USE OF LOAD DEPENDENT RITZ VECTORS IN MODAL PUSHOVER ANALYSIS

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ABSTRACT:

The performance of a structural system can be estimated using a non-linear static analysis which has found widespread use in performance based seismic design due to its simplicity in estimating inelastic structural response. Modal Pushover Analysis (MPA) has been suggested to increase the accuracy of Pushover analysis; but it fails in some cases of the irregular structural systems (i.e. stiffer lower stories). The objective of this paper is to present the use of Load Dependent Ritz vectors (LDR) which takes into account the spatial distribution of dynamic force; instead of commonly used eigen-mode shape in the MPA in order to improve the accuracy of calculated response of irregular systems when limited number of modes is to be considered, especially for stiff systems where higher mode effects cannot be ignored. The numerical results have indicated that using LDR vectors, in case of stiffer lower stories, increase the accuracy of force response significantly (because of inclusion of higher mode effects without really computing them). There are also some suggestions about choosing adequate number of required Ritz vectors for vertically regular or irregular structures considered.

KEYWORDS: Modal Pushover Analysis, Ritz vectors, LDR, Nonlinear Static Analysis, Stiffness Irregularity

1. INTRODUCTION

Evaluation of structural response is a key concept in performance based seismic engineering. Structures can be analyzed using either linear or nonlinear methods. Nonlinear dynamic analysis is considered as ‘exact’ method, however, it is very time consuming and may not be suitable for everyday engineering practice; the NSP which is a simple method has been preferred in practice. Although it may provide good approximation for some types of structures (mainly regular structures), due to limitations in its fundamental concepts like the first mode distribution of load, good estimates would not be promised for all types of structures especially for structures with considerable higher-mode contribution to response.

In order to overcome this limitation, MPA procedure has been proposed by Chopra [3] which offers several attractive features, for instance, it retains the conceptual simplicity and computational ease of current pushover procedures with invariant force distributions. In the MPA procedure, the seismic demand due to individual terms in the modal expansion of the effective earthquake forces is determined by nonlinear static analysis using the inertia force distribution for each mode. These “modal” demands due to the first few terms of the modal expansion are then combined by the CQC rule to obtain an estimate of the total seismic demand for inelastic systems. A step-by-step summary of the MPA procedure to estimate the seismic demands for a multistory building is presented in References [2] and [3].

The accuracy of MPA must be evaluated for a wide range of structural systems and ground motions to identify the conditions under which it is applicable for seismic evaluation of structures. By studying the bias of this approximate procedure, MPA has been shown to be accurate enough in estimating seismic demands for seismic evaluation of “regular” buildings. Because vertical irregularities significantly influence the seismic demands on buildings, the next step is to determine whether or not the MPA can estimate seismic demands of irregular buildings to a degree of accuracy which is considered sufficient for practical application? Chopra and Chintanapakdee [2] have shown that: “The MPA procedure is less accurate relative to the reference *regular* *frame* in estimating the seismic demands of frames with strong or stiff-and-strong first story; soft, weak, or soft-and-weak lower half; stiff, strong, or stiff-and-strong lower half". As this method considers the first few modes it is susceptible to lack of accuracy for the cases in which higher-mode contribution cannot be neglected.

The objective of the proposed method is to improve the accuracy of MPA for the vertically irregular frames with stiffer lower stories for the estimation of the seismic demands. Comparison of the seismic demands in irregular frames determined by MPA procedure with the "exact" nonlinear response history analysis response, shows that the use of LDR vectors in MPA procedure can improve the accuracy of MPA for the case of irregular structures.

2. PROPOSED METHOD: MPA USING LDR OR PSUEDO STATIC VECTOR INSTEAD OF EIGENVECTOR

In 1982 Wilson et al. [8] introduced a new dynamic analysis method based on Rayleigh-Ritz method, which took into account the effects of spatial distribution of the dynamic loading and yielded much better results in less computational time than use of exact eigenvectors known as WYD Ritz vectors (Wilson, Yuan and Dickens) or LDR (Load Dependent Ritz) vectors. Consider the input force as $\{F(s,t)\}=\{f(s)\}.g(t)$ ($\{f(s)\}$ is the spatial distribution of excitation), the first Ritz vector which is the static response of the time independent portion of load vector, $\{f(s)\}$ can be computed as: $\{\bar{X}_1\}=[K]^{-1}\{f\}$ (which is known as pseudo static vector in mode-acceleration procedure). Since this vector will be used for extraction of other vectors it would be impossible to generate vectors which are not excited by the assumed loading. The obtained response format is analogous to pseudo static vector used in the "mode-acceleration" based response spectrum approach [5].

- Static correction

$$\{U(t)\}=\sum_{i=1}^{r}\{\varphi_i\}y_i(t)+[K]^{-1}(\{f(s)\}-\{f_r(s)\})g(t)$$

- Pseudo static vector

$$\{U(t)\}=\sum_{i=1}^{r}\{\varphi_i\}y_i(t)+([K]^{-1}-[K_r]^{-1})\{f(s)\}g(t)$$

$$\{U_s\}=[K]^{-1}\{f(s)\}$$

- LDR vectors

$$\{U(t)\}=\sum_{i=2}^{r}\{X_i\}y^*_i(t)+\frac{\bar{X}_1}{(\bar{X}_1^T[M]\bar{X}_1)^{1/2}}y^*_1(t)$$

In each of the first two summation methods the full loading vector has been accounted for either in the dynamic manner by first term or in a static manner by the second term, which is not the case for LDR vectors; since the first vector corresponds to static solution and additional vectors represent dynamic contribution neglected by the static solution. This is the major benefit of using LDR vectors in MPA approach where first few modes are usually considered, and implementing LDR vectors ensures the consideration of higher-mode participation in force or displacement response calculation (displacement response converges faster than force response so accurate displacement response do not guarantee accurate force responses due to higher modes). It should be noted that the first pseudo static vector, $\{U_s\}$, is identical to the starting vector of LDR vector approach. Thus in the continuation of the paper, only the results for LDR has been presented.

3. NUMERICAL ANALYSIS

To evaluate the effectiveness of using LDR or $\{U_s\}$ vectors; two types of structural systems were considered in this research: 1) Regular moment resisting frame systems and 2) Irregular frame systems with stiffer lower stories. Both type of systems were designed according to Iranian National Building code which is similar to AISC-89 (ASD method) for PGA=0.3g, soil type III and Response reduction factor R=6. The regular and irregular frames and their respective periods for the first 3 modes are shown in Figure 1.

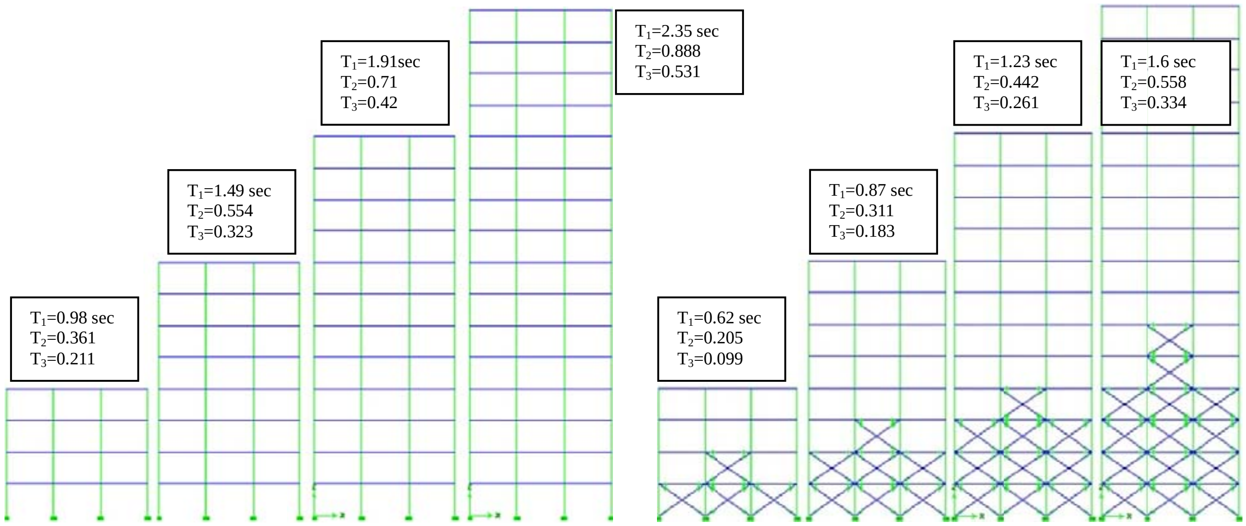


Figure 1 Regular and Irregular frame models that have been used in the application of MPA with LDR

For the input ground motion, a set of near-field ground motions recorded on soft soil (type 3) scaled to PGA of 0.7g (to ensure the nonlinear response) with impulsive character which can cause considerable response and contribution of higher modes has been used.

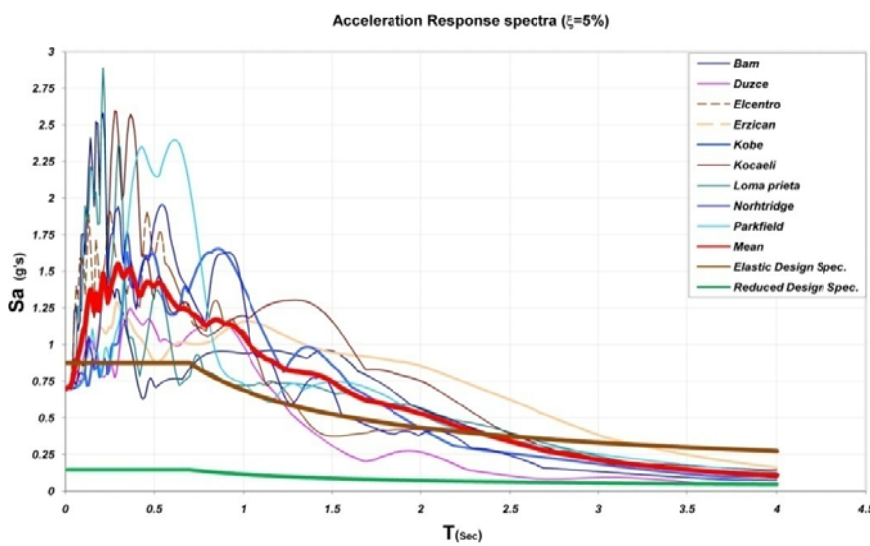


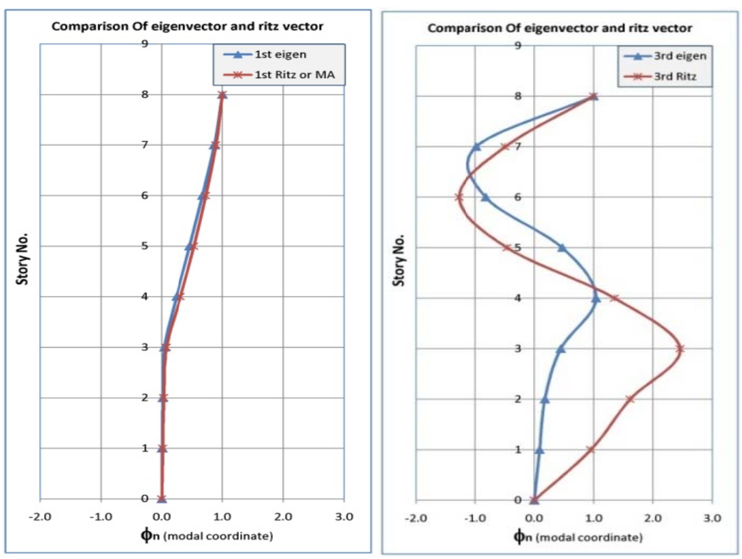
Figure 2 Spectral accelerations for selected ground motions and mean spectrum

Considering the effect of higher modes on response for the case of stiffer lower stories, the use of LDR vectors and pseudo static vector for such irregular systems (also for regular systems for measuring the accuracy, in line with nonlinear time history analysis which is assumed to be exact response) have been examined.

Figure 3 shows sample modal properties for the 8-story irregular frame which have been obtained and used for the evaluation of two approaches (LDR vectors and eigenvectors). Rayleigh damping ratio has been used for calculating damping in each mode for the analysis of SDOF (Single Degree of freedom) systems which are required in MPA approach. It is important to consider rational damping ratios since it is not known whether the properties are in the velocity sensitive region or not. Different systems have been coded according to the number of stories, regularity, type of vectors (LDR or eigenvector) and number of modes that have been used; For example 4sIr-e(2) represent the results for a 4 story Irregular-frame, using 2 eigen-modes). It should be noted that since spectral content of the first vector in LDR vector approach is spread among all the basis vectors, shape of each computed vector depends on the requested number of LDR vectors. Appropriate selection of required number of LDR vectors can significantly reduce the computational effort needed for MPA.

	Eigenvectors			Story Mass	First LDR vector	LDR vectors (set of 2)		LDR vectors (set of 3)		
Story	Mode 1	Mode 2	Mode 3		or MA	Mode 1	Mode 2	Mode 1	Mode 2	Mode 3
8	1.0000	1.0000	1.0000	2.8824	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0.8694	0.1582	-0.9790	3.2983	0.8956	0.8720	0.3513	0.8694	0.2140	-0.4901
6	0.6715	-0.6269	-0.8185	3.3230	0.7286	0.6762	-0.4772	0.6715	-0.6090	-1.2733
5	0.4626	-0.8768	0.4713	3.3634	0.5355	0.4672	-1.0364	0.4626	-0.9474	-0.4575
4	0.2459	-0.6313	1.0388	3.3779	0.3109	0.2484	-1.1278	0.2459	-0.7315	1.3520
3	0.0463	-0.1414	0.4411	6.4759	0.0784	0.0459	-0.6691	0.0463	-0.1664	2.4579
2	0.0162	-0.0473	0.1764	6.5786	0.0330	0.0158	-0.3626	0.0162	-0.0445	1.6150
1	0.0070	-0.0228	0.0854	6.6444	0.0162	0.0067	-0.2022	0.0070	-0.0171	0.9485
T_n	0.8722	0.3111	0.1828		0.8521	0.8721	0.2378	0.8722	0.3091	0.1406
ω_n	7.2040	20.1995	34.3776		7.3739	7.2045	26.4241	7.2040	20.3260	44.6966
Γ_n	1.3849	-0.6154	0.4665		1.3834	1.3805	-0.8068	1.3848	-0.6006	0.4208
Mass Participation %	41.6916	8.7989	8.5818		45.9568	41.7163	28.9610	41.6916	9.3196	38.5239
Cumulative MP %	41.6916	50.4905	59.0723		45.9568	41.7163	70.6773	41.6916	51.0112	89.5351

Figure 3 Sample Modal properties for 8 story irregular frame for eigen-modes and LDR vectors



(a) (b)

Figure 4 Comparison of Eigenvectors and LDR vectors (a) First LDRvector vs. 1st eigenvector (b) 3rd vector comparison

Figure 4(a) shows the difference between First LDR vector or Pseudo static vector and first eigenvector. As it can be seen there is no significant participation of lower stories in either case. Figure 4(b) shows the comparison between 3rd eigenvector and 3rd LDR vector (in a set of three).It is apparent that LDR vectors have activated the stiffer part of the structure and as a result their contribution to the final response can be captured by using only first few modes.

3.1. Story Shear and Moments

In the case of shear response of irregular structure, to satisfy 90% of mass participation requirement, 8 eigen-modes should be considered for 16-story irregular frame but in case of using LDR vectors, 3 vectors would be sufficient. Figure 5 and Table 1 show the advantage of MPA analysis with LDR vectors over eigenvector with respect to nonlinear time history analysis (NLTHA) for various types of frames. The first story shear of 16 story irregular structure after using 4 eigen-modes will have 40% error while using 3 LDR vectors only cause 10% error. This trend can be seen for stiffer part of structures because these parts are not excited till the higher modes. In case of the regular structures use of LDR vectors does not cause significant improvement but fewer vectors would be needed to satisfy 90% participation of mass.

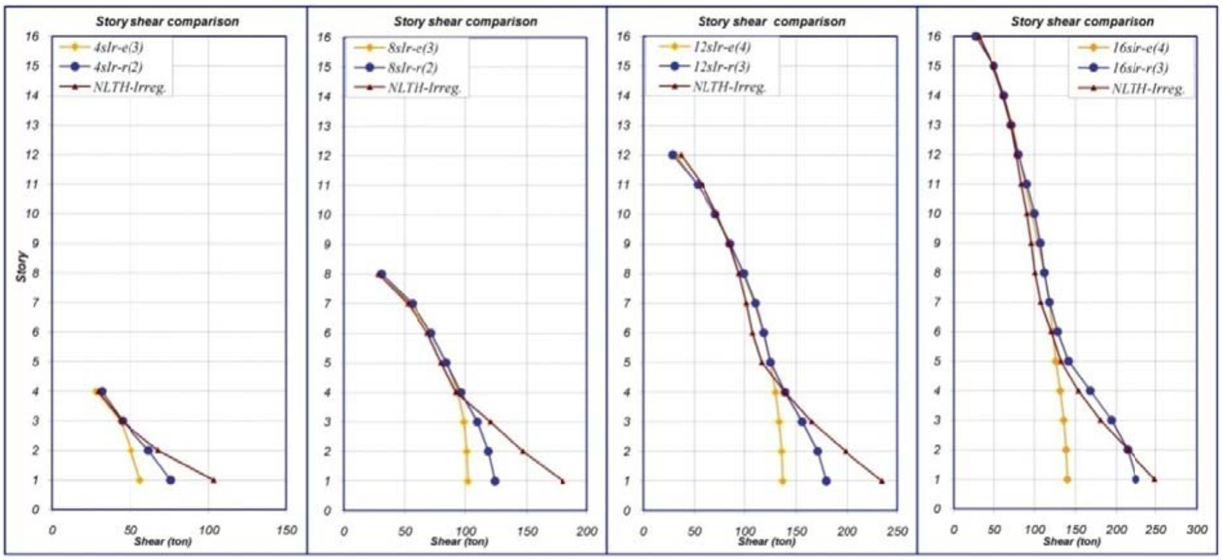


Figure 5 Comparison of story shear between NLRHA and MPA using eigenvectors and LDR vectors

Table 1 MPA error percentage in Story Shear response using LDR vectors or eigenvectors

Story	4 Story-Irregular		8 Story-Irregular		12 Story-Irregular		16 Story-Irregular	
	Eigen	LDR	Eigen	LDR	Eigen	LDR	Eigen	LDR
16	-	-	-	-	-	-	-4.48	-14.16
15	-	-	-	-	-	-	2.47	1.58
14	-	-	-	-	-	-	1.57	2.38
13	-	-	-	-	-	-	2.76	0.45
12	-	-	-	-	-14.55	-22.13	3.97	3.06
11	-	-	-	-	-7.88	-7.29	5.79	8.14
10	-	-	-	-	-1.57	-1.83	6.84	10.38
9	-	-	-	-	1.74	1.13	9.12	11.71
8	-	-	11.85	10.24	4.95	5.35	11.08	11.56
7	-	-	2.16	6.99	8.01	8.88	9.72	10.01
6	-	-	4.19	4.70	10.00	10.41	1.63	6.55
5	-	-	5.86	5.43	6.76	7.10	-4.64	7.12
4	-5.14	7.67	1.70	4.27	-6.52	0.03	-14.55	9.49
3	-1.76	0.93	-17.96	-8.91	-19.09	-5.52	-24.86	7.56
2	-25.55	-8.75	-31.24	-19.16	-31.54	-13.89	-35.42	-0.26
1	-45.27	-26.34	-43.31	-31.02	-41.70	-23.69	-43.67	-9.59

Figure 6 shows the similar results for the story moments. Using LDR vectors for low-rise regular structures has no noticeable benefit due to the fact that LDR vectors for these kinds of structures rapidly converge to exact eigenvectors. As the structures get taller, more accurate results can be obtained by using less number of LDR vectors than eigen-modes, still there is no significant difference. In the case of irregular frames considered the superiority of LDR vectors is noticeable.

3.2. Drift Response

Figure 7 and Table 2 show the results for drift ratio ($\frac{\Delta_i - \Delta_{i-1}}{h_i}$) of the irregular structures. It can be seen that as the height of structure increases, the use of LDR vectors will lead to better estimates of exact values. In the case of regular structures there is no important difference in using either LDR vectors vs. eigenvectors.

Table 3 shows the accuracy of the proposed method for the "Overall Drift Index (Δ_{TOP}/H)" where H is overall height of structure. However, it should be noted that higher modes have little influence on displacement response of a structure, but in some irregularities (like the one considered here) they may become more important. Use of LDR vectors for regular frames, does not have any superiority over the use of eigenvectors in the obtained results (except for the time of calculations and number of vectors required). But like other parameters, hitherto studied, for irregular frames especially for taller frames LDR vectors will produce more accurate results, even with smaller number of LDR vectors used for the analysis. This is mainly due to the benefits of including higher mode responses (via static correction concept) in the starting vector for LDR vectors algorithm.

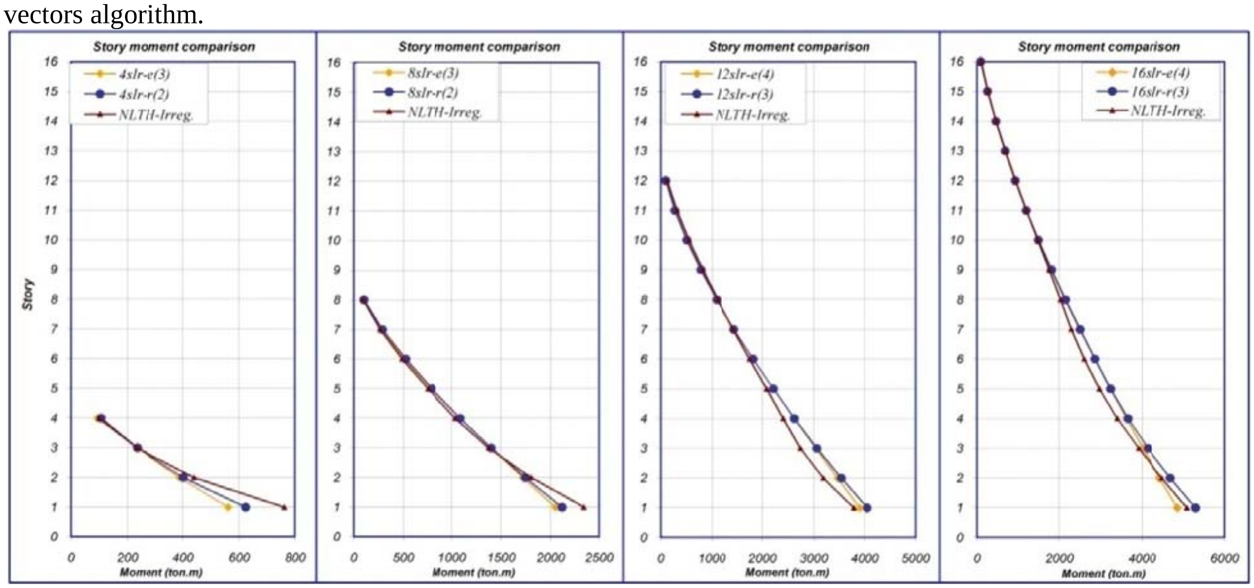


Figure 6 Comparison of story moment between NLRHA and MPA using eigenvectors and LDR vectors

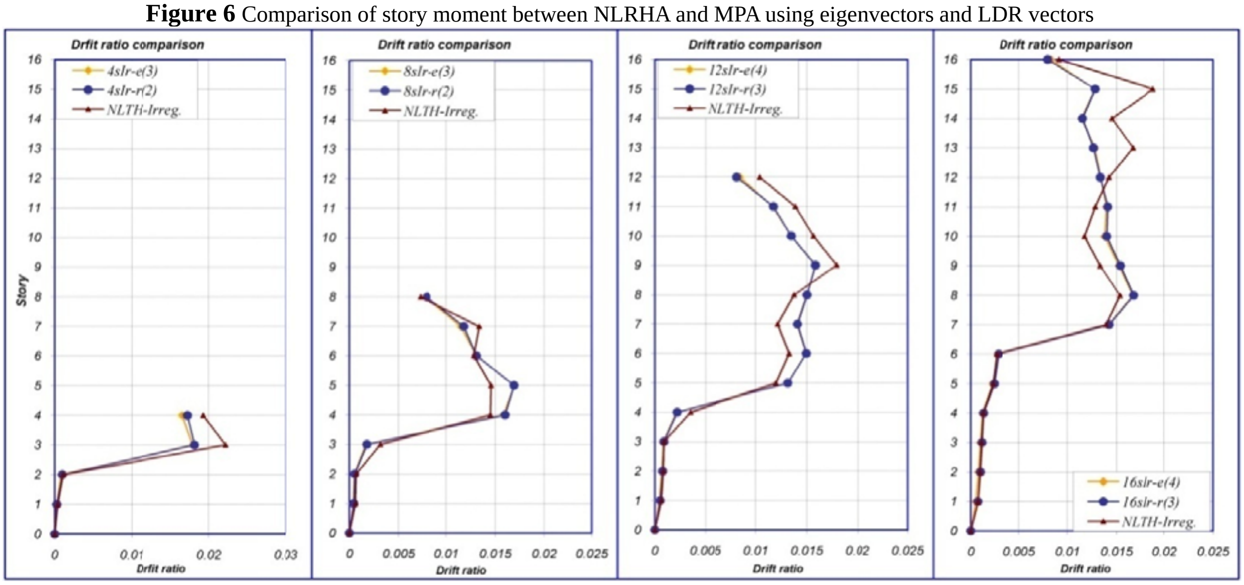


Figure 7 Comparison of drift ratio between NLRHA and MPA using eigenvectors and LDR vectors

Table 2 MPA error percentage in drift ratio using LDR vectors or eigenvectors

Story	4 Story-Irregular		8 Story-Irregular		12 Story-Irregular		16 Story-Irregular	
	Eigen	LDR	Eigen	LDR	Eigen	LDR	Eigen	LDR
16	-	-	-	-	-	-	-9.08	-12.66
15	-	-	-	-	-	-	-32.01	-31.86
14	-	-	-	-	-	-	-21.42	-20.71
13	-	-	-	-	-	-	-23.45	-24.19
12	-	-	-	-	-18.55	-21.58	-5.80	-6.08
11	-	-	-	-	-16.00	-15.45	8.92	10.08
10	-	-	-	-	-13.88	-13.80	17.39	19.27
9	-	-	-	-	-11.39	-11.55	14.46	15.71
8	-	-	8.12	9.21	8.97	9.15	8.61	9.12
7	-	-	-14.14	-11.76	15.66	16.06	1.91	2.39
6	-	-	2.15	2.11	12.58	12.73	2.77	6.77
5	-	-	16.29	16.13	9.49	9.66	-2.06	5.84
4	-13.99	-10.43	9.23	10.48	-39.74	-37.28	-4.30	4.53
3	-18.98	-17.93	-47.65	-43.46	-4.37	2.12	-9.48	5.27
2	-22.42	-7.49	-26.62	-18.82	-16.99	-6.39	-17.16	5.28
1	-45.41	-26.92	-38.07	-27.43	-32.77	-17.84	-32.40	-2.47

Table 3 MPA error percentage in drift index using LDR vectors or eigenvectors

Story	4 Story-Irregular		8 Story-Irregular		12 Story-Irregular		16 Story-Irregular	
	Eigen	LDR	Eigen	LDR	Eigen	LDR	Eigen	LDR
16	-	-	-	-	-	-	2.71	2.95
15	-	-	-	-	-	-	2.56	2.81
14	-	-	-	-	-	-	6.72	6.97
13	-	-	-	-	-	-	8.29	8.61
12	-	-	-	-	3.50	3.51	10.90	11.31
11	-	-	-	-	4.81	4.83	12.15	12.58
10	-	-	-	-	7.70	7.69	12.13	12.50
9	-	-	-	-	11.37	11.40	9.76	10.21
8	-	-	3.80	3.76	16.12	16.19	7.10	7.96
7	-	-	3.66	3.50	12.60	12.70	4.26	6.72
6	-	-	7.25	7.25	8.01	8.25	-4.79	5.22
5	-	-	9.00	9.71	-1.05	-0.27	-8.06	5.23
4	-16.62	-16.01	0.83	2.65	-26.94	-21.56	-12.39	4.50
3	-18.90	-17.41	-41.57	-35.98	-16.42	-6.76	-16.56	4.56
2	-28.84	-13.15	-32.09	-22.97	-23.86	-11.30	-22.58	3.95
1	-45.32	-27.01	-38.51	-27.53	-32.97	-17.64	-32.28	-1.48

From the above mentioned results, the following observation can be made for regular and irregular structures:

Regular structures:

- Best results are obtained using first set of LDR vectors with more than 90% cumulative mass participation.
- For most of low rise to medium rise regular models ($H/B \leq 3$) first mode is dominant for response calculation. As the height of structure increases, neglecting upper modes may cause serious errors in

estimation of the response parameters. For this group of models using LDR vectors has no meaningful superiority over eigenvectors (except for the number of vectors required in some cases).

- Use of LDR vectors (because of their nature) usually leads to over estimated results for lower stories but slightly underestimated results for higher stories comparing to use of eigenvectors.
- For the case of low rise to medium rise regular models, using more than 2 or 3 LDR vectors would not lead to better results. Since, at the end of LDR vectors algorithm an eigenvalue problem is solved and first few LDR vectors (depending on the size of the system) will converge to the exact eigenvectors.

Irregular structures:

- Best response estimates are obtained using the first set of LDR vectors with less than 90% cumulative mass participation. One should keep in mind that the shapes of LDR vectors are dependent on number of vectors requested, as this number increases shapes of first few vectors will be the same as eigenvectors, so it is important to choose adequate number of LDR vectors.
- Use of LDR vectors provides better estimates of response calculation in comparison with use of eigen-modes, since the starting vector in LDR vector generation algorithm is equivalent to the static correction concept. Considering the effect of higher (rigid) modes to the response of the irregular frames, their effects can be compensated by the first vector of LDR.
- In the case of using single ground motion, results are more scattered. Best results are obtained using first set of LDR vectors with more than 90% cumulative mass participation.

4. CONCLUSION

The paper proposed an MPA procedure using LDR vectors. Use of the LDR vectors, was investigated for structures with stiffer lower stories. From the numerical results it can be seen that for this type of irregularity, use of LDR vectors is superior to use of exact eigenvectors based modal properties. As the simplicity and speed, besides accuracy, are the most important factors in selecting an approach, use of LDR vectors can supersede the use of exact eigenvectors for this special kind of irregularity. For the case of regular structures there were no meaningful differences in using either set of vectors. It should be kept in mind that the rule proposed here for selecting the number of required Ritz vectors is a rule of thumb and it needs more investigation.

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