vertical component) and two-component (excluding vertical component) of earthquake excitations is investigated. The effect of vertical component of Tabas 1978 earthquake motions on the response of this idealized structure is illustrated for a wide range of structural parameters.

## 2. MATHEMATICAL MODELING OF XY-FP BEAIRBG

Because of the similarity between the XY-FP bearings and the friction pendulum bearings equations, this section will first review the friction pendulum bearings relations and then the XY-FP bearings relations will be examined. The force-displacement relationship of FP bearing undergoing unidirectional excitation can be described by (Zayas et al., 1987)

$$F_b = \frac{N}{R} u_b + \mu N \operatorname{sgn}(\dot{u}_b) \tag{1}$$

Where  $F_b$  is the bearing resisting force,  $u_b$  is the bearing displacement, N is the normal load on the bearing, R is the curvature radius of sliding surface,  $\mu$  is friction coefficient, and sgn is the signum function. The coefficient of sliding friction between the PTFE and stainless steel is known to be velocity dependent, which can be modeled as (Mokha et al., 1988)

$$\mu = f_{\max} - (f_{\max} - f_{\min}) \exp(-a|\dot{u}_b|)$$
<sup>(2)</sup>

Where  $f_{\min}$  is the coefficient of friction at a large sliding velocity,  $f_{\max}$  is the coefficient of friction at a low sliding velocity,  $\dot{u}_b$  is the sliding velocity, and *a* is a constant that controls the variation of the coefficient of friction with sliding velocity.



Roussis (Roussis, 2004) showed that for XY-FP bearings the bi-directional interaction between the shear force in one direction with the friction force in the other direction in small, so the bearing behavior can be modeled as a two independent FP bearings which are laid in two orthogonal directions. Therefore the force-displacement relationship of a sliding XY-FP bearing in each direction can be easily derived from Eqn. 1, and the general relationship takes the following matrix form:

$$\begin{bmatrix} F_{xb} \\ F_{yb} \end{bmatrix} = N \begin{bmatrix} u_{xb}/R_x \\ u_{yb}/R_y \end{bmatrix} + N \begin{bmatrix} \mu_x \operatorname{sgn}(\dot{u}_{xb}) \\ \mu_y \operatorname{sgn}(\dot{u}_{yb}) \end{bmatrix}$$
(3)

Where  $\begin{bmatrix} F_{xb} & F_{yb} \end{bmatrix}^T$  is the resisting force vector of XY-FP bearing;  $u_x$  and  $u_y$  are the bearing displacement in x and y directions, respectively;  $R_x$  and  $R_y$  are the radius of curvature of the rails oriented in x and y directions, respectively; and  $\mu_x$  and  $\mu_y$  are the coefficient of frictions of the rails laid in x and y directions, in the same order. Similar to Eqn. 2 the friction coefficients for each sliding direction can be computed using the followings equations:

$$\mu_x = f_{x\max} - \left(f_{x\max} - f_{x\min}\right) \exp\left(-a|\dot{u}_{xb}|\right) \tag{4.1}$$

$$\mu_{y} = f_{y\max} - \left(f_{y\max} - f_{y\min}\right) \exp\left(-a\left|\dot{u}_{yb}\right|\right)$$
(4.2)

The parameters presented in these equations have the same meaning as those defined for Eqn. 1. Herein, the subscripts x, and y stand for x-direction, and y-direction. In the current study, it is supposed that the sliding surfaces in both directions are identical, i.e.  $R_x = R_y$ ,  $f_{x\min} = f_{y\min}$ ,  $f_{x\max} = f_{y\max}$ , and  $f_{\min}$  and a for both surfaces are considered to be constant and equal to 0.03 and 100 s/m, respectively (Fenz et al., 2006 & Tsai et al., 2005).

The isolation period  $(T_i)$  of isolated structure using XY-FP bearing with identical sliding surfaces is independent of the superstructure weight and can be expressed using the following equation:

$$T_i = 2\pi \sqrt{\frac{2R}{g}} \tag{5}$$

Where R, having in mind that both surfaces are identical, is the radius of curvature of upper or lower rail (see Fig. 2).

#### 3. MODELING OF BASE-ISOLATED BUILDING

Fig. 2 represents the assumed structural system, which is an idealized three-dimensional single-story building model, mounted on a XY-FP bearing. The top mass  $m_s$  and base mass  $m_b$  are rigid decks supported on axially inextensible mass-less columns. The superstructure is assumed to be linear elastic. This is a reasonable assumption, since the purpose of the base isolation is to reduce the earthquake forces on the structure. The center of mass (CM) of the top deck and the base deck are assumed to be vertically aligned. As a result, there is no torsional coupling.  $\beta$  Represents the ratio between the vertical and horizontal vibration frequency of the structure - typical values for  $\beta$  in frame buildings range between 5 and 15 and  $\beta = 7$  is selected for the present study (Almazan *et al.*, 1998).





Figure 2 Idealized thee-dimensional single-story structure resting on XY-FP bearing

The dynamic behavior of the investigated system subjected to earthquake excitation can be described by the following six degrees of freedom:  $u_{xs}$ ,  $u_{ys}$  and  $u_{zs}$  are the displacement of the superstructure at the center of top deck relative to the base deck and  $u_{xb}$ ,  $u_{yb}$  and  $u_{zb}$  are the base displacement at the center of base deck relative to the ground in x-, y- and z- directions, respectively. The equation of motion for the structure in vertical direction can be expressed in matrix form as:

$$\begin{bmatrix} m_s & m_s \\ 0 & m_b \end{bmatrix} \begin{bmatrix} \ddot{u}_{zs} \\ \ddot{u}_{zg} + \ddot{u}_{zb} \end{bmatrix} + \begin{bmatrix} \beta c & 0 \\ \beta c & 0 \end{bmatrix} \begin{bmatrix} \dot{u}_{zs} \\ \dot{u}_{zg} + \dot{u}_{zb} \end{bmatrix} + \begin{bmatrix} \beta^2 k & 0 \\ -\beta^2 k & 0 \end{bmatrix} \begin{bmatrix} u_{zs} \\ u_{zg} + u_{zb} \end{bmatrix} = \begin{bmatrix} -w_s \\ N - w_b \end{bmatrix}$$
(6)

Where  $w_s$  is the weight of superstructure deck and  $w_b$  is the weight of base deck;  $u_{zg}$  and  $\dot{u}_{zg}$  are displacement and velocity of the ground in vertical direction, respectively; and  $\ddot{u}_{zb}$  is the acceleration of base slab relative to the ground which can be written in the following form (Rabiei, 2008):

$$\ddot{u}_{zb} = \ddot{v}_x + \ddot{v}_y \tag{7}$$

Where:

$$\ddot{v}_x = \ddot{u}_{xb} \sin \frac{u_{xb}}{2R} - \frac{\dot{u}_{xb}^2}{2R} \cos \frac{u_{xb}}{2R} \tag{8}$$

$$\ddot{v}_{y} = \ddot{u}_{yb} \sin \frac{u_{yb}}{2R} - \frac{\dot{u}_{yb}^{2}}{2R} \cos \frac{u_{yb}}{2R}$$
 (9)



Where  $u_{xb}$ ,  $\dot{u}_{xb}$  and  $\ddot{u}_{xb}$  are displacement, velocity and acceleration of the base slab relative to the ground in x direction and  $u_{yb}$ ,  $\dot{u}_{yb}$  and  $\ddot{u}_{yb}$  are displacement, velocity and acceleration of the base slab relative to the ground in y direction; and R is the curvature radius of sliding surface. The corresponding equation of motion for the top deck can be expressed by:

$$\begin{bmatrix} m_s & 0\\ 0 & m_s \end{bmatrix} \begin{bmatrix} \ddot{u}_{xs}\\ \ddot{u}_{ys} \end{bmatrix} + \begin{bmatrix} c & 0\\ 0 & c \end{bmatrix} \begin{bmatrix} \dot{u}_{xs}\\ \dot{u}_{ys} \end{bmatrix} + \begin{bmatrix} k & 0\\ 0 & k \end{bmatrix} \begin{bmatrix} u_{xs}\\ u_{ys} \end{bmatrix} = -\begin{bmatrix} m_s & 0\\ 0 & m_s \end{bmatrix} \begin{bmatrix} \ddot{u}_{xb} + \ddot{u}_{xg}\\ \ddot{u}_{yb} + \ddot{u}_{yg} \end{bmatrix}$$
(10)

Also the equation of motion for the base deck can be written in the form of

$$\begin{bmatrix} m_s & 0\\ 0 & m_s \end{bmatrix} \left\{ \begin{bmatrix} \ddot{u}_{xs}\\ \ddot{u}_{ys} \end{bmatrix} + \begin{bmatrix} \ddot{u}_{xb} + \ddot{u}_{xg}\\ \ddot{u}_{yb} + \ddot{u}_{yg} \end{bmatrix} \right\} + \begin{bmatrix} m_b & 0\\ 0 & m_b \end{bmatrix} \begin{bmatrix} \ddot{u}_{xb} + \ddot{u}_{xg}\\ \ddot{u}_{yb} + \ddot{u}_{yg} \end{bmatrix} + \begin{bmatrix} F_{xb}\\ F_{yb} \end{bmatrix} = 0$$
(11)

Where  $\begin{bmatrix} F_{xb} & F_{yb} \end{bmatrix}^T$  is the bearing resisting force vector according to Eqn. (3). Eqns. (6) to (11) are the governing equations of motion for whole system. Finally the coupled differential equations of motion for the considered system are solved in the incremental form using Newmark's average acceleration method of integration.

Based on the above equations, a computer program was written in Matlab to investigate the effect of vertical ground motions on the response of isolated structures using XY-FP system. The results obtained by this program are discussed.

#### 4. NUMERICAL STUDY

The response of three-dimensional single-story building resting on XY-FP bearing subjected to two and three component of earthquake excitations has been investigated. The effect of vertical component of earthquake on the peak bearing displacement and base shear of the isolated structure are illustrated. The damping ratio of the structure is assumed to be 2 percent of critical damping. For the present study the mass ratio  $m_s/m_b$  is supposed to be constant with  $m_s/m_b = 1$ . The parametric studies are carried out, by applying the 1978 Tabas earthquake records (Tabas receiving station). A small time interval  $\Delta t = 0.001s$  is employed for the computations.

Fig. 3 shows the hysteretic loops of the bearing (normalized with the total weight of the structure), with and without vertical component of earthquake. In the small bearing displacement, because of the insignificance of the vertical acceleration of the ground, the obtained hysteretic diagrams are of the three component earthquake matching the two component earthquake, but when the displacement grows, because of the increase in the variations of the vertical acceleration of the ground, the variations in the diagrams of the 3-component earthquake begin, and the diagrams start to depart. Fig. 4 shows the time variation of the bearing displacement of a structure isolated by the XY-FP bearing under Tabas record. Diagrams for bearing displacement under two and three components are identical along both directions. And these diagrams indicate the negligible effect of vertical component of earthquake on the bearing displacement. In Fig. 5, there is a significant difference in the peak base



shear for two-component and three-component of earthquake. According to this figure for this case the error in x and y directions are -4 and -16 percent respectively.

In Fig. 6 the error variation, as a result of ignoring vertical component, of the peak base shear and bearing displacement are plotted against the superstructure period (fixed-base period,  $T_s$ ), isolation period ( $T_i$ ) and maximum coefficient of friction ( $f_{\rm max}$ ), respectively. It can be seen in all of these diagrams that the bearing displacement affected insignificantly by the vertical component of earthquake, and the maximum recorded bearing displacement error is less than 4 percent that is negligible and can be omitted in design of isolated structures with XY-FP bearing. Fig. 6.a shows that increase of superstructure period results in decrease of the peak base shear error. For example, for  $T_s = 0.1Sec$  the error level is 17 percent and it decreases up to 3 percent as the superstructure period results 1.5 Sec.

The effects of isolation period on the base shear error of the isolated structure with XY-FP bearing is shown in Fig. 6.b. It can be clearly seen that the level of error caused by the failure to incorporate vertical component in calculating the base shear of the structure decreases, when there is an increase in the isolation period. For instance, there has been an error of 31 percent for the  $T_i = 1.5$  Sec, where the value of the error decreases to 3 percent when the period increases up to 3 seconds. Fig. 6.c illustrates the insignificant effect of coefficient of friction on the error of both bearing displacement and base shear. For  $f_{\text{max}} = 0.03$  the base shear error is about 15 percent and it increases slightly up to 22 percent as the friction coefficient approaches 0.15. Looking at the diagrams relating to the base shear error, one can observe that the base shear error is a negative one and this indicates the fact that neglecting the vertical component of earthquake in determining the peak base shear of the structure results in its underestimation.



Figure 3 Hysteresis loops of bearing in X and Y directions under two and three components of Tabas earthquake excitation ( $T_i = 2s$ ,  $T_s = 0.1s$ ,  $f_{max} = 0.1$ )



## **5.** Conclusions

Effects of vertical ground motions on an idealized three-dimensional single-story building resting on XY-FP bearing subjected to Tabas earthquake motion has been investigated. This study considers the superstructure period, the isolation period, and the maximum value of bearing coefficient of friction as the variable parameters of the XY-FP base-isolated structure, and attempt has been made to investigate the effect of variation in these parameters on the effects of vertical ground motions on the response of the isolated structure. From the trends of the results of the present study, following conclusions are drawn.

- 1. The maximum error caused by neglecting the vertical component of earthquake in determining the peak bearing displacement and base shear of the structure are 4 and 31 percent, respectively.
- 2. When the superstructure period increases, the error caused by ignoring the vertical component of earthquake in determining the maximum base shear of structure, decreases. It means that the effect of vertical component on the base shear of the structure is less for significant superstructure period.
- 3. Increasing of isolation period results in decreasing of the peak base shear error.
- 4. Failure to incorporate the vertical component of earthquake results in an underestimation of the base shear of structure compared with its exact value.

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Figure 4 Time variation of bearing displacement in (a) X and (b) Y directions under Tabas earthquake excitation ( $T_i = 2s, T_s = 0.1s, f_{max} = 0.1$ )



Figure 5 Time variation of base shear in (a) X and (b) Y directions under Tabas earthquake excitation  $(T_i = 2s, T_s = 0.1s, f_{max} = 0.1)$ 





Figure 6 Error variation of the peak bearing displacement and base shear versus (a) superstructure period  $(T_s)$  (b) isolation period  $(T_i)$  (c) maximum coefficient of friction  $(f_{max})$