RESPONSE OF STRUCTURES UNDER BIDIRECTIONAL HORIZONTAL SEISMIC EXCITATIONS

A. Pozos-Estrada¹, H.P. Hong² and J.A. Escobar³

¹¹²Dept. of Civil and Environmental Engineering, The University of Western Ontario, Canada
³Institute of Engineering, National Autonomous University of Mexico, Mexico

Email: ¹apozoses@uwo.ca, ²hongh@eng.uwo.ca, ³jess@pumas.iingen.unam.mx

ABSTRACT:
The Arias intensity tensor is used to define the principal axes of seismic excitation. These axes are employed to characterize the seismic excitations and often interpreted as if their orientations represent the axes of the maximum, intermediate and minimum pseudo-spectral acceleration (PSA) responses. The development of the extended complete quadratic combination (CQC) rule proposed in the literature for evaluating the responses of structures under multicomponent orthogonal seismic excitations and used to select the critical angle of seismic incidence is based on the assumption of the existence of the principal axes. However, the orientation of the axis associated with the maximum PSA response often does not coincide with the orientation of the major principal axis and is period-dependent. The impact of this on the extended CQC rule is unknown and deserves to be investigated. Moreover, to the author’s knowledge, there is no guideline on how to select the magnitude of the response spectra for the two orthogonal horizontal directions that are used with the extended CQC rule. These encouraged us to investigate the accuracy of this rule for estimating the responses under bidirectional horizontal seismic excitations, and their use together with uniform hazard spectra or design response spectrum. The study uses about 600 actual strong ground motion records for such an investigation. The analysis results form the basis for providing recommendations on how to define the spectra for two orthogonal horizontal directions and how to correct the biases in these rules.

KEYWORDS: Earthquakes, Directionality, Spectra, Seismic design, Seismic effects.

1. INTRODUCTION

Engineered structures are subjected to multicomponent seismic excitations. The orthogonal horizontal ground excitations can be characterized along the principal directions defined by the Arias intensity tensor (Arias 1970, Penzien and Watabe 1975, Kubo and Penzien 1979). Several authors have adopted this seismic excitation model and it seems that the major, intermediate, and minor principal axes are taken to be as if they represented the axes with the major, intermediate, and minor responses for selecting the critical angle of seismic incidence (Smeby and Der Kiureghian 1985, Menun and Der Kiureghian 1998, Lopez et al. 2001, Anastassiadis et al. 2002). However, recent findings showed that the orientation of the axis associated with the maximum pseudo-spectral acceleration (PSA) response often does not coincide with the orientation of the major principal axis and is period-dependent (Hong and Goda 2007, Pozos-Estrada et al. 2007). Therefore, this seismic excitation model which was adopted for developing the extended complete quadratic combination (CQC) rule for multicomponent excitations (Smeby and Der Kiureghian 1985), and the accuracy of the extended CQC rule should be verified or validated using sufficient number of actual strong ground motion records. The validation can only be carried out in statistical sense since the excitations are uncertain. Furthermore, to the authors’ knowledge, in the literature there is no guideline on how to select the response spectra for the two orthogonal horizontal directions and how to relate them to the uniform hazard spectra (UHS) or the design spectrum for randomly oriented systems.

This study aims at carrying out a statistical assessment of the peak response of structures under two orthogonal horizontal ground excitations; to assess the accuracy of the extended CQC rule (Smeby and Der Kiureghian 1985),
and to provide recommendations for selecting the response spectra in two orthogonal horizontal directions that are to be used in conjunction with the extended CQC rule. To facilitate the assessment, the basic assumptions leading to the extended CQC rule are summarized in the following section. For the assessment, sets of observed strong ground motion records are employed, and the implications of the results in estimating the maximum responses of structures under orthogonal bidirectional horizontal excitations are discussed.

2. ASSUMPTIONS LEADING TO THE EXTENDED COMPLETE QUADRATIC COMBINATION RULE FOR MULTICOMPONENT SEISMIC EXCITATIONS

The development and usefulness of the complete quadratic combination (CQC) rule for estimating the peak structural responses under seismic excitations have been given by Der Kiureghian (1981), the adequacy of using the CQC rule with the uniform hazard spectrum (UHS) is also discussed in the literature (Hong and Wang 2002; Wang and Hong 2005). Smeby and Der Kiureghian (1985) extended the CQC rule for estimating the peak response of structures under multicomponent excitations. The basis for this extended CQC rule is summarized in the following. Consider that a structure modeled as a linear elastic multi-degree-of-freedom (MDOF) system is subjected to multicomponent seismic excitations. A structural response of interest, $R(t)$, can be expressed as a combination of the nodal displacements, denoted by $U$, which can be obtained by solving the following equation of motion,

$$M\ddot{U} + C\dot{U} + KU = -MI\ddot{U}_g$$  \hspace{1cm} (2.1)

where $M$, $C$, and $K$ are the mass, damping and stiffness matrices; dots above a symbol indicate its time derivatives; $I$ is an influence matrix and $\ddot{U}_g$ is the vector of no more than three translational ground excitations.

By considering that the excitations along the principal axes are independent (Arias 1970, Penzien and Watabe 1975), Smeby and Der Kiureghian (1985) developed the extended CQC rule to estimate the mean peak response of $R(t)$, $\mu_R$, that are functions of $n$, $C_{ij,k}$, $\mu_{i,k}$, $\rho_{ij}$, and $\theta$ where $n$ is the total number of vibration modes, $C_{i,k}$ is the effective participation factor of $i$-th mode associated with $k$-th component of ground motion, $\mu_{i,k}$ denotes the mean peak modal response for the $i$-th mode with $k$-th component of ground motion in which $k = 3$ represents the vertical component, $\rho_{ij}$ is the modal correlation coefficient of the responses between the $i$-th and $j$-th modes (Der Kiureghian 1981), and $\theta$ is the angle between the set of structural principal axes and the set of seismic excitation principal axes in the horizontal plane. The extended CQC rule is referred to as the CQC3 rule by Menun and Der Kiureghian (1998). Smeby and Der Kiureghian (1985) stressed that the developed rule is specific to the adopted seismic ground excitation model.

It was observed (Hong and Goda 2007, Pozos-Estrada et al. 2007) that the major principal axis does not coincide with the axis along which the peak response of a single-degree-of-freedom (SOF) system is maximum (i.e., major response axis); and the ratio of the peak response for a SDOF with an arbitrary orientation to the maximum peak response (i.e., maximum resultant response) for a record in the horizontal plane cannot be adequately represented by an ellipsoid. These are illustrated in Figure 1 for an arbitrarily selected record considering the excitation in the horizontal plane only. The first observation can be explained by noting that the major response axis is associated with the energy at a particular frequency whereas the major principal axis, calculated based on the Arias intensity tensor, is a measure of the sum of the energy along the frequencies. The second observation simply reflects the nonstationarity of the process, and that the accuracy of the extended CQC rule which assumes that the spectra for the two horizontal axes have the same spectral form needs to be evaluated. In other words, there is a need for assessing or evaluating the rule using the actual strong ground motion records. The need for such an assessment is further justified by noting that many studies (Menun and Der Kiureghian 1998, Lopez et al. 2001, Anastassiadis et al. 2002) use the predicted responses from the extended CQC rule as benchmarks to assess the accuracy of the approximate combination rules such as the SRSS rule and, the percentage rules.

It should be noted that the accuracy of the CQC rule for estimating the fractile of peak response is similar to that for
estimating the mean peak response for structures under a single component of horizontal ground excitations (Hong and Wang 2002, Wang and Hong 2005). However, whether this conclusion is valid for structures under multicomponent seismic excitations is unknown, although it is implied that the extended CQC rule could be used in estimating the fractiles of peak response in some applications. With this observation, one could relate the peak response of $R(t)$, $r_c$, to the peak modal responses, $r_{i,k}$, for the same given probability level by replacing $\mu_R$ and the mean peak modal response $\mu_{i,k}$ with $r_c$ and $r_{i,k}$, respectively, where $r_{i,k}$ represents the peak modal response (with the effective participation factor equal to one) for the $i$-th mode with $k$-th component of ground motion.

In particular, if the spectral shape for the two horizontal axes are the same (i.e., $r_{i,2} = \gamma \times r_{i,1}$) and the vertical component is not considered, the extended CQC rule reduces to (Menun and Der Kiureghian 1998),

$$r_c = \left( R_{i1}^2 + R_{i2}^2 - (1 - \gamma^2) \left( R_{i1}^2 - R_{i2}^2 / \gamma^2 \right) \right) \sin^2 \theta + \left( (1 - \gamma^2) / \gamma^2 \right) R_{i2} \sin 2\theta \right)^{1/2},$$

(2.2)

where $R_k^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{i,k}C_{j,k}r_{i,k}r_{j,k}$ and $R_{kl} = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{i,k}C_{j,l}r_{i,k}r_{j,l}$, and the critical angle of seismic incidence $\theta_c$ is given by,

$$\theta_c = \left( \tan^{-1} \left( \left(2R_{i2} / \gamma \right) \left( R_{i1}^2 - R_{i2}^2 / \gamma^2 \right) \right) \right) / 2.$$

In codified design, the use of the UHS is recommended (e.g., NBCC 2005) which is commonly developed based on the seismic source zone models, the earthquake occurrence modeling and the ground motion prediction equation (GMPE) (i.e., attenuation relations of the PSA) (Frankel et al. 1996, Adams and Atkinson 2003). The evaluation of UHS is for a randomly oriented SDOF system rather than the maximum PSA among all possible orientations (Boore et al. 2006, Hong and Goda 2007). Consequently, the seismic design spectrum or the UHS determined in such a manner does not represent the spectrum for any of the principal axes in the horizontal plane. That is, the critical response for a structure under orthogonal horizontal ground motions, which is calculated using the extended CQC rule and assuming identical spectral shape for the two principal horizontal axes, needs to be validated.

### 3. ANALYZED STRUCTURES AND STRONG GROUND MOTION RECORDS USED

It is noted that a comprehensive and systematical assessment of the extended CQC rule requires the consideration of an extremely large number of combination of parameters or factors. To simplify the parametric analysis, this

![Figure 1 Illustration of response axes, principal axes and orientation effects on the records and the PSA](image)
study considers only single storey symmetric buildings under two horizontal orthogonal excitations. The idealized single storey building to be analyzed has a rigid slab as illustrated in Figure 2.

Figure 2 Single story symmetric system

To facilitate the parametric study, we consider that the system is characterized using

$$\omega_x = \frac{k_x}{m}, \quad \eta_y = \frac{\omega_y}{\omega_x},$$

and $$\omega_y = \frac{k_y}{m}$$ where $$k_x$$ and $$k_y$$ represent the stiffness along the x and y-directions, and $$m$$ is the lumped mass of the system; and that the classical damping matrix of the structure can be derived based on the modal damping coefficient leading to the modal damping ratio equal to $$\xi_x$$ and $$\xi_y$$ for the vibration along the x- and y-directions, respectively. This leads to,

$$\begin{cases}
\frac{\ddot{u}_x}{\ddot{u}_y} + 2\omega_x \left[ \begin{array}{cc}
0 & 0 \\
\xi_x & \eta_y
\end{array} \right] \begin{bmatrix}
\dot{u}_x \\
\dot{u}_y
\end{bmatrix} + \omega_x \left[ \begin{array}{cc}
1 & 0 \\
0 & \eta_y
\end{array} \right] \begin{bmatrix}
u_x \\
u_y
\end{bmatrix} = - \begin{bmatrix}
\ddot{u}_x(t) \\
\ddot{u}_y(t)
\end{bmatrix},
\end{cases}$$

where $$u_x$$ and $$u_y$$ denote the displacement in x- and y-directions respectively; a dot over a symbol represents its temporal derivative. A modal damping ratio of 5% is considered for all the numerical analyses in this study.

Given a record, Eqn. (3.1) can be solved using the Newmark method, and the maximum of a response of interest, $$R(t)$$, $$r_{\text{max}}$$, can be calculated using the obtained response time history of the structure. Consequently, statistics of $$r_{\text{max}}$$ can be obtained for a selected set of records.

Numerical analyses are to be carried out using a judiciously selected set of 592 actual strong ground motion records from 39 earthquakes from the NGA database for California earthquakes (PEER, 2006), for which the GMPEs have already been developed (Hong and Goda 2007) considering the following functional form,

$$\ln Y = f(M, d) + \varepsilon,$$  

where $$Y$$ (g) represents the PSA, $$A(T_n)$$, for a randomly oriented SDOF system with vibration period $$T_n$$ or the maximum resultant response $$A_{\text{Max}}(T_n)$$; $$f(M, d)$$ is a function of earthquake magnitude and source to site distance and other model parameters; and $$\varepsilon$$ denotes the sum the intra-event variability $$\varepsilon_c$$ and the inter-event variability $$\varepsilon_e$$. For the considered records, their obtained standard deviations of $$\varepsilon_c$$, $$\varepsilon_e$$ and $$\varepsilon$$ are 0.21, 0.53 and 0.61 for $$T_n$$ equal to 0.5(s), and are 0.33, 0.56 and 0.69 for $$T_n$$ equal to 1.0(s). Note that the GMPEs for $$A(T_n)$$ are often developed based on the geometric mean, $$A_{\text{GM}}(T_n)$$, with a correction to the standard deviation of the error term $$\varepsilon$$ (Boore et al. 1997) and, used for developing the UHS. The relations developed based on $$A_{\text{Max}}(T_n)$$ represent the response for the SDOF systems oriented along the critical angle of seismic incidence.

The assessments of the mean of the ratios between $$A_{\text{Max}}(T_n)$$ and $$A(T_n)$$, between the PSA for the minor response axis $$A_{\text{Min}}(T_n)$$ and $$A(T_n)$$, between the PSA for the major principal axis $$A_{\text{Pma}}(T_n)$$ and $$A(T_n)$$, and between the PSA for the minor principal axis $$A_{\text{Pmi}}(T_n)$$ and $$A(T_n)$$, are carried out and the results are illustrated in Table 3.1 for $$T_n$$ equal to 0.5 and 1(s). The table indicates that the means of the ratios $$A_{\text{Max}}(T_n)/A(T_n)$$, $$A_{\text{Min}}(T_n)/A(T_n)$$, $$A_{\text{Pma}}(T_n)/A(T_n)$$, and $$A_{\text{Pmi}}(T_n)/A(T_n)$$ vary with vibration frequency. For convenience, if one is interested in using a
single value for each ratio, one could consider 1.15 and 0.95 for the means of \( \frac{A_{\text{MaxR}(T_n)}}{A(T_n)} \) and \( \frac{A_{\text{MinR}(T_n)}}{A(T_n)} \), and 1.30 and 0.70 for the means of \( \frac{A_{\text{MaxR}(T_n)}}{A(T_n)} \) and \( \frac{A_{\text{MinR}(T_n)}}{A(T_n)} \).

<table>
<thead>
<tr>
<th>Ratio</th>
<th>( T_n ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{\text{MaxR}(T_n)} / A(T_n) )</td>
<td>1.29</td>
</tr>
<tr>
<td>( A_{\text{MinR}(T_n)} / A(T_n) )</td>
<td>0.69</td>
</tr>
<tr>
<td>( A_{\text{MaxR}(T_n)} / A(T_n) )</td>
<td>1.17</td>
</tr>
<tr>
<td>( A_{\text{MinR}(T_n)} / A(T_n) )</td>
<td>0.93</td>
</tr>
</tbody>
</table>

4. STATISTICS OF THE RESPONSES UNDER BIDIRECTIONAL SEISMIC EXCITATIONS

4.1 Characteristics of the responses

Consider that the structure of Figure 2 is under bidirectional excitations with governing equation shown in Eqn. (3.1), and that the response of interest \( R(t) \) can be expressed as,

\[
R(t) = C_x \ddot{u}_x + C_y \ddot{u}_y = C_y \left( \zeta_x u_x + u_y \right),
\]

where \( C_x \) and \( C_y \) are the coefficients relating the displacements to the response of interest, and \( \zeta_x = C_x/C_y \). For simplicity and without loss generality, \( C_y \) is set equal to one.

Consider that the structure is subjected to the excitations of an arbitrarily selected ground motion record shown in Figure 1. For the moment, it is considered that the structural principal axes coincide with the orientations of the recording sensors. The maximum absolute value of \( R(t) \), \( r_{\text{max}} \), obtained by solving Eqns. (3.1) and (4.1) is illustrated in Figure 3a for \( \zeta_x = 1, T_{\text{max}} = 0.5, \) and \( \eta_y = 1 \). Since the structural principal axes may not coincide with the orientations of the recording sensors, the evaluation of \( r_{\text{max}} \) is also carried out by varying the angle between the structural principal axes and the ground motion recording axes \( \theta \) (i.e., rotate structure counterclockwise, or rotate the orientations of the recording sensors clockwise) and the results are also shown in Figure 3a. Similar analysis is carried out and the results are shown in Figure 3b but for \( \zeta_x = 1, T_{\text{max}} = 1.0, \) and \( \eta_y = 1.0 \). Figure 3 indicates that the maximum of \( r_{\text{max}} \), denoted by \( r_{\text{max},c} \), occurs at different angles of seismic incidence for the two considered cases; the values of \( r_{\text{max}} \) in the horizontal plane does not resemble an ellipsoid and; the critical angle of seismic incidence does not coincide with the principal axes of the record.

Figure 3 Illustration of the effect of angle of seismic incidence on the structural response considering bidirectional excitation (there is periodicity of 180° in \( r_{\text{max}} \)).

To assess the characteristics of \( r_{\text{max}} \) for all possible angles of seismic incidence, values of \( r_{\text{max}} \) and of \( r_{\text{max}} / r_{\text{max},c} \) are calculated for each of the 592 records considering the structure shown in Figure 3a; the obtained results are shown in Figure 4 considering that the axes of the seismic excitations are rotated clockwise.
Figure 4a indicates that by considering the randomness in the recording orientations and the structural principal axes, the mean of the $r_{\text{max}}$ (considering all possible structural principal axis orientations) is about 0.8 of $r_{\text{max,c}}$. As shown in Figure 4b, the ratio falls within a circle of radius 1.0 but almost always outside of two inner mutually exclusive small circles of radius 0.5, which is termed as the “goggle” phenomenon observed for the SDOF systems (Hong and Goda 2007). The mean of the ratio is about 0.62 along the $Y_0$-axis. The results shown in Figure 4c suggest that the mean of the ratio is about 0.8 along $Y_0$-axis, and 0.8 along $X_0$-axis.

Figure 4. Effect of angle of seismic incidence on $r_{\text{max}}/r_{\text{max,c}}$ for bidirectional excitation: a) the structural principal axes and the recording orientations initially coincide; b) initially the structural principal axes and the recording orientations are placed in such a way that $r_{\text{max,c}}$ is attained; c) the structural principal axes and the excitation principal directions coincide initially and $r_{\text{max}}$ is the greatest for all possible combinations of such coincidences.

4.2 Evaluation of the combination rule

It is noted that the extended CQC rule is often used for design purposes, and most recent design spectra are developed based on the GMPEs for $A(T_n)$. Consequently, it is of interest to assess the characteristics of the peak response of $R(t)$ defined in Eqn. (4.1) using the extended CQC rule together with the predicted PSA values associated with principal directions. However, the design spectra or the PSA values suggested in the design codes represent the PSA for randomly oriented SDOF systems rather than those for the principal directions. Since this is likely to be the case at least in the near future, it is desirable to take this into account in the following assessment.

Note that for given structure (i.e., $\zeta_x$, $T_{nx}$, and $\eta$), one can calculate the $r_{\text{max}}$ and $r_{\text{max,c}}$ for each considered record whose earthquake magnitude and distance ($M$, $d$) are known. One can also calculate the peak response of $R(t)$, $r_c$, using the CQC rule together with the PSA values predicted by the GMPE for the same ($M$, $d$). The critical response $r_c$, (i.e., the maximum of $r_c$) and its corresponding critical angle of incidence $\theta_c$ can be found using Eqn. (2.2).

For the moment, consider that Eqn. (3.2) for $A(T_n)$ can be used to predict the PSA for the two orthogonal horizontal orientations. This consideration is based on the observation that the GMPEs are employed to develop the UHS and design spectrum. To investigate the accuracy of the extended CQC rule (which reduces to the well-known SRSS rule for this case since the PSA along the structural principal axes are considered to be the same, $r_c$ given in Eqn. (2.2) is independent of the angle of seismic incidence) an assessment of statistics of the ratio of the peak response obtained from time history under bidirectional excitations to the peak response predicted by the CQC rule is carried out. Inspection of the calculated samples by considering the structures with characteristics shown in Figure 4 suggests that the logarithmic of the ratios could be assumed to be linearly uncorrelated with $M$ and $d$. Therefore, similar to the assessment of the GMPE (Joyner and Boore 1993, Boore et al. 1997), one could carry out regression analysis to assess this variability of the ratio by considering,

$$\ln(r_{\text{max}}) = \ln(r_c) + a + \eta,$$

where $a$ is a regression coefficient, $\eta = \eta_e + \eta_r$ in which $\eta_e$ represents the zero mean inter-event variability and $\eta_r$ represent the zero mean intra-event variability. For the considered structures, the obtained $a$, the standard deviation of $\eta_e$, $\sigma_{\eta_e}$, and the standard deviation of $\eta_r$, $\sigma_{\eta_r}$, are shown in Table 4.1.
The results presented in the table indicate that for this case the obtained $\sigma_{pe}$ and $\sigma_{pe}$ are similar to the standard deviations of $\varepsilon_1$ and $\varepsilon_2$ mentioned earlier; and that the bias ranges about -0.4% to 0.8% 0.6% (i.e., exp(-0.004)-1 to exp(0.008)-1). This implies that on average the extended CQC rule (together with the PSA predicted by the GMPE) provides sufficient accurate estimates of the peak response considering the random orientation effects.

To discuss the critical response $r_{cr}$ associated with the critical angle of incidence $\theta_{cr}$, one must consider that the spectra differ for two orthogonal directions. However, the uniform hazard spectra and the design spectrum given in design codes are developed based on $A(T_n)$ for randomly oriented SDOF systems and do not represent the responses in two orthogonal directions. To overcome this and aimed at developing a simple representation of the response spectra for the two response axes or for the two principal directions, one could directly relate the response spectra for orthogonal horizontal direction to $A(T_n)$. The response spectra for the two response axes could be given in terms of $A_{Max}(T_n)$ and $A_{Min}(T_n)$ which can be approximated by $1.30 A(T_n)$, $0.70 A(T_n)$ as shown in Table 3.1; while the spectra for the two principal axes are given in terms of $A_{PmA}(T_n)$ and $A_{PmP}(T_n)$ and can be approximated by $1.15 A(T_n)$ and $0.95 A(T_n)$.

Table 4.1 Obtained parameters for the regression equations

<table>
<thead>
<tr>
<th>Structure characteristics</th>
<th>ln($r_{max}/r_{rule}$)</th>
<th>PSA defined by ($A_{Max}(T_n), A_{Min}(T_n)$)</th>
<th>PSA defined by ($A_{PmA}(T_n), A_{PmP}(T_n)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_s$</td>
<td>$T_{nx}$</td>
<td>$m_\gamma$</td>
<td>$a$</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>0.006</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>1</td>
<td>-0.040</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.008</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

It is emphasized that the PSA calculated based on the ground motion prediction equation (GMPE) (or attenuation relation) of randomly oriented SDOF systems is used to develop the UHS and the design spectra and differs from the PSA along the principal directions and along the response axes.

The extended CQC rule provides biased estimates of the maximum peak responses under bidirectional horizontal excitations. The bias depends on the structural characteristics, and may be corrected using scaling factors. It is propose that to estimate the maximum peak responses or critical peak responses under bidirectional excitations, the response spectra for the two orthogonal horizontal directions can be defined using $(1.30 A(T_n)$, $0.70 A(T_n)$) or $(1.15 A(T_n)$, $0.95 A(T_n)$) shown in Table 4.1 are for cases where the responses depend on the bidirectional excitations.
The former is based on the response spectra along the response axes, while the latter is based on the principal axes. Use of the former provides less biased estimate of the critical responses than the latter.

ACKNOWLEDGMENTS
The financial supports of the Natural Science and Engineering Research Council of Canada and the National Council of Science and Technology of Mexico are gratefully acknowledged.

REFERENCES