UTILIZATION OF RESIDUAL DISPLACEMENTS IN THE POST-EARTHQUAKE ASSESSMENT

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ABSTRACT:

A new post-earthquake seismic performance assessment method which takes into account residual displacements attained by the damaged structures is presented. The post-earthquake residual displacements and the visual inspection of the damage to the structure are considered to obtain improved estimates of the maximum deformations experienced by the structure. The improved estimates of the maximum deformations are expected to yield a better picture of the extent of damage to the structure. As a result, better informed decisions regarding the post-earthquake usability and repairability of the structure can be made. Both the uncertainties associated with the structural properties and the probable errors due to the imperfectness of the analytical models are considered in the proposed method. A trial application of the method to a real structure tested on a shaking table is presented.

KEYWORDS: post-earthquake assessment; residual displacement; damage assessment; response simulation error; Monte Carlo simulation; Bayesian analysis

1. INTRODUCTION

An essential part of the post-earthquake rebuilding process is the assessment of the damage to the structures. An accurate assessment of the extent of the damage to structures is critical to establish a reliable basis for taking decisions regarding their post-earthquake usability and/or repairability. However, due to the prevailing uncertainties related to the structure, the site and the seismic event, a reliable assessment of the damage is a challenging task. In order to determine the optimal allocation of the resources for rebuilding, all the important sources of uncertainties should be considered in the assessment. Furthermore, all the available information, i.e. evidence related to the seismic performance of the structure should be taken into account for a complete description of the problem. The residual displacements are the permanent displacements attained by the structure after a damaging earthquake and typically, they are the only measurable indicators of the damage to the structure after an earthquake. A new post-earthquake assessment method that takes the observable damage and the residual displacements into account, is presented in this study. Additionally, a more elaborate discussion of the method is available by Yazgan and Dazio (2008).

Direct consideration of residual displacements in the seismic design and assessment has been the focus of many studies (Kawashima *et al.*, 1998; Christopoulos *et al.*, 2003; Ruiz-Garcia, 2004, e.g.). Moreover, several methods have been proposed to utilize the residual displacements in the post-earthquake damage assessment. Mackie and Stojadinovic (2004) proposed a decision-making framework for reinforced concrete highway bridges based on the loss of load carrying capacity. Extending this framework, they developed a decision-making tool to assess the condition of highway bridges based on their residual displacements. Bazzurro *et al.* (2004) and Luco *et al.* (2004) proposed a post-earthquake performance assessment method focusing on the assignment of occupancy statuses to damaged buildings. They proposed a non-linear static analysis method that considers the residual displacement while estimating the aftershock ground motion intensity the building can resist without collapsing.

The previous studies briefly presented above have all provided valuable insight into the various aspects of the residual displacements and introduced different ways of considering these displacements in seismic design and assessment. However, the uncertainty due to the errors associated with the adopted response prediction models were not directly taken into account. The results presented in this study suggests that the probable errors associated with the predicted residual displacements are larger compared to the peak displacements. In view of this fact, the method proposed here directly takes into account the uncertainty in the predicted response quantities due to the imperfectness of the adopted analytical model. Another novel aspect of the study is the direct consideration of visual damage indicators (e.g. the presence or absence of cracks, ruptured reinforcement bars) in the updating of the uncertainties related to the extent of damage.

2. POST-EARTHQUAKE ASESSMENT METHOD

The proposed method aims to estimate, after a damaging earthquake, the peak displacement experienced by the structure during the earthquake. First, observable damage indicators and, subsequently, known residual drift ratios are taken into account in the method to improve the estimates of the peak deformations. The method takes into account the uncertainties arising from the following sources: (1) limited knowledge about the properties of the structure and the site, (2) the errors associated with the models used to predict the seismic response and the deformation limits, and (3) uncertainties regarding the excitation that caused the damage. The proposed method can be applied to estimate any deformation parameter of interest relevant to the considered structure, and that for this reason in this section a general average drift ratio will be addressed.

The experienced maximum average drift DM is considered as a random variable. Application of the method leads to the estimation of the probability of DM being in a specific drift interval dm_i, conditional on the observed damage and the measured residual displacement. This event, M_i is defined as follows:

$$M_i = \{DM > dml_i \cap DM \leq dmu_i\} \tag{2.1}$$

where dml_i and dmu_i are the lower and upper limits of the maximum drift interval dm_i. The five steps comprising the proposed method are presented below.

2.1. Step 1: Modeling of the structure

First, a deterministic analytical model of the structure is established. The model should allow the estimation of the maximum and the residual displacements experienced by the structure based on a given set of parameters. Any suitable modeling approach can be adopted into the proposed method. The accuracy of the model in terms of predicting the maximum and the residual displacements is directly taken into account in the assessment process. The method takes into account the probable error associated with the simulated maximum and residual drift ratios through random variables referred to as correction factors. Within this context, the correction factor is the ratio of the *true* value of the response parameter divided by the value that is simulated by adopting a specific modeling approach. The best practice to identify the probabilistic character of the correction factors, is to compare the simulation results against shaking table experiments for a set of test units. In the following, the random variables FM and FR are the correction factors associated with the predicted maximum and the residual average drift ratios, respectively.

2.2. Step 2: Estimating the prior probability distribution of the maximum drift ratio

In order to understand the probabilistic character of the structural response quantities, all uncertainties associated with the structure, site, the ground motion and the probable errors associated with the employed analytical models have to be considered in the assessment process. The Monte Carlo simulation method is a useful tool for numerically investigating this probabilistic character. To conduct the Monte Carlo simulations, the probability distributions of the model parameters (*e.g.* material model parameters, loading conditions, damping level) needs to be identified. Accordingly, sets of N random values are generated for the uncertain model parameters and the simulation is carried out for each realization. Based on the results of N simulations, the probability of M_i can be calculated as follows:

$$\Pr(M_i) = \sum_{n=1}^{N} \Pr(M_i|S_n)\Pr(S_n) \tag{2.2a}$$

$$\text{where } \Pr(M_i|S_n) = \int_{\frac{dml_i}{sm_n}}^{\frac{dmu_i}{sm_n}} f_{FM}(x)dx \tag{2.2b}$$

In the equation above the probability $\Pr(S_n)$ is equal to $1/N$, since all simulations are assumed to yield equally likely predictions; $\Pr(M_i|S_n)$ is the probability of M_i given the n^{th} simulation S_n; dmu_i and dml_i are the upper and lower limits of the drift interval dm_i; sm_n is the simulated maximum drift value obtained for the n^{th} simulation; $f_{FM}(x)$ is the probability density function of the correction factor for the simulated maximum drift ratio.

2.3. Step 3: Updating the distribution of maximum drift ratio based on observable damage indicators

In the light of the observed damage —such as presence of cracks in the regions of plastic hinging, spalled cover concrete, crushed core concrete, ruptured reinforcement bars or indications of pounding— the permissible range of maximum drift values can be identified. This can be achieved by estimating the probability of specific damage indicators being detected or not detected. These probabilities can be estimated for a given maximum drift ratio using the models that relate deformation limits of the components to specific damage indicators (Fardis and Biskinis, 2003, e.g.). In essence, the process of updating the probability of M_i based on observable damage indicators can be performed for any number of damage indicators. Here, only the case featuring two indicators is demonstrated: (1) indicators of deformation beyond yielding DI_y (*i.e.* the presence of unclosed cracks or spalled cover concrete) and (2) indicators of deformation beyond ultimate deformation capacity DI_u (*i.e.* the presence of ruptured reinforcements or crushed confined concrete). As an example case, the situation when during the post-earthquake inspection the indicator DI_y is detected and DI_u is not detected is considered here. Given that the experienced maximum drift DM is in the interval dm_i (i.e. given M_i), the probabilities of DI_y being detected and DI_u not being detected can be estimated as:

$$\Pr(I_y|M_i) \cong F_{L_y}(m) \tag{2.3}$$

$$\Pr(\overline{I_u}|M_i) \cong 1 - F_{L_u}(m) \tag{2.4}$$

where I_y and $\overline{I_u}$ are the events $I_y = \{DI_y = \text{'}detected\text{'}\}$ and $\overline{I_u} = \{DI_u = \text{'}not\ detected\text{'}\}$; L_y and L_u are the drift limits beyond which the indicators DI_y and DI_u become present; F_{L_y} and F_{L_u} are the probabilities of L_y and L_u being smaller or equal to the central value m of the considered drift interval dm_i. After the inspection is conducted, the posterior probability of M_i conditional on the inspection results can be calculated according to the Bayes' Theorem as follows:

$$\Pr(M_i|I) = \Pr(M_i|I_y \cap \overline{I_u}) = \frac{\Pr(\overline{I_u}|M_i \cap I_y)\Pr(M_i|I_y)}{\sum_j \Pr(\overline{I_u}|M_j \cap I_y)\Pr(M_j|I_y)} \tag{2.5a}$$

$$\text{where } \Pr(M_i|I_y) = \frac{\Pr(I_y|M_i)\Pr(M_i)}{\sum_k \Pr(I_y|M_k)\Pr(M_k)} \tag{2.5b}$$

where I is the inspection results (i.e. $I = \{I_y \cap \overline{I_u}\}$). Note that for a damaged structure, the indicator DI_u cannot be detected if the DI_y is not present. Therefore, $\Pr(\overline{I_u}|M_i \cap I_y)$ in essence equals to $\Pr(\overline{I_u}|M_i)$ and Eqn. 2.4 can be directly substituted into Eqn. 2.5a.

2.4. Step 4: Establishing the joint probability distribution of the maximum and residual drift ratio values

The residual displacements attained by a structure subjected to a damaging earthquake can be utilized to update the estimated maximum drift ratio distribution. The residual displacement DR attained by the structure is considered as a random variable within this context. The event R_j is the residual drift DR being in the residual drift interval dr_j and is defined as follows:

$$R_j = \{DR > drl_j \cap DR \le dru_j\} \tag{2.6}$$

where drl_j and dru_j are the lower and upper limits of the residual drift interval dr_j. In order to update the estimates of the maximum drift ratio based on the measured residual drift, the joint probability distribution of the maximum and residual drift ratio values is established. The results of the Monte Carlo simulations are utilized for this purpose. The joint probability of the experienced maximum and the residual drift ratios being in the drift intervals dm_i and dr_j, is estimated as follows:

$$\Pr(M_i \cap R_j) = \sum_{n=1}^{N} \Pr(M_i \cap R_j | S_n) \Pr(S_n) \tag{2.7a}$$

$$\text{where } \Pr(M_i \cap R_j | S_n) = \int_{\frac{dml_i}{sm_n}}^{\frac{dmu_i}{sm_n}} \int_{\frac{drl_j}{sr_n}}^{\frac{dru_j}{sr_n}} f_{FM,FR}(x,y)dxdy \tag{2.7b}$$

In the equation above $\Pr(M_i \cap R_j|S_n)$ is the joint probability of M_i and R_j given the n^{th} simulation S_n; the probability $\Pr(S_n)$ was already defined in Eqn. 2.2a; sm_n and sr_n are the maximum and residual drift ratios resulting from the n^{th} simulation; dmu_i, dml_i, dru_j and drl_j are the limits of the drift intervals dm_i and dr_j, as defined in Eqn. 2.1 and 2.6; $f_{FM,FR}(x,y)$ is the joint probability density function of the correction factors FM and FR associated with the simulated maximum and residual drift ratios respectively.

The results of the visual inspection of the damage can be reflected to this joint probability utilizing the Bayes' Theorem with similar approach to that pursued in Step 2 (Eqn. 2.5a). However, in this case the joint probability of M_i and R_j is considered instead of M_i. In the updating process, the optimal approach would be to adopt the deformation limit prediction models that relate the maximum and residual deformations to the observable damage indicators. However, since such models are currently not available, those solely based on the maximum drift (Eqn. 2.3 and 2.4) can be used instead. The resulting joint probability of M_i and R_j conditional on the inspection results I (*i.e.* $\Pr(M_i \cap R_j|I)$) is utilized in the next step.

2.5. Step 5: Estimating the maximum drift ratio based on the known residual drift ratio

The precision of the identified residual drift ratios is usually limited due to a number of factors such as: (1) the geometric accuracy of the adopted photogrammetric data acquisition technology, (2) the site conditions, (3) the complexity of the deformation pattern of the structure and (4) the limited knowledge on the pre-earthquake geometry of the structure. Therefore, inevitably there is uncertainty related to the *actual* value of the residual displacement even after the measurement is conducted. Taking into account this uncertainty, the probability $\Pr(R_j|MR)$ of R_j conditional on the measurement method MR can be estimated after the measurement.

The measured residual displacement can be utilized to improve the estimates of the experienced maximum drift ratio. In order to achieve this, the probability of the maximum drift DM being in the drift interval dm_i conditional on the measurement MR and the inspection results I is estimated. This probability is calculated as follows:

$$\Pr(M_i|I \cap MR) = \sum_j \frac{\Pr(M_i \cap R_j|I)}{\Pr(R_j|I)} \Pr(R_j|MR) \tag{2.8a}$$

$$\text{where } \Pr(R_j|I) = \sum_k \Pr(M_k \cap R_j|I) \tag{2.8b}$$

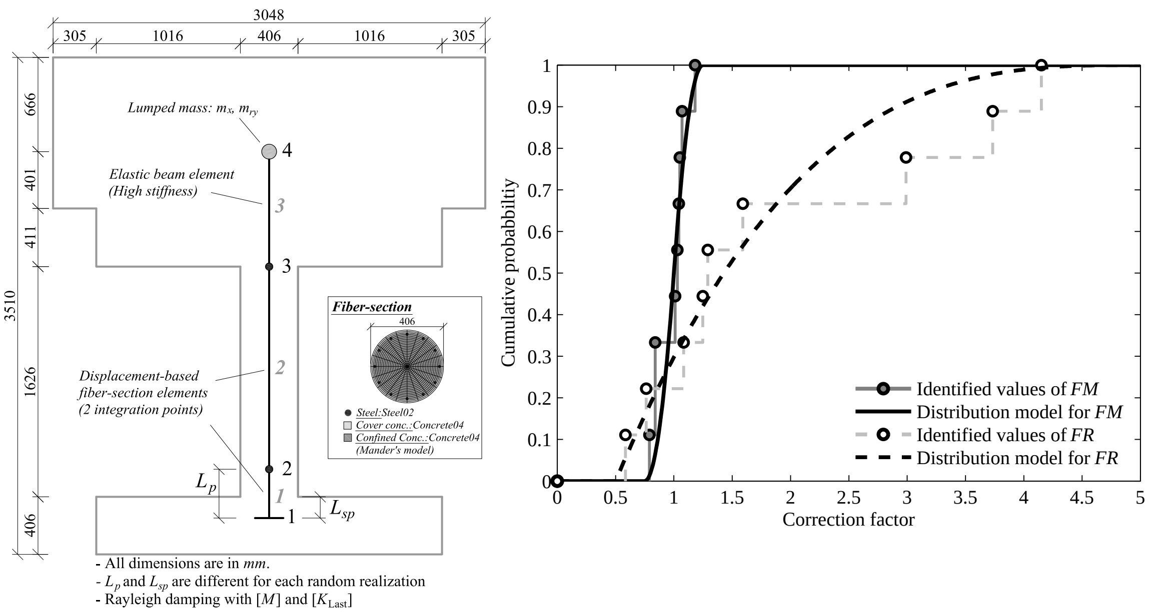
This resulting probability reflects both the uncertainties due to the limited knowledge related to model parameters and the imperfectness of the model that is used to predict the seismic response. Moreover, the evidences collected during damage inspection and the measurement of residual displacement are also accounted for in the estimates.

3. SAMPLE APPLICATION

The procedure presented in the previous section is now applied to a reinforced concrete (RC) bridge pier model tested on a shaking table. The test unit is assumed to be a structure that is to be assessed after the earthquake. The peak drift ratio experienced by the unit is assumed to be unknown and its value is to be estimated by taking into account the information (i.e. the structural drawings, the classification of the construction materials according to the adopted design code, magnitude and distance of the damaging earthquake) which may be available in a typical post-earthquake evaluation situation.

The RC column A1 tested on the shaking table by Hachem *et al.* (2003) is considered as the sample structure. During the test "Run 2" the unit was subjected to the stronger horizontal component of the *Olive View, Northridge 1994* record scaled to match the design intensity for the model. The response of the unit recorded by Hachem *et al.* (2003) exhibits a peak drift ratio of 5.1% and a residual drift ratio of 0.55%. In the following, only the fundamental points are presented, the details of the application are available in Yazgan and Dazio (2008).

***Step 1*:** The finite element model of the column is presented in Figure 1a. The analyzes were carried out using OpenSees (McKenna *et al.*, 2007). In order to identify the accuracy of the adopted modeling approach, the shaking table tests for 9 different RC test units were numerically reproduced by following the same modeling strategy. The simulated maximum and residual displacements were compared against the measured values to establish a set of correction factors related to the predicted maximum and the residual displacements. The probabilistic characters of the maximum drift correction factor FM and the residual drift correction factor FR, are identified based on this set of values (Figure 1b). At the moment, a comprehensive investigation aiming to identify the performances of different modeling approaches in terms of predicting the residual displacements exhibited by RC structures has been undertaken by the authors.

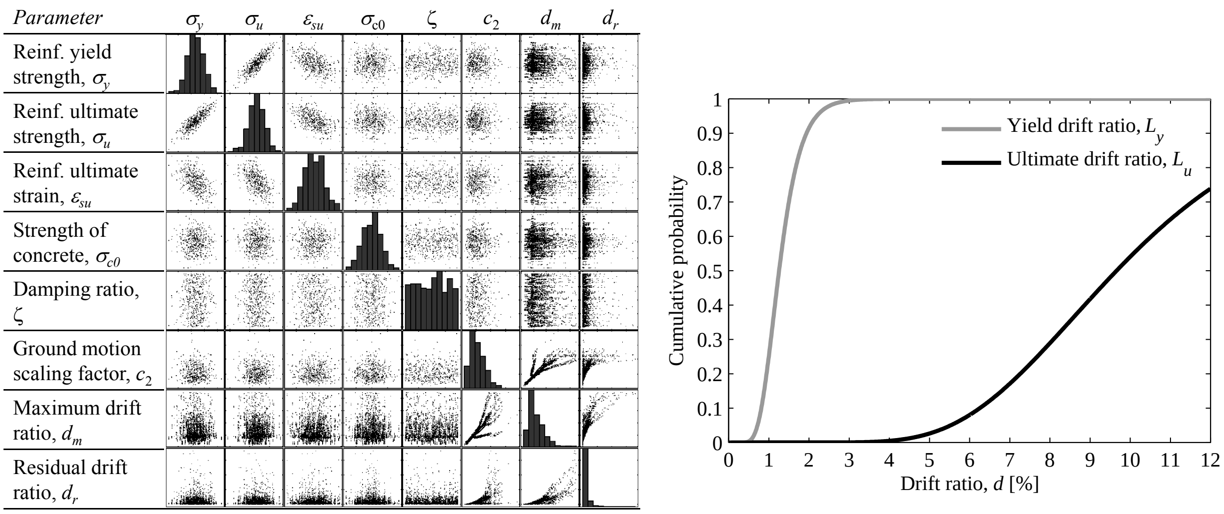


(a) Finite element model

(b) Maximum drift ratio correction factor, FM and the residual drift ratio correction factor, FR

Figure 1: Numerical model and the correction factors

***Step 2*:** It is assumed that —during the post-earthquake assessment— only the grades of the reinforcement steel and the concrete are known for the structure, and the actual material properties are assumed to be unknown. The recommendations by the Joint Committee on Structural Safety (2001) are followed to estimate the probabilistic characters of: the yield and the ultimate tensile strengths of the steel, the ultimate tensile strain capacity of the steel



(a) Scatter plot of the model input and output parameters

(b) Drift capacities for the yield and the ultimate deformation limits

Figure 2: Scatter plot matrix of the model parameters and probability distributions estimated for the yield and ultimate drift ratios

and the strength of concrete (Figure 2a). The damping ratios are assumed to be uniformly distributed within the range of 1–5% of the critical damping. It is assumed that the fundamental properties the seismic event and the site are known but a reliable record of the ground motion is not available. A set of six representative ground motions —that are recorded during similar events— is established to capture the uncertainty due to the exact ground motion being unknown. The ground motions are scaled by a log-normally distributed scaling factor estimated using the attenuation relationship by Campbell (1997). This scaling aims to capture the uncertainty in the intensity of shaking at the site. The scaling factor assumed for the shaking table test, is also taken into account and the time and the amplitude scaling of the ground motions are made accordingly. The results of 3000 nonlinear time-history analysis and the probability distribution of FM are used to establish the prior-probability distribution of the maximum drift ratio *DM* according to Eqn.2.2a (Figure 3, 1st series).

***Step 3*:** In Hachem *et al.* (2003), the visible damage to the test unit after the test *Run 2* is reported. Based on the reported damage, it is possible to infer that the system has deformed beyond the yielding. Furthermore, the reported damage also suggests that the ultimate deformation capacity of the test unit was not exceeded during the test “Run 2”. The probabilistic character of the yield and the ultimate drift limits are identified based on the study by Fardis and Biskinis (2003) (Figure 2b). In this sample application, the definition of the drift limits L_y and L_u, and the inspection I are the same as those defined in Section 2.3. The known inspection results I are reflected on the prior-distribution of *DM* according to Eqn. 2.5a. The resulting probabilities $\Pr(M_i|I)$ are presented in Figure 3 (2nd series).

***Step 4*:** The joint probability distribution of *DM* and *DR*, is established using Eqn. 2.7a. The resulting distribution is updated by taking into account the results of the inspection I according to the method described in Section 2.3. As a result, the joint probabilities $\Pr(M_i \cap R_j|I)$ are established.

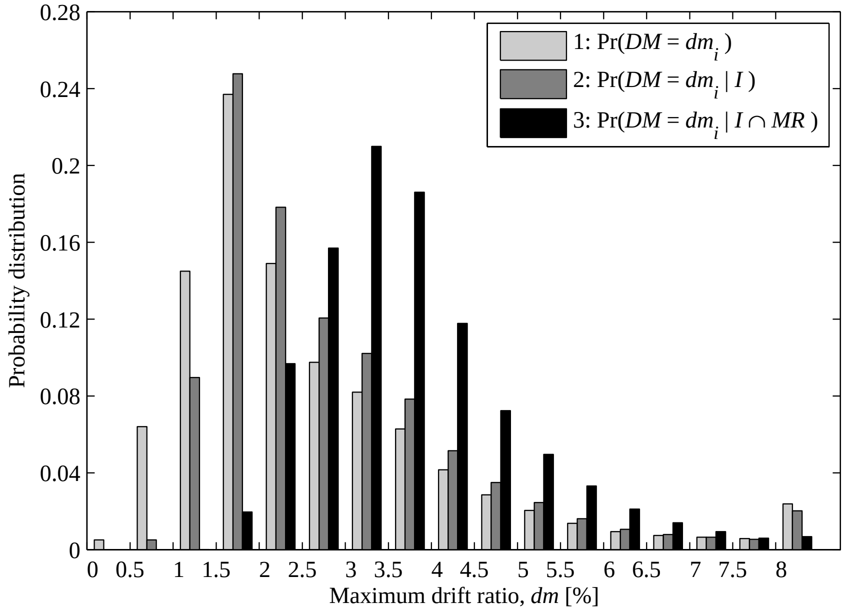


Figure 3: Probability distributions of the maximum drift ratio identified for the different levels of information

***Step 5*:** The residual displacement reported by Hachem *et al.* (2003) is employed as the measured residual displacement of the structure after the damaging earthquake. Based on the joint probabilities $\Pr(M_i \cap R_j | I)$ and the measurement results *MR*, the probabilities $\Pr(M_i | I \cap MR)$ can be calculated using Eqn. 2.8a. The resulting probability distribution is presented in Figure 3 (3$^{\text{rd}}$ series). This posterior distribution of the estimated for the experienced maximum drift is expected to reflect the uncertainty related to the model and the knowledge gained from the visual inspection and the residual displacement measurement.

4. DISCUSSION OF THE RESULTS

The probability distributions obtained based on different sets of evidence are noticeably different from each other (Figure 3). Initially, significantly higher prior probabilities $\Pr(M_i)$ are assigned to the smaller drift intervals ($DM <$ 1.5%). This is a result of the adopted attenuation model underestimating the true intenstity of the shaking for the specific case considered. Subsequently when the probabilities are conditioned on the visual inspection results $\Pr(M_i | I)$, the probabilities associated with the drift values smaller than 1.5% decseases and those associated with the range 1.5 to 7% increases. Finally, when the information gained by measuring the residual drift of the structure is considered to improve the estimates, the probabilities $\Pr(M_i | I \cap MR)$ are obtained. The consideration of the residual drift as an additional source of information results in very significant increases in the probabilities assigned to the drift intervals in the range 2.5–5.5%. The mode of the distribution shifts from the 1.5–2% drift interval to the 3–3.5% interval. With the application of the proposed method the prior distribution is effectively updated and the resulting estimates are in better aggreement with the recorded maximum drift ratio of 5.1%.

5. SUMMARY AND CONCLUSIONS

A post-earthquake assessment method is presented in this paper. The method can be applied to obtain improved estimates of the damage to a structure through a better prediction of the maximum drift attained during the earthquake. This goal is reached by taking both observable damage and post-earthquake residual displacements directly into account. The resulting improved damage estimates are expected to lead to better informed decisions regarding the post-earthquake usability and/or repairability of the structure.

The method is formulated to allow direct consideration of the uncertainties —related to the excitation, the material properties and the dynamic response— in the assessment process. Furthermore, the probable errors associated with the adopted models are also directly taken into account. The method allows information from different sources being considered to improve the estimates of the maximum drift ratio.

The results obtained for the sample application proves the effectiveness of the method. When utilized in post-earthquake assessment applications where a reliable record of the damaging ground motion is not available, the method successfully updates the probability distribution of the maximum drift ratio based on the available information. Moreover, the results indicate that the residual displacements have the potential of being a valuable performance indicator in post-earthquake applications.

The approach proposed in this study can be utilized as a sub-component in any post-earthquake decision-making process. The improved estimates of the damage to the structure would allow a better assessment of the risks associated with the post-earthquake performance of a structure.

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