SIMPLIFIED STOCHASTIC ANALYSIS OF REINFORCED CONCRETE FRAMES UNDER SEISMIC LOADS

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ABSTRACT :

Proper seismic performance of a structure is highly dependent on the mechanical properties of structural materials. A stochastic analysis allows us to evaluate the effect of variability of such mechanical properties on seismic performance of structures. The goal of this investigation was to perform a simplified stochastic analysis of reinforced concrete frames under seismic loads. Simplified analyses were performed based on Point Estimate Method, considering compressive concrete strength (f'_c) and reinforcement steel yield stress (f_y) as independent stochastic variables. Some application examples are presented concerning reinforced concrete cross-sections, structural members and frame structures. Seismic performance of frame structures was evaluated with pushover analyses. Results obtained with Point Estimate Method were validated with results obtained with Monte Carlo Simulation Method. It was concluded that Point Estimate Method can be used to perform simplified probabilistic analyses of structures under seismic actions. More refined results can be obtained with Monte Carlo Simulation Method, which requires a greater amount of numerical evaluation of seismic responses.

KEYWORDS: reinforced concrete, frames, seismic behavior, Point Estimate Method, Monte Carlo Simulation Method

1. INTRODUCTION

Modeling, design, detailing, and construction of a reinforced concrete structure are performed to guaranty its adequate structural and seismic performance. However, some aspects of performance such as strength, ductility, and absorbing-dissipating energy capacity depend upon the mechanical properties of structural materials. Usually there is a great amount of uncertainty due to the variability of such mechanical properties. A stochastic analysis allows us to evaluate the effect of such variability on the seismic performance of the structure.

Monte Carlo Simulation Method (MCSM) is a widespread and relatively simple method to perform stochastic analyses, but it has the disadvantage of requiring an enormous amount of numerical effort to obtain reliable results. Otherwise, there are methods which require lesser numerical efforts to perform simplified, but equally useful, stochastic analyses. Point Estimate Method (PEM) is one of such methods. The goal of this investigation is to perform a simplified stochastic analysis of reinforced concrete frames under seismic loads. Simplified analyses are performed based on Point Estimate Method, considering compressive concrete strength (f'_c) and reinforcement steel yield stress (f_y) as independent stochastic variables. Special attention is focused on non-linear behavior of reinforced concrete structures when subjected to seismic loads. Results obtained with Point Estimate Method are validated with results obtained with Monte Carlo Simulation Method.

2. STOCHASTIC ANALYSES

2.1. Monte Carlo Simulation Method

MCSM is a numerical method to solve problems through simulation of stochastic variables. MCSM has been widely used in civil engineering problems (see for instance Benjamin and Cornell, 1981, and Ang and Tang, 1984). More recently MCSM has been used to analyze behavior of structures under seismic loads. Esteva *et al*. (2002) obtained mean values and standard deviations for structural responses. Towashiraporn *et al*. (2002) calculated probabilities of damage for unreinforced masonry structures with and without passive energy dissipation devices. Vukazich *et al*. (2002) calculated probabilities of failure for structures. Chen and Li (2004) studied reliability for non-linear behavior of structures. Marubashi *et al*. (2004) studied the effect of strength variability of structural members on seismic behavior of structures. Biondini y Toniolo (2004) validated seismic design criteria for frame structures with results obtained from pseudo static tests performed on natural scale structures.

Results obtained with MCSM can be considered as “exact results”, since they are among the best available approximation to “actual results”. However, MCSM has the disadvantage of usually requiring an enormous amount of numerical evaluation of structural responses. Details for implementation of MCSM can be found in technical literature (see for instance Melchers, 1999).

2.2. Point Estimate Method

PEM was originally proposed by Rosenblueth (1975) and it basically considered that a function of one stochastic variable (y = f(x)) can be lumped at two points:

$$
\begin{aligned}
y_+ &= f(x_+) = f(m_x + s_x) \\
y_- &= f(x_-) = f(m_x - s_x)
\end{aligned}
\tag{1}
$$

Where m_x is the mean and s_x is the standard deviation of the variable x. The mean and variance of y can be obtained by using the following expressions:

$$m_y = y_+ \cdot P_+ + y_- \cdot P_- \qquad (2)$$

$$s_y^2 = (y_+ - m_y)^2 \cdot P_+ + (y_- - m_y)^2 \cdot P_- \qquad (3)$$

Where $P_- = P_+ = P = ½$ if the probability function of x is considered symmetrical. This concept can be generalized for functions of several stochastic variables. If the function y involves n stochastic variables, the number of terms will be equal to 2^n and the probability P will be $P = 1/2^n$ assuming that the n variables are independent and have symmetric probabilistic distributions.

MEP has the advantage of requiring a lesser amount of numerical evaluation of responses than MCSM. However, it has the disadvantage of requiring knowing the probabilistic distribution of responses for probability evaluation purposes.

3. APPLICATION EXAMPLES

Some application examples of MCSM and PEM are presented herein, considering compressive concrete strength (f'_c) and reinforcement steel yield stress (f_y) as independent stochastic variables. Special attention is focused on non-linear behavior under seismic actions.

3.1. Example 1 - Reinforced Concrete Cross-Section

Reinforced concrete section considered in this example was a 0.40m x 0.40m square section, reinforced with eight Nº 7 bars as showed in Figure 1. The variables considered for section analyses were yielding moment and curvature (M_y and ϕ_y), ultimate moment and curvature (M_u and ϕ_u), and curvature ductility (μ_ϕ). Analyses were performed with a computational code developed by the author in *Matlab* environment (MathWorks, 1999). The code considered a stress-strain curve for unconfined concrete, as proposed by Hognestad, with ultimate strain ε_{cu}=0.003, and a bilinear stress-strain curve for reinforcing steel with a post-yield slope of 0.02 E_s (see for instance Park and Paulay, 1975).

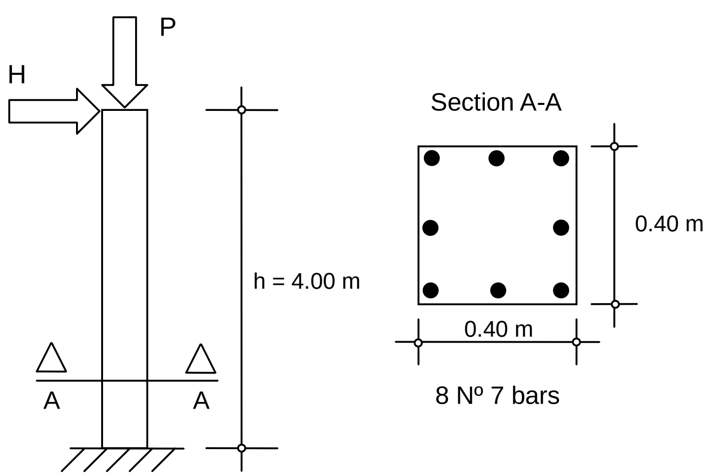

Figure 1 - Reinforced concrete column under seismic loads

3.1.1. Materials

Mean values, coefficients of variation, and probabilistic distributions considered for f'_c and f_y are presented in Table 1, according to Melchers (1999). One hundred values of f'_c and f_y were generated with MCSM, whose histograms are showed in Figure 2. Table 2 contains the mean values and coefficients of variation obtained for values of f'_c and f_y generated with MCSM.

Table 1 - Mean values, coefficients of variation and probabilistic distribution for f'_c and f_y

Variable	Mean (MPa)	CV	Distribution
f'_c	25	0.15	Normal
f_y	420	0.05	Normal

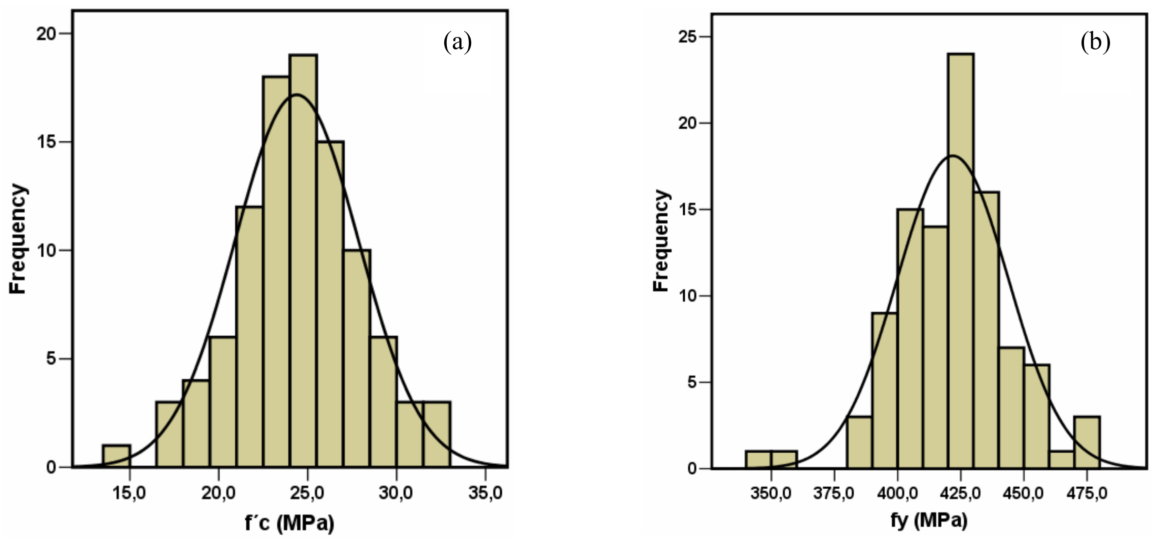

Figure 2 - Histograms for (a) f'_c and (b) f_y generated with MCSM

Table 2 - Mean values and coefficients of variation for f'_c and f_y generated with MCSM

Variable	Mean (MPa)	CV
f'_c	24.4	0.143
f_y	421.8	0.052

3.1.2. Section without Axial Load

The reinforced concrete section was first analyzed without axial load (P=0), as a usual condition for reinforced concrete beams. MCSM results were obtained after performing one hundred evaluations using the f'_c and f_y values previously obtained. PEM results were obtained with only four evaluations as it was considered two stochastic variables. Table 3 contains the results obtained with MCSM and PEM. Results obtained with PEM showed a good agreement with those obtained with MCSM.

Table 3 - Results obtained with MCSM and PEM for a reinforced concrete section without axial load

Variable	Units	MCSM		PEM	
		Mean	CV	Mean	CV
My	N m	168265	0.055	167863	0.057
ϕy	rad/m	8.57E-03	0.054	8.50E-03	0.065
Mu	N m	218429	0.047	218560	0.049
ϕu	rad/m	3.77E-02	0.076	3.83E-02	0.100
$\mu\phi = \phi u / \phi y$	-	4.43	0.119	4.54	0.155

3.1.3. Section with Axial Load

The section was then analyzed with axial load (P=1000 kN), as a usual condition for reinforced concrete columns. Table 4 contains the results obtained with MCSM and PEM. Results obtained with PEM showed a good agreement with those obtained with MCSM.

Table 4 - Results obtained with MCSM and PEM for a reinforced concrete section with axial load

Variable	Units	MCSM		PEM	
		Mean	CV	Mean	CV
My	N-m	283046	0.041	283805	0.043
ϕy	rad/m	1.17×10^{-2}	0.062	1.15×10^{-2}	0.082
Mu	N-m	302105	0.058	304153	0.063
ϕu	rad/m	1.92×10^{-2}	0.087	1.96×10^{-2}	0.104
$\mu\phi = \phi u / \phi y$	-	1.66	0.139	1.71	0.176

3.2. Example 2 - Reinforced Concrete Column

The reinforced concrete column showed in Figure 1 was analyzed with an axial load (P=1000 kN) under the action of a lateral load (H). The seismic responses considered were maximum lateral load (H_{max}), maximum lateral displacement at the top of the column (Δ_{max}), and displacement ductility (μ_Δ).

Maximum lateral load was evaluated with Equation 4, considering flexural strength at the base of the column. Maximum lateral displacement at the top of the column was evaluated with Equation 5, which considers in a simplified way the non-linear behavior of the column (see for instance Paulay and Priestley, 1992). Plastic hinge length (l_p) in Equation (5) was evaluated with Equation (6), where d_b is the diameter of reinforcing steel bars (Paulay and Priestley, 1992).

$$H_{max} = \frac{M_u}{h} \tag{4}$$

$$\Delta_{max} = \left(\frac{\phi_y \cdot h^2}{3}\right) + (\phi_u - \phi_y) \cdot l_p \cdot \left(h - \frac{l_p}{2}\right) \tag{5}$$

$$l_p = 0.08 \cdot h + 0.022 \cdot d_b \cdot f_y \qquad \text{(in MPa)} \tag{6}$$

Table 5 contains the results obtained with MCSM and PEM. MCSM results were obtained after performing one hundred evaluations, using the ϕ_y and ϕ_u values previously obtained with the analysis of the reinforced concrete section. PEM results were obtained with four evaluations, as only f'_c and f_y were considered as stochastic variables in the analysis. Results obtained with PEM showed a good agreement with those obtained with MCSM.

Table 5 - Results obtained with MCSM and PEM for a reinforced concrete column

Variable	Units	MCSM		PEM	
		Mean	CV	Mean	CV
H_{max}	kN	75.5	0.058	76.0	0.063
Δ_{max}	m	0.0772	0.028	0.0774	0.031
$\mu_\Delta = \Delta_{max} / \Delta_y$	-	1.24	0.068	1.26	0.089

3.3. Example 3 - Reinforced Concrete Frame

The reinforced concrete frame selected for this example is showed in Figure 3, which was typical for reinforced concrete structures designed in Venezuela in mid-1950s. First floor columns were 0.30m x 0.30m reinforced with four Nº 5 bars. First floor beam was 0.30m x 0.60m reinforced with three Nº 8 bars at bottom and two Nº 8 + two Nº 4 bars at top. Second floor columns were 0.25m x 0.25m reinforced with four Nº 4 bars. Second floor beam was 0.25m x 0.50m reinforced with four Nº 8 bars at bottom and two Nº 8 + two Nº 4 bars at top. The mean values considered for f'_c and f_y were 23 MPa and 276 MPa, respectively, while the coefficients of variation and the probabilistic distributions were the same showed in Table 1.

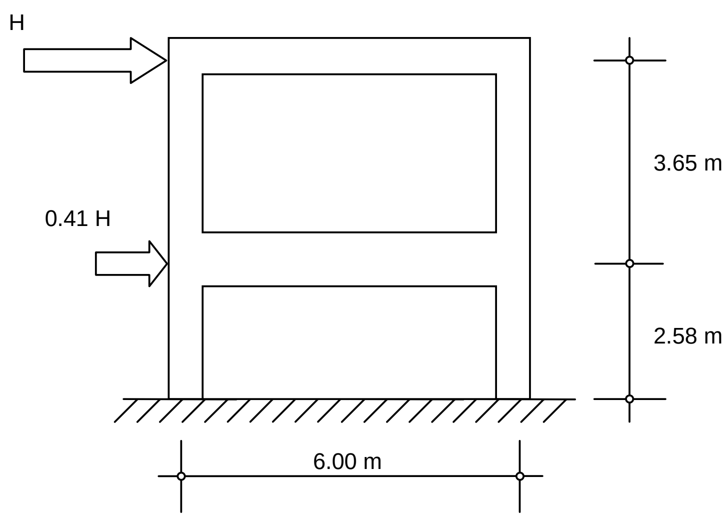

Figure 3 - Reinforced concrete frame under seismic loads

Non-linear seismic behavior was evaluated performing pushover analyses with commercial software SAP2000 (Computers and Structures, 2006). The pattern of lateral loads considered for pushover analyses was triangular as showed in Figure 2. Distributed gravity loads acting on first floor and second floor beams were 54.8 kN/m and 24.5 kN/m, respectively. Plastic hinges for beams were modeled with moment-curvature diagrams. Plastic hinges for columns were modeled with interaction diagrams and moment-curvature diagrams associated to axial load. Potential plastic hinges were located at the ends of all beams and columns. The seismic responses reported herein are maximum shear at the base (V_{max}) and maximum lateral displacement at the top of the structure (Δ_{max}).

Table 6 contains the results obtained with MCSM and PEM. The histograms obtained for V_{max} and Δ_{max} are showed in Figure 4. MCSM results were obtained after performing one hundred fifty evaluations. PEM results were obtained with four evaluations, as only f'_c and f_y were considered as stochastic variables in the analysis.

Mean values obtained with PEM showed an excellent agreement with those obtained with MCSM, while coefficient of variation showed errors not greater than 15%.

Table 6 - Results obtained with MCSM and PEM for a reinforced concrete frame

Variable	Units	MCSM		PEM	
		Mean	CV	Mean	CV
V_{max}	kN	45.8	0.076	45.8	0.083
Δ_{max}	m	0.155	0.107	0.154	0.123

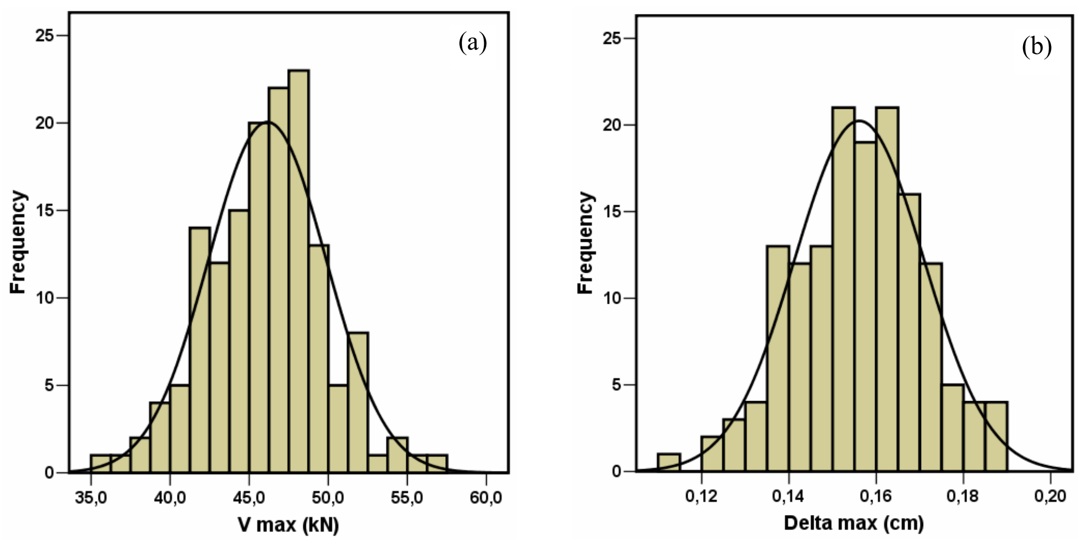

Figure 4 - Histograms for (a) V_{max} and (b) Δ_{max} generated with MCSM

4. CONCLUSIONS

Point Estimate Method can be used in practice to perform simplified stochastic analyses, even considering non-linear behavior of structures, when reinforced concrete frames are subjected to seismic loads. More refined results can be obtained with Monte Carlo Simulation Method, which requires a greater amount of numerical evaluation of seismic responses.

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