EFFECT OF INTERACTION BETWEEN PRIMARY AND SECONDARY SYSTEMS ON FLOOR RESPONSE SPECTRA

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ABSTRACT:

A parametric study on the interaction between primary structure and secondary components is presented in this paper. A simplified model of multi-story buildings is used to model the primary structure. The secondary component is modeled as a single degree of freedom object with known period of vibration, mass and damping ratio. A formulation already developed in another study has been employed to compute floor response spectrum of components that their characteristics affect the response of the primary structure. The advantage of this method to other proposed methods is that the primary structure is modeled as an MDOF which can capture higher mode effects. Effect of secondary component with different mass ratios on modified acceleration transfer functions of different modes of the primary structures as well as the secondary component has been shown. The ranges of mass ratios where the interaction can be ignored have also been presented. Effect of location of the secondary component has also been presented. The building models have been subjected to white noise as well as filtered white noise and floor response spectra for different mass ratios mounted at different heights of the primary structures have been investigated.

KEYRORDS: interaction, secondary systems, transfer function, mass ratio, floor response spectrum

1. INTRODUCTION

A common approach to obtain the input load of a secondary system attached to a primary structure is by using the floor response spectrum computed from the acceleration time history at the floor where the secondary system is located. This method, sometimes referred to as a "cascade approach", has been widely used by engineers for the design of secondary systems and their supports in buildings. The basic assumption of this method is that the secondary system does not interact with the primary structure and that the dynamic properties of the primary system are not affected by the presence of the secondary system. This assumption may not be valid when the secondary system is heavy and the primary and secondary systems may affect the response of each other and therefore it is necessary to consider the combined system (Penzien and Chopra 1965, Kelly and Sackman 1978). When the secondary system is tuned to one of the modes of the primary structure and its mass is not negligible, a considerable error may be produced in estimation of demand in the secondary system (Kapur and Shao 1973, Singh 1975, Gupta and Tembulkar 1984, Hadjian 1977) and the interaction between the primary and secondary system has to be considered. Neglecting the interaction in general leads to an overestimation of the demand on secondary systems and therefore to an overly conservative design.

Interaction effects and dynamic properties of the combined system have been studied by several researchers considering a combined oscillator-structure model. First order perturbation methods have been used by many researchers to evaluate the modal properties of the combined system (Kelly and Sackman 1978, Sakman and Kelly 1979, Sakman et al. 1983, Der Kiureghian et al. 1983, Igusa and Kiureghian 1985, Vilaverde and Newmark 1980). Second order perturbation methods have also been used to evaluate the eigen properties and floor spectra of combined primary-secondary systems (Singh and Suarez 1986, Suarez and Singh 1987a). A more accurate approach for incorporating the dynamic interaction between primary and secondary systems has also been developed by using the combined modal properties of the primary and secondary system for both classically damped (Suarez and Singh 1987b) and non-classically damped systems (Singh and Suarez 1987c).

Gupta (1997) presented a method to evaluate the transfer function and root mean square response of the secondary systems considering the interaction between the primary and secondary system. In his method, the mode shapes of the primary system are approximated with the mode shapes of a decoupled primary system. The accuracy of the method was examined by comparing the transfer functions obtained with the simplified method to those computed using exact mode shapes. Other researchers have also proposed decoupling criteria using the transfer functions (Chen 1998, Chen and Wu 1999).

In this paper, the formulation proposed by Gupta (1997) is employed to study the interaction effects between the primary and secondary system, by considering the continuous beams developed by Miranda and Taghavi (2005) as the primary system. The effects of various parameters such as fundamental period of vibration of the primary system, the lateral stiffness ratio and the level of damping on transfer functions relating response of secondary systems to ground motion and also to the root mean square response of secondary systems are investigated.

2. FORMULATION

In the approximate method presented by Gupta (1997), the dynamic properties of the coupled primary and secondary system are approximated by the dynamic properties of the primary system. In the other words, the dynamic interaction effects are first neglected. Then to account for the dynamic interaction between the primary system and single-degree-of-freedom (SDOF) secondary system, the force between the primary and secondary system at the degree of freedom in which the two are connected is applied to the primary structure.

Developing equation of motion of the primary structure and applying the force between the secondary component and primary structure, modified transfer function of the r^{th} mode of primary structure, $\acute{H}_r(\omega)$, is defined as:

$$\acute{H}_r(\omega) = \frac{1+\frac{\phi_p^r}{\alpha_r} M_s U_{bs}\left[1+\omega^2 H_s(\omega)\right]}{1-\omega^2 M_s[1+\omega^2 H_s(\omega)]\left[\sum_{r=1}^{n} \phi_p^{r\,2} H_r(\omega)\right]} H_r(\omega) \tag{1}$$

Ms is the mass ratio (mass of secondary system to the mass of supporting floor) of the secondary system and M_p is the mass of p^{th} floor. $H_r(\omega)$ is the transfer function for the r^{th} mode of the primary structure neglecting the interaction effects which is given by:

$$H_r(\omega) = \frac{1}{\omega_r{}^2-\omega^2+2i\xi_r\omega_r\omega} \tag{2}$$

H_s is defined similarly to H_r and is the transfer function of the secondary system. α_r is defined as:

$$\alpha_r = \Phi^{\mathrm{r}} \mathrm{M_p} \mathrm{U_{bp}} \tag{3}$$

Writing the absolute acceleration of the secondary system as the sum of relative acceleration of the secondary system with respect to its support and absolute acceleration of the support, the transfer function relating the response of the secondary system to the base excitation becomes:

$$H(\omega) = [1+\omega^2 H_s(\omega)]\left[1+\omega^2 \textstyle\sum_{r=1}^{n} \alpha_r \phi_p^r \acute{H}_r(\omega)\right] \tag{4}$$

It should be noted that equation 6.10 is reduced to the transfer function of the primary system with no interaction when the mass of the secondary system is zero. The root mean square response of the secondary system when subjected to a ground motion with power spectral density (PSD) function G(□) is then computed as:

$$\Phi_{\mathrm{R.M.S}}(\omega) = \mathrm{G}(\omega)|\mathrm{H}(\omega)|^2 \tag{5}$$

3. PARAMETRIC STUDY

In this parametric study, the effects of various parameters including fundamental period of the structure, lateral stiffness ratio and damping ratio of the secondary system are investigated. Dynamic characteristics of the primary structure are approximated with those of the continuum model developed by Miranda and Taghavi

(2005) for estimation of dynamic characteristics of the structure. Uniform and modified Kanai-Tajimi power spectral density functions are used to calculate the root mean square response of secondary systems. The fundamental period of the primary structure was varied from 0.5 s to 3.5 s with 1.0 s increments. Lateral stiffness ratios of 0, 2, 4, 8 and 20 were considered. The damping ratio of the primary structure was assumed to be 5 percent, while the damping ratio of the secondary system was assumed to be 2, 5 and 10 percent. The results were computed at 25, 50, 75 and 100 percent of the height of the structure. The results will be presented for white noise as well as modified K-T base excitation. The parameters of modified K-T PSD were selected to match the PSD of the ground motions used in this study.

4. TRANSFER FUNCTIONS

4.1. Effect of Mass Ratio on Transfer Function of Primary Structure

Fig-1 shows the transfer function of the displacement of the primary system relative to the base excitation for the first three modes when a secondary system with damping ratio of 5 percent located at the roof of the structures is tuned to the first mode of the structure. Mass ratios of zero (no interaction), 0.01 and 0.1 have been considered. It is seen that the response of the primary system is affected by the secondary system if the frequency of the input motion is around the frequency of the secondary system. For example, according to Fig-1, transfer functions of the first three modes are only affected by interaction of the two systems around the first mode of the primary system which is same as the frequency of the secondary system. They remain unaffected by interaction at frequencies that not very close to the frequency of the secondary system. It is interesting to note that a mass ratio of 1 percent (M_s = 0.01) has very little effect on transfer functions which means that for such light components, the interaction effects can be neglected and a decoupled analysis can be performed. Interaction effects become clear when the mass of the component is 10 percent of the floor mass.

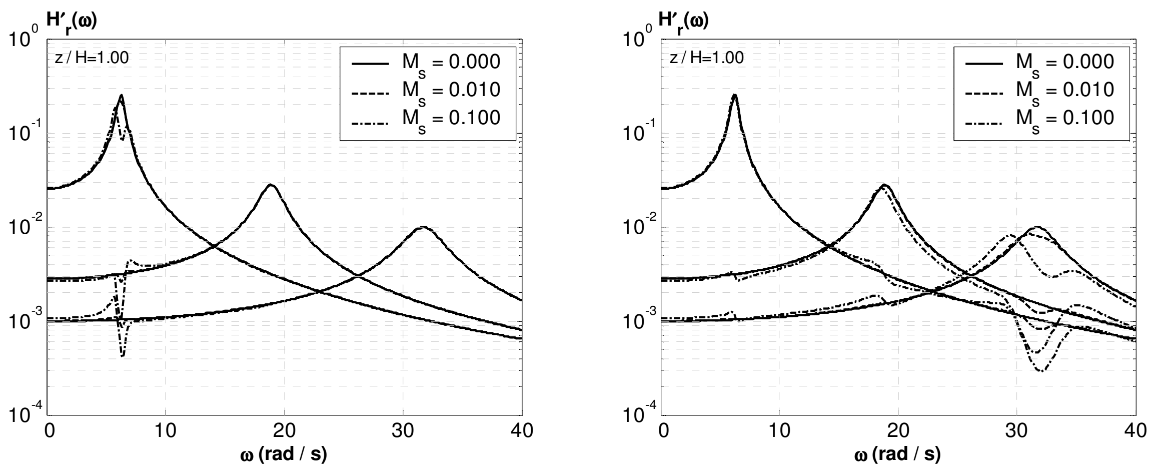

Figure 1 - Modified transfer function of the primary system for different mass ratios of secondary systems located at the roof of a structure when they are tuned to the first (fundamental) mode of vibration the structure

Fig-2 shows the ratio of the modified transfer function of the first mode of the primary system ($H'_r(\omega)$) due to interaction effects to the original transfer function ($H_r(\omega)$) at mid height and roof when the secondary component is tuned to the first, second or third mode of the structure. At roof (left), it is seen that for a mass ratio of 10 percent, the amplitude of the transfer function of the first mode is reduced to about 30 percent of its original value around the frequency of the secondary system. For mass ratio of 1 percent, the amplitude of the transfer function is reduced to about 80 percent of the original amplitude. The modified transfer function rapidly approaches the original transfer function as the frequency of the input motion departs from the frequency of the secondary system. As mentioned before, the effect of dynamic interaction is significantly smaller at lower heights. The plot on the right side of Fig-2 shows the ratio of modified to original transfer function of the first mode for components installed at mid height and tuned to the first, second or third mode of the primary system.

As shown in the figure, a mass ratio of 1 percent has almost no effect on the transfer function, while a mass ratio of 10 percent reduces the amplitude of the transfer function to about 50 percent of its original amplitude.

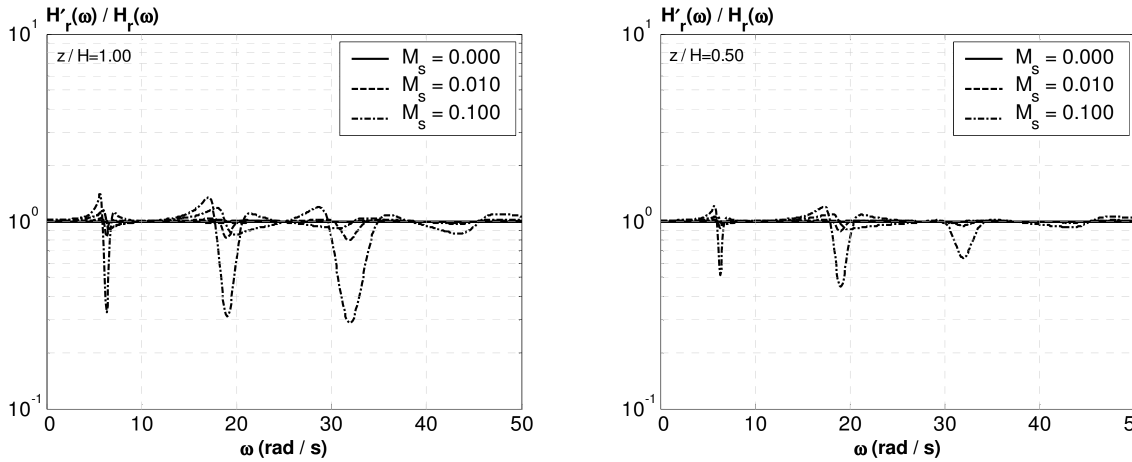

Figure 2 – Normalized modified transfer function of the third mode to the original transfer function with secondary system tuned to the first, second or third mode of the primary system for different mass ratios

4.2. Effect of Mass Ratio on Acceleration Transfer Function of Secondary System

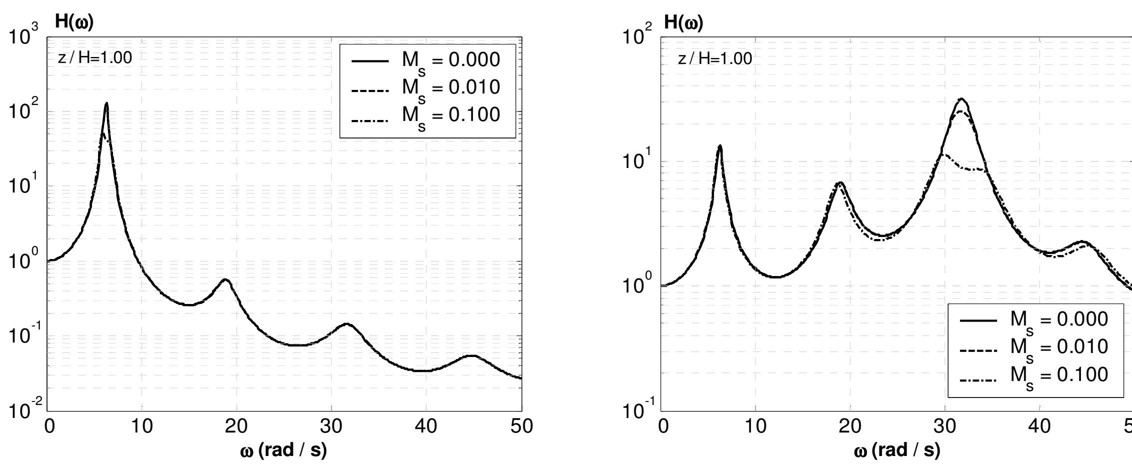

Figure 3 – Modified transfer function of the secondary system tuned to the first and third modes of the primary system for different mass ratios, roof

Fig-3 shows the acceleration transfer function of a secondary system tuned to the first and third modes of the primary system for different mass ratios at roof level of a moment frame structure (α_0 = 20). It is seen from the graph on the left that a mass ratio of 1 percent does not change the transfer function at all and coupling can be ignored for such light systems. A mass ratio of 10 percent decreases the amplitude of the response of the secondary system around its period of vibration which is same as the natural period of the structure. The amplitude of the transfer function reduces from about 140 for an uncoupled system to about 30 for a coupled system with a mass ratio of 10 percent. As soon as the frequency of the input motion departs from the frequency of the secondary system, the coupling effect disappears and the system behaves like a zero mass secondary system mounted on a primary system. The graph on the right shows the acceleration transfer function of a secondary system tuned to the third mode of the structure and mounted at roof level. It is shown that a mass ratio of 1 percent has a small effect on transfer function around the period of the secondary system and rapidly disappears as the frequency of the input motion departs from the frequency of the component. The reduction at the tuned frequency is about 20 percent. Larger difference occurs as the mass ratio increases. The reduction is as high as 70 percent for mass ratio of 10 percent. For large mass ratios, peaks of the transfer function move slightly to the left and right but this effect is negligible.

The general observation of the study has been that at roof level, for mass ratio of 10 percent, the transfer function of the coupled system is about 30 percent of that of the uncoupled system for periods of the secondary system tuned around one of the modes of the primary system and rapidly approaches to the uncoupled transfer function as the frequency of the input motion deviates from the frequency of the secondary system. For mass ratio of 1 percent, the transfer function is reduced to about 80 percent of the original amplitude. Again the effect of dynamic interaction at mid height is smaller compared to the effect of dynamic interaction when the secondary system is on the roof.

5. FLOOR RESPONSE SPECTRUM

In the following section, the root mean square response of the coupled primary-secondary system is computed using equation 5. Floor response spectrum which is the spectrum of peak responses of the secondary system to the input motion can be computed by dividing the root mean square response to the peak factor. Two types of input motions are considered in this study. First, the system is subjected to a white noise and effects of mass ratio, damping ratio of the secondary system and lateral stiffness ratio on the response of the coupled primary-secondary system is investigated. Then, ground motions compatible with the modified K-T power spectral density are applied to the system and the effect of the above parameters as well as the period of the primary system are studied. The first five modes of the primary system are considered in the calculations.

5.1. White Noise Excitation

A white noise with amplitude of 0.005 is applied to the system with the lower cuff-off frequency of 0.5 rad/s and upper cut-off frequency of 150 rad/s. For a peak factor of 3, the peak ground acceleration corresponding to this motion is 0.26g. The peak factor of the response is also assumed to be 3 (Gupta 1997).

5.1.1. Effect of mass ratio and elevation on FRS

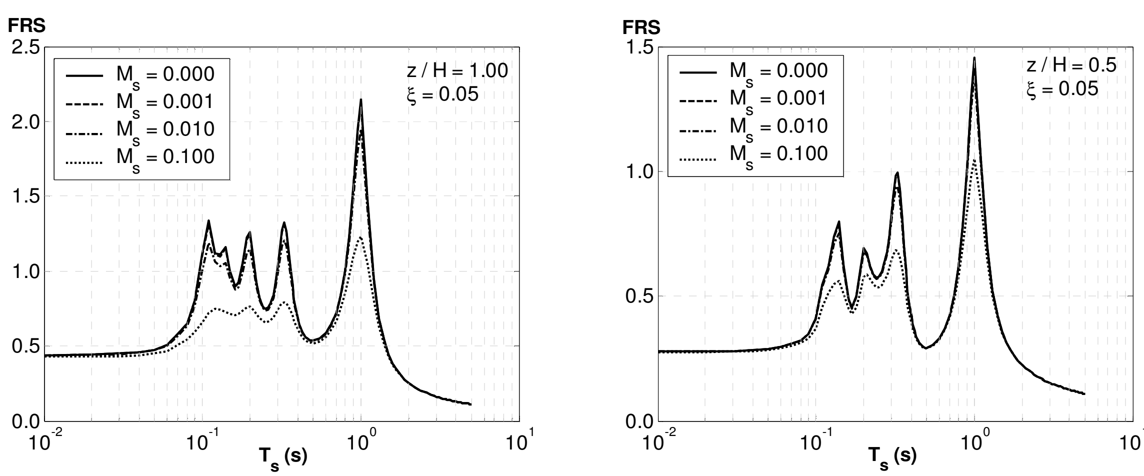

Figure 4 – Effect of mass ratio on floor response spectrum at roof and mid-height.

Fig-4 shows roof and mid-height response spectra of a component with damping ratio of 5 percent and different mass ratios for secondary systems mounted at roof level (z/H = 1.0) of a moment resisting frame building (α_0 = 20). It can be seen that for this special case, a mass ratio of 0.001 has almost no effect on FRS. As the mass ratio increases, FRS starts to show the coupling effects between the primary and secondary systems. For mass ratio of 1 percent, the peaks of the FRS reduce about 10 percent for all the modes, while it has practically no effect for periods of vibration that are not very close to the modal periods of the primary structure. In other words, the coupling effect for mass ratio of 1 percent is only important if the secondary system is tuned to one of the modes of the primary structure and otherwise, the uncoupled analysis provides accurate results. When the mass ratio is increased to 10 percent, considerable difference is observed between the coupled and uncoupled systems. Unlike the mass ratio of 1 percent, for heavy components, coupling of the primary-secondary systems affects the FRS between the modes of the primary structure. In secondary components with periods of vibration larger than the fundamental period of the structure, the coupled FRS approaches the uncoupled FRS. It can also be seen that the coupling effect is also negligible for relatively rigid components. The interaction effect is lower at mid-height compare to roof.

Fig-5 shows the ratio of coupled to uncoupled FRS in a moment resisting frame structure. For a component with mass ratio of 10 percent, the ratio of coupled to uncoupled FRSO at 25, 50, 75 and 100 percent of the height is 0.92, 0.73, 0.61 and 0.57, respectively. For secondary systems tuned to the second mode of the structure, a different behavior is observed. FRSO at the second mode is shown in Fig-5 (right) for components tuned to the second mode. It is seen that the FRSO ratio is the almost the same for z/H = 0.25 and 0.50, very close to one (small reduction) for z/H = 0.75 and small (large reduction) at z/H = 1.00. The difference in reduction between the first and second modes can be explained by the mode shapes of the structure.. It is seen that the first mode shape increases with height but the second mode shape has a node at about 70 percent of the height in moment frame structures. Therefore, the second mode shape at z/H = 0.75 is very small and so the contribution of the second mode in the response of the primary system. As a result, a component tuned to the second mode near this height is similar to a component which is not tuned to any of the modes and therefore, the ratio of the coupled to uncoupled FRSO is close to one. Similarly, the small difference between this ratio at z/H = 0.25 and 0.50 can be explained since the second mode shape at these two heights have almost the same amplitude. In general, the reduction of the FRSO is approximately proportional to the corresponding mode shape.

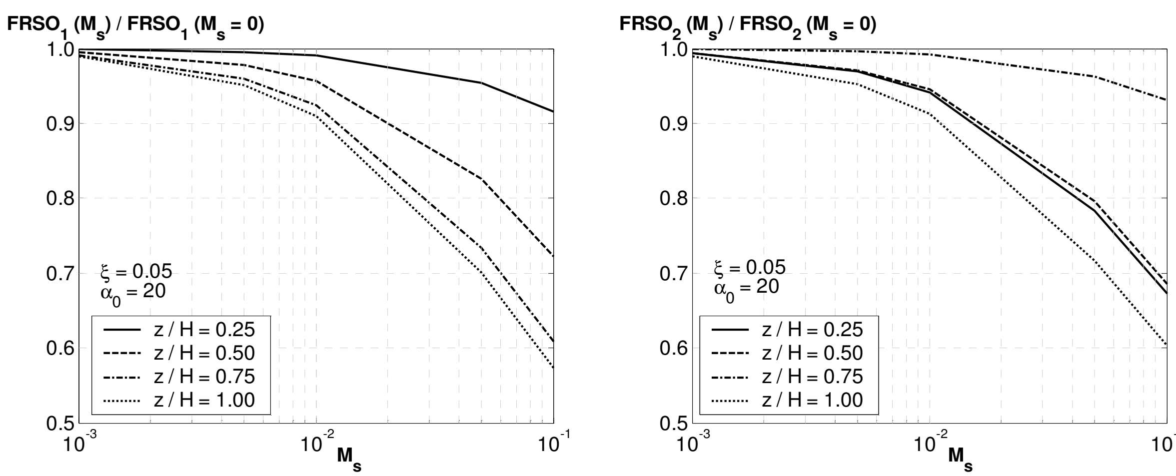

Figure 5 – Effect of mass ratio on the ratio of coupled to uncoupled FRSO for systems tuned to the first and second modes at four heights for moment frame buildings

5.1.2. *Effect of mass ratio, damping and lateral resisting system on FRS*

Fig-6 (left) shows the ratio of coupled to uncoupled FRSO of the first mode as a function of damping ratio of the secondary system for components installed at floors with z/H = 0.25, 0.50, 0.75 and 1.0. It can be seen that in components with smaller damping, the interaction between the primary and secondary system is more critical compared to that highly damped secondary components. For damping ratio of 2 percent and □0 = 0, the ratio of coupled to uncoupled FRSO at the first mode is about 97, 69, 44 and 31 percent for z/H = 0.25, 0.50, 0.75 and 1.0, respectively while the ratio of coupled to uncoupled FRSO for secondary components with 10 percent damping increase to 1.00, 0.92, 0.74 and 0.58, at z/H = 0.25, 0.50, 0.75 and 1.0, respectively. The other observation is that not only the components with lower damping ratio have larger coupling effect, but also this effect is larger at upper floors compared to lower levels. For example, at z/H = 0.25, this ratio decreases from 1.00 to 0.97 for a 10 percent damped component (3 percent change) while at z/H = 1.00, this ratio decreases from 0.58 to 0.31. Same observations are valid for moment frame structures.

The interaction between the primary and secondary components is different in structures with moment resisting frames and shear wall buildings. The variation of coupling effect with changes in lateral stiffness ratio is shown in Fig-6 (right). It is clear that the lateral stiffness ratio affects the coupled to uncoupled FRSO ratio and as the lateral stiffness ratio increases, the interaction between the two systems may increase or decrease, depending on the height. For lower levels, increase of lateral stiffness ratio will increase the coupling effect for both light components (Ms = 1 percent) and heavy components (Ms = 10 percent) while at the top level, structures with larger lateral stiffness ratio are less sensitive to the interaction between the primary and secondary systems. The ratio coupled to uncoupled FRSO at the ordinate of the floor response spectrum corresponding to the first mode will decrease as the lateral stiffness ratio increases for floors with z/H < 0.75 and increase for floors with z/H > 0.75.

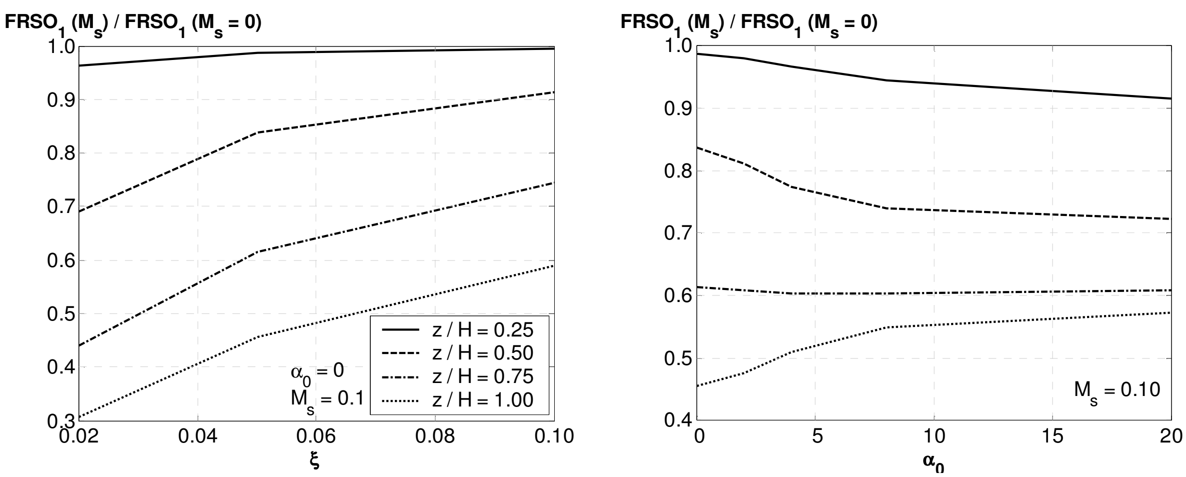

Figure 6 – effect of mass ratio on the first ordinate of FRS for different levels of damping ratio and different lateral resisting systems

5.2. Modified Kanai-Tajimi Base Excitation

The fundamental period of vibration of structures subjected to white noise ground motions does not have any effect of the response since the ground motion has the same power at all frequencies. To study the effect of interaction between the primary and secondary systems in structures with different periods of vibration, structures were subjected to modified K-T compatible ground motions. The parameters of the power spectral density function were selected such that they match the average power spectral density function of a set of ground motions recorded in firm soils. The central frequency (ω_g) and band width (ξ_g) parameters of the modified K-T PSD are 1.5 Hz and 0.7, respectively and ω_f and ξ_f parameters are 0.1 Hz and 1.0.

5.2.1. Effect of mass ratio and period on FRS

Fig-7 shows the ratio coupled to uncoupled roof response spectrum of a moment frame building (α_0 = 20). A similar behavior can be observed between the combined effects of mass ratio and fundamental period of vibration for flexural and shear beams. It is seen that in long period structure (T_1=3.5 s), all the modes show practically the same reduction in FRSO. The reason is that in moment frame structures, the periods of vibrations of different modes are closer to each other compare to shear wall structures and therefore, all of the 5 modes used in the analysis fall within the high power region of the ground motion spectrum unlike the shear wall building where modes are more separated and higher modes could fall in low power regions. The more the power of ground motion is in a particular mode, its contribution is higher in total response and as we know, the more one mode contributes in the response, and coupling effect are higher.

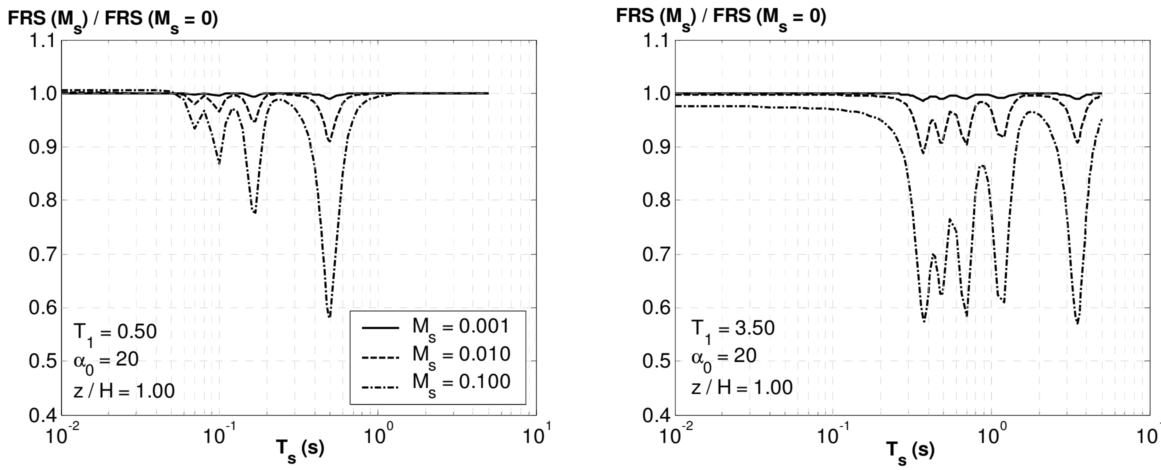

Figure 7 – Ratio of coupled to uncoupled FRSO at roof level of moment resisting frames.

6. SUMMARY AND CONCLUSIONS

In summary, the dynamic properties of the primary system such as fundamental period of vibration as well as lateral stiffness ratio as well as the period and damping ratio of the secondary system have a significant effect on the seismic demand of secondary systems. Contrary to results of some previous investigations, the effects of dynamic interaction are not the same for secondary systems tuned to different modes of vibration. In general the coupling effects are smaller for secondary systems tuned to higher modes than the effects of secondary systems tuned to the fundamental mode. For components with mass ratios of equal or smaller than 1 percent dynamic interaction effects are relatively small (less than 10%) and therefore may be neglected. However, for heavier components the effects may be very large for systems tuned to any of the modes of the primary structure. In these cases the effects of dynamic coupling vary along the height of the structure.

Dynamic coupling effects are smaller in components with higher damping ratio than in lightly damped components. The coupling effect changes with the lateral stiffness ratio and depending on the floor in which the secondary system is attached to the primary system. Increasing the lateral stiffness ratio may increase or decrease the interaction. The interaction between the primary and secondary system is proportional to the amplitude of floor response spectrum. This means that the first ordinate of the FRS is more coupled with the secondary system at higher floors compared to lower floors since the first mode shape and therefore the FRSO increases along the height. On the other hand, the second ordinate changes similar to the second mode shape and therefore the coupling effect changes following the second mode shape which means the interaction is the least at or near the node of the second mode which is located approximately around 70 percent of the height. As the fundamental period of the structure increases, the effects of dynamic coupling also occur in higher modes. The interaction between the primary and secondary system when the component is tuned to the first mode of the structure does not change with variations of fundamental period of vibration while the second ordinate is less sensitive to coupling in short period structures compare to long period structures.

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