PROBABILISTIC SEISMIC ASSESSMENT OF RESIDUAL DRIFT DEMANDS IN EXISTING BUILDINGS

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ABSTRACT :

This paper presents a probabilistic approach to estimate residual drift demands (e.g. residual roof, residual drift at specific stories, and maximum residual drift over all stories) during the seismic performance-based assessment of existing multi-story buildings. The suggested approach combines residual drift demand fragility curves with very recently introduced maximum inelastic displacement seismic hazard curves to obtain site-building-specific residual drift demand hazard curves which express the mean annual frequency of exceeding residual drift demands. In particular, functional models that capture the variation of central tendency and dispersion of residual drift demands with changes in the ground motion intensity are proposed. It is shown that the proposed procedure can be very helpful during the performance-based seismic assessment of existing multi-story building frames since it incorporates explicitly the epistemic uncertainty (i.e. record-to-record variability) inherent in the estimation of residual drift demands at the end of the seismic excitation

KEYWORDS: Probabilistic assessment, residual drift, seismic assessment, inelastic intensity measure

1. MOTIVATION

Post-earthquake field reconnaissance have evidenced that residual (permanent) lateral displacement demands after earthquake excitation might have an impact in the decision-making process of retrofitting/demolishing man-made engineered structures such as buildings and bridges. For example, Okada et al. (2000) reported that several low-rise reinforced concrete (RC) buildings suffered light structural damage and experienced relatively large residual deformations as a consequence of the 1995 Hyogo-Ken Nambu earthquake even though they had sufficient deformation capacity. Similarly, many RC bridge piers of the Hanshin Expressway viaduct were demolished in Kobe after the aforementioned earthquake for the elevated cost that would be required to repair piers with large permanent drifts (Kawashima, 2000; Fujino, et al., 2005). Thus, the estimation of residual drift demands should also play an important role in the evaluation of structural performance of existing structures in addition to maximum (transient) lateral displacement demands and peak floor acceleration. Motivated by recent post-earthquake field reconnaissance observations, several researchers have performed analytical investigations aimed at gaining further understanding on the parameters that influence the amplitude and height-wise distribution of residual drift demands in multi-degree-of-freedom (MDOF) systems (e.g. Pampanin et al., 2003, Ruiz-Garcia and Miranda, 2005, 2006). They have reported that the residual drift demand amplitude and distribution over the height mainly depends on the component hysteretic behavior, building frame mechanism, structural overstrength as well as the ground motion intensity. In particular, Ruiz-Garcia and Miranda (2005, 2006) have noted that the evaluation of residual drift demands in regular moment-resisting frame models involves large levels of uncertainty (i.e. record-to-record variability) in its estimation and, moreover, this uncertainty is larger than that associated to the estimation of maximum (transient) drift demands. Thus, the evaluation of residual deformation demands into seismic performance-based assessment methodologies requires a probabilistic approach where the record-to-record variability is explicitly incorporated.

The main purpose of this paper is to present a procedure aimed at obtaining probabilistic descriptions of residual drift demands to be used during the seismic performance-based assessment of existing multi-story frame buildings. In this context, the following specific goals were stated: a) to investigate a suitable intensity measure, b) to select a parametric probability distribution in order to characterize the empirical probability distribution of residual drift demands, and c) to characterize the variation of central tendency and dispersion (i.e. record-to-record variability) of residual drift demand s with changes in the ground motion intensity. For that purpose, three regular multi-story on-bay generic frame models having 3, 9 and 18 stories subjected to earthquake ground motions from a small distance-large magnitude seismic environment were employed in this investigation.

2. PROBABILISTIC ESTIMATION OF RESIDUAL DRIFT DEMANDS

2.1. Probabilistic framework

In agreement with modern performance-based seismic assessment procedures, such as that developed at the Pacific Earthquake Engineering Center and later implemented in FEMA 356 (Cornell et al., 2002; Deierlein, 2004), the site-specific mean annual frequency (MAF) of exceeding a given residual drift demand, Δ_r, for an existing building can be obtained as follows:

$$\nu(\Delta_r > \delta_r) \cong \int_0^{\infty} P\left(\Delta_r > \delta \mid IM = im; T_1, C_y\right) \times \left|\frac{d\nu(IM)}{dIM}\right| dIM \tag{2.1}$$

In the above expression, $P(\Delta_r > \delta \mid IM = im; T_1, C_y)$ is the probability of Δ_r exceeding a defined residual deformation demand conditioned on the fundamental period of vibration of the existing building, T_1, the yielding strength coefficient, C_y, and the ground motion intensity measure, IM, evaluated at level im. In addition, $\nu(IM)$ refers to the mean annual frequency of exceedance of the IM, which also represents the seismic hazard at a specific site. In this context, while the first term in the right-hand side of Equation (2.1) can be obtained from probabilistic estimates of the Δ_r of interest (i.e. residual roof drift ratio, residual drift at specific stories or maximum residual inter-story drift ratio over all stories), the second term in Equation (2.1) represents the slope in the seismic hazard curve, which can be computed from conventional Probabilistic Seismic Hazard Analysis (PSHA), evaluated at the ground motion intensity level im.

2.2. Selection of intensity measure

An important component in Equation (2.1) is the selection of an appropriate parameter to characterize the intensity of the ground motion, which is also known as intensity measure (IM). In this study, the spectral displacement, S_d, of a linear elastic 5% damped single-degree-of-freedom (SDOF) system having a fundamental period of vibration of the structure, T_1, as ground motion intensity measures (i.e. $S_d(T_1)$) and the maximum inelastic displacement of an equivalent elastoplastic SDOF system having the same initial lateral stiffness (i.e., fundamental period of vibration, T_1), $\Delta_i(T_1)$, were chosen as candidates $IM\acute{}s$. For instance, a comparison of the variation of median maximum residual drift demand at all stories, $RIDR_{\max}$, as well as counted 16th and 84th percentile bands, with changes in the ground motion intensity computed from the response of a 18-story one-bay generic frame model (T_1=2.0s) using both $S_d(T_1)$ and $\Delta_i(T_1)$ as $IM\acute{}s$ is shown in Figure 1. From the figure, it can be seen that the record-to-record variability is not constant and it tends to increase as the intensity of the ground motion increases, but the use of $\Delta_i(T_1)$ as IM leads to smaller levels of record-to-record variability as compared to $S_d(T_1)$. Moreover, the number of outliers (i.e. very large values compared to the rest of the sample) is considerable reduced when using $\Delta_i(T_1)$. Similar observations were found for other one-bay generic frame models having different story-height. Thus, it is believed that the uncertainty in the estimation of residual drift demands is reduced when $\Delta_i(T_1)$ is employed as IM and Equation (2.1) can be expressed in the following form:

$$\nu(\Delta_r > \delta_r) = \int_0^{\infty} P\left(\Delta_r > \delta \mid \Delta_i(T_1) = \delta_i; T_1, C_y\right) \times \left| \frac{d\nu(\Delta_i(T_1))}{d\Delta_i(T_1)} \right| d\Delta_i(T_1) \qquad (2.2)$$

In the above alternative expression $\nu(\Delta_i(T_1))$ is the site-specific maximum inelastic displacement demand hazard curve, as a function of the specific fundamental period of vibration of the system and the yield strength coefficient of the structure, which should be available.

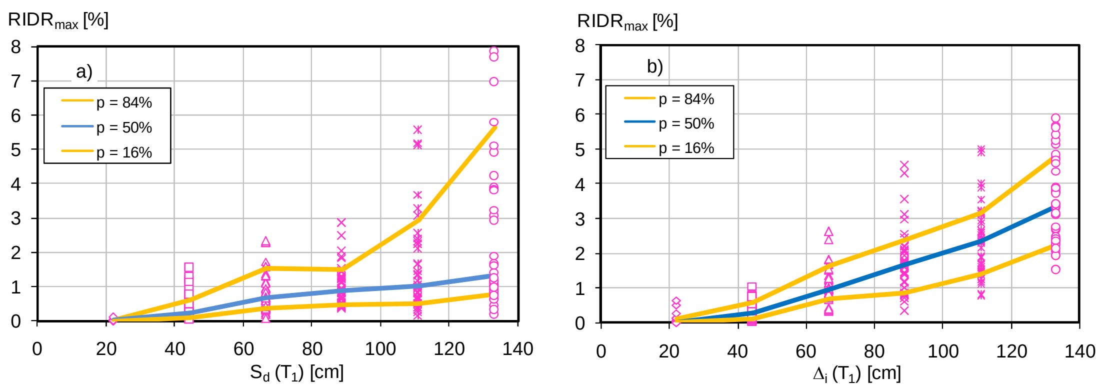

Figure 1 Variation of $RIDR_{\max}$ with changes in IM : a) $S_d(T_1)$, b) $\Delta_i(T_1)$

2.3. Parametric probability distribution f residual drift demands

Next step in the development of Equation (2.2) consists on obtaining a probabilistic description of the distribution of residual drift demands. For instance, the empirical probability distribution of $RIDR_{\max}$ corresponding to a 9-story one-bay generic stiff frame model (T_1 = 1.185s) is shown in Figure 2a while an analogous distribution for a flexible counterpart (T_1 = 1.902s) is shown in Figure 2b. The empirical cumulative probability distribution of $RIDR_{\max}$ was obtained by considering drift values as independent outcomes. Sample data was then sorted in ascending order and plotted with a probability equal to $i/(n+1)$, where i is the position of the drift ratio and n is the size of the sample.

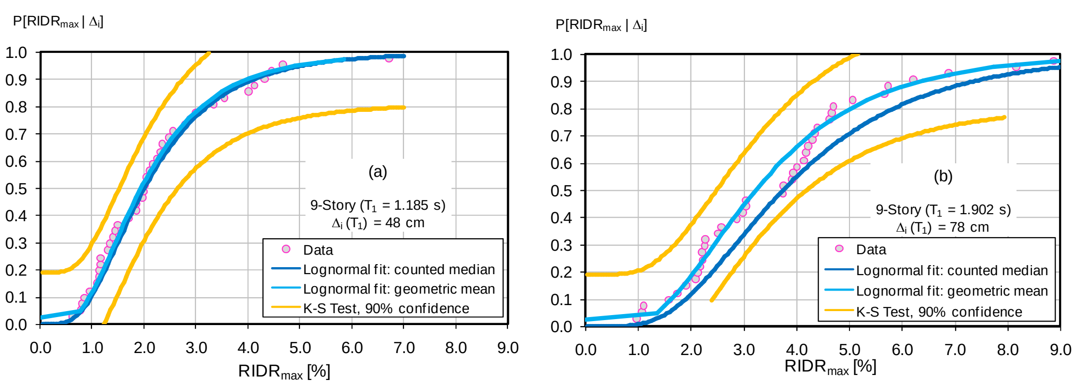

Figure 2 Variation of $RIDR_{\max}$ with changes in IM : a) $S_d(T_1)$, b) $\Delta_i(T_1)$

It can be seen that in both cases the empirical distribution is not symmetric with respect to the 50th percentile (i.e. sample median) and they have longer tails moving towards upper values. Thus, right-skewed parametric probability distributions such as lognormal, Weibull or Gumbel could be adequate to characterize the empirical cumulative probability distribution. For example, Figures 2a and 2b show also the fitted probability distribution using two-parameter lognormal probability distribution. To verify whether the parametric distributions is adequate to characterize the empirical cumulative probability distribution of residual drift demands the

Kolmogorov-Smirnov (K-S) goodness-of-fit test was used in this investigation and a graphic representation of the K-S test (at a 10% significance level) is also shown in Figures 2a and 2b. It can be seen that the lognormal distribution is adequate since all data points lies between the K-S test bands for both building models. Similar plots were obtained for other parametric probability distribution and generic frame models. Even though other probability distributions might satisfy the K-S test, the lognormal probability distribution has the convenience that can be fully defined from two parameters which explicitly represent the central tendency and the dispersion (i.e. record-to-record variability) of the sample distribution. Thus, the left-hand side term in the integrand of Equation (2.2) can be obtained as follows:

$$P\left(\Delta_r > \delta_r \mid \Delta_i(T_1) = \delta_i; T_1, C_y\right) = 1 - \Phi\left(\frac{\ln(\delta_r) - \mu_{\ln \delta_r}}{\sigma_{\ln \delta_r}}\right) \tag{2.3}$$

However, it should be noted that the sample geometric mean and the standard deviation of the natural logarithm of the data as parameters of central tendency and dispersion of residual drift demands provide better fitting with respect to the sample distribution. For example, Figures 2a and 2b show a comparison of the fitted lognormal probability distribution of $RIDR_{\max}$ for both 9-story generic frame models employing the sample geometric mean and the counted median as a measure of central while the logarithmic standard deviation is employed as a measure of dispersion.

2.4 Statistical parameters of residual drift demands as a function of the ground motion intensity

Ruiz-Garcia and Miranda (2005, 2006) showed that the sample statistical measures (i.e. central tendency and dispersion) of residual drift demands change with variation of the ground motion intensity. For instance, the variation of median $RIDR$ and $\sigma_{\ln RIDR}$ for 5 different story levels of a 9-story generic frame model (T_1=1.185s) with changes in the ground motion intensity is illustrated in Figure 3. It can be seen that median $RIDR$ computed in the seventh- and ninth-story level grows nonlinearly as the ground motion intensity increases, whereas median $RIDR$ of the first- and third-story grows almost linearly at a much faster rate than the aforementioned stories with changes in the ground motion intensity. The latter trend is a reflex of residual drift concentration in the bottom stories as the ground motion intensity increases and, in general, both trends reflex the type of frame mechanism. On the other hand, dispersion seems to increase or to decrease depending on the level of ground motion intensity and location along the height. For example, dispersion tends to decrease for $\Delta_i(T_1)$ between 20 and 30 cm, but it tends to increase for $\Delta_i(T_1)$ smaller than 20 cm.

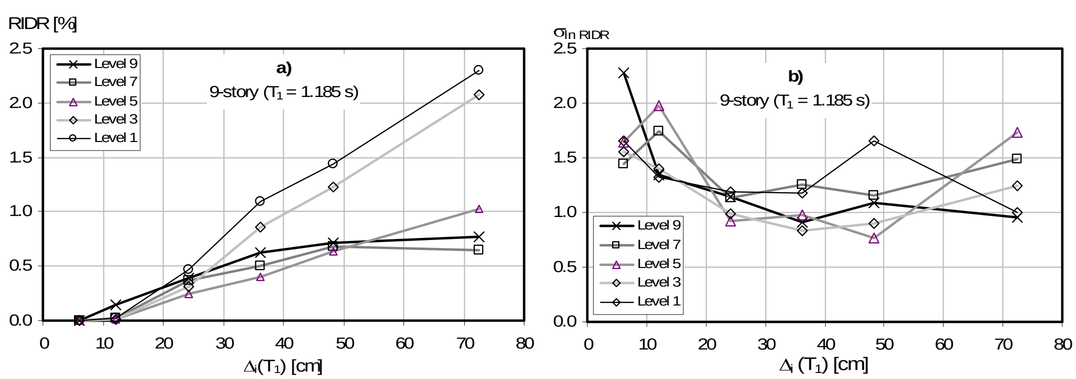

Figure 3 Variation of central tendency and dispersion of $RIDR$ computed for five stories of the GF-9R building model: (a) Median $RIDR$; and (b) dispersion of $RIDR$ ($\sigma_{\ln RIDR}$)

Therefore, the variation of sample statistical measures with changes in the ground motion intensity should be reflected in the parameters employed to estimate the building-specific conditional probability of exceeding a given residual drift demand threshold given in Equation (2.2), Thus, the following functional are employed in this investigation for describing the variation of the central tendency of drift demands with changes in the intensity measure:

$$\tilde{\mu} = \alpha_1 \alpha_2^{IM} (IM)^{\alpha_3} \tag{2.4}$$

$$\tilde{\mu} = a(IM)^b \tag{2.5}$$

While coefficients $\alpha_1, \alpha_2, \alpha_3$ in Equation (2.4) can be obtained from nonlinear regression analysis, coefficients a and b in Equation (2.5) are obtained from conventional linear regression analysis in the log-log domain that, indeed, implies a linear relationship between $\tilde{\mu}_r$ and IM. The fitted variation of geometric mean of $RIDR_{\max}$ with changes in the ground motion intensity using Equations (2.4) and (2.5) obtained for a short-period (3-story, T_1 =0.5s) and a long-period (18-story, T_1 =2.0s) frame models is shown in Figure 5. It can be seen that the functional form of Equation (2.4) captures reasonable well the variation of median $RIDR_{\max}$ with changes in the intensity of the ground motion. However, the use of Equation (2.5) might lead to underestimations or overestimations, depending on the level of ground motion intensity, to predict median $RIDR_{\max}$ for both building models.

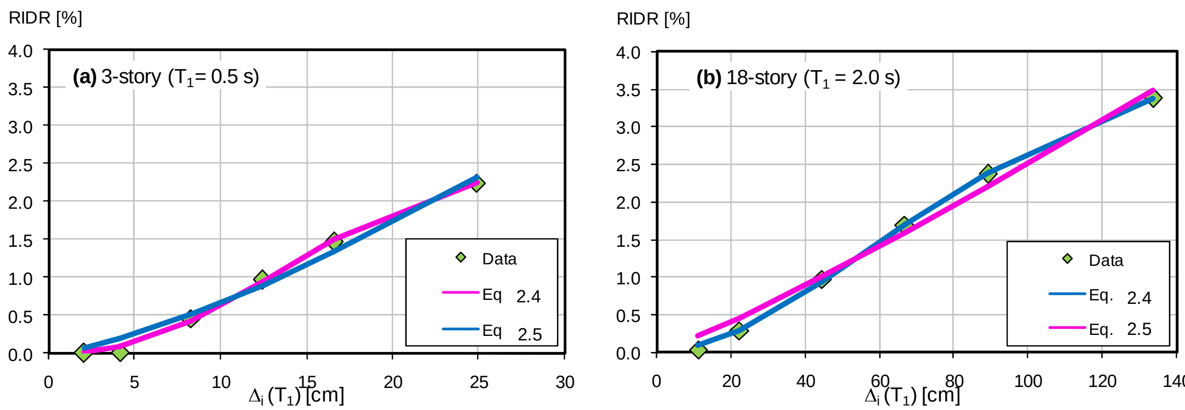

Figure 4 Evaluation of Equations 2.4 and 2.5 to estimate $RIDR_{\max}$ for two generic frame models: a) 3-story (T_1 =0.5s); and b) 18-story (T_1 =2.0s).

In addition of evaluating changes of central tendency, the feasibility of using the functional form given by Equations (2.4) to characterize the variation of dispersion as a function of ground motion intensity was also investigated (Ruiz–Garcia and Miranda, 2005). It was found (not shown due space limitations) three-parameter functional forms provides a reasonable fit of the variation of $\sigma_{\ln RIDR_{\max}}$ with changes on $\Delta_i(T_1)$. It is important to mention that in order to employ the functional form given in Equation (2.4), at least three different levels of ground motion intensity should be used to obtain the parameter estimates. It is also recommended that two of these levels of ground motion intensity correspond to approximately the limits of the range of interest (Aslani and Miranda, 2005). In this investigation, it was found that the use of functional form of Equation (2.4) leads to adequate probability parameter estimates and, thus, to a reasonable representation of the probability of exceeding $RIDR_{\max}$ (Ruiz–Garcia and Miranda, 2005).

3. ILLUSTRATION OF RESIDUAL DRIFT DEMAND HAZARD CURVES

3.1 Residual drift demand hazard curves

The improved procedure allows obtaining different residual drift demand hazard curves that would be useful for decision making process during the seismic assessment of existing structures. For example, building-specific hazard curves of maximum residual roof drift, $\nu(\theta_{r,roof})$, maximum residual inter-story drift ratio over all stories, $\nu(RIDR_{\max})$, or residual inter-story drift ratio at selected story levels, $\nu(RIDR_i)$ can be obtained, which can be directly compared with their maximum drift demand hazard curves counterparts developed in parallel. This information is very useful since it can be related to transient and residual drift limit-states associated to different structural performance levels, as those provided in the FEMA 356 (2000) recommendations in the United States, during the performance-based seismic assessment phase of existing structures. To illustrate the proposed probabilistic approach, residual drift demand hazard curves for a 3-story (T_1=0.5s, C_y=0.8) and a 18-story (T_1=2.0s, C_y=0.2) one-bay generic frame models previously studied by the authors (Ruiz-Garcia and Miranda, 2006) were computed by performing numerical integration of Equation (2.2). It should be noted that the proposed probabilistic approach requires that a site-fundamental period-yield strength-specific maximum inelastic displacement hazard curve, $\nu(\Delta_i)$ is available. These $\nu(\Delta_i)$ curves can be obtained from the recently proposed approach suggested by Ruiz-Garcia and Miranda (2007). For example, maximum inelastic displacement hazard curves obtained for systems having a fundamental period of 0.5s corresponding to five different yield strength coefficients are shown in Figure 5a.

A comparison of the resulting residual drift hazard curves $\nu(\theta_{r,roof})$ and $\nu(RIDR_{\max})$ obtained for the 3-story frame model is shown in Figure 5b. The residual drift hazard curves shown in these figures allow a probabilistic assessment of residual drift demands, in which both the epistemic uncertainty in the ground motion hazard at a given site and the epistemic uncertainty (i.e. record-to-record variability) in the seismic response of a specific building are explicitly taken into account.

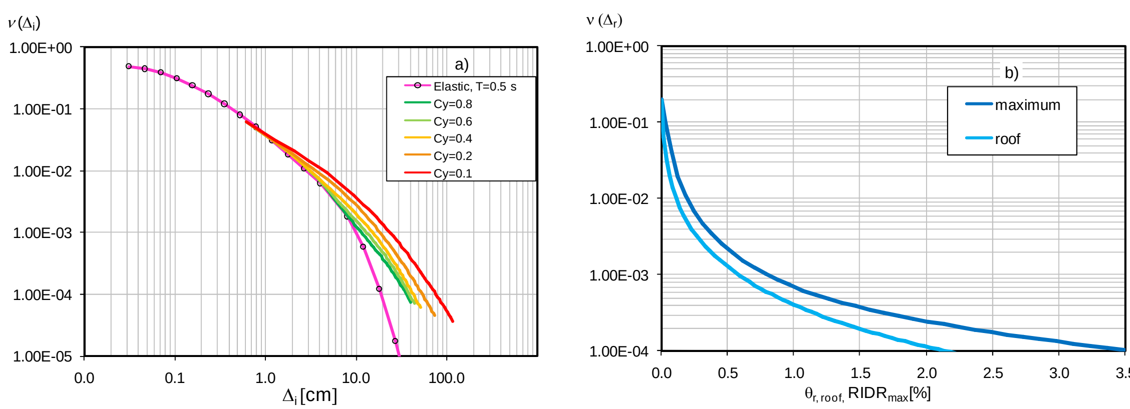

Figure 5 Variation of $RIDR_{\max}$ with changes in IM : a) $S_d(T_1)$, b) $\Delta_i(T_1)$

It is interesting to examine the influence of assuming that dispersion in the estimation of residual drift demands do not change with changing level of the ground motion intensity. For example, Figure 6a and 6b shows $\nu(\theta_{r,roof})$ and $\nu(RIDR_{\max})$ hazard curves computed for the 18-story stiff generic frame model assuming dispersion varying with the ground motion intensity and two levels of constant dispersion with changes in the ground motion intensity. From the figures, it can be seen that assuming constant dispersion might lead to significant difference in the MAF of exceeding $\theta_{r,roof}$ when assuming constant dispersion. Therefore, it is believed that the variation of both central tendency and dispersion of residual drift demands with changes in the intensity of the ground motion should be addressed while computing residual drift hazard curves.

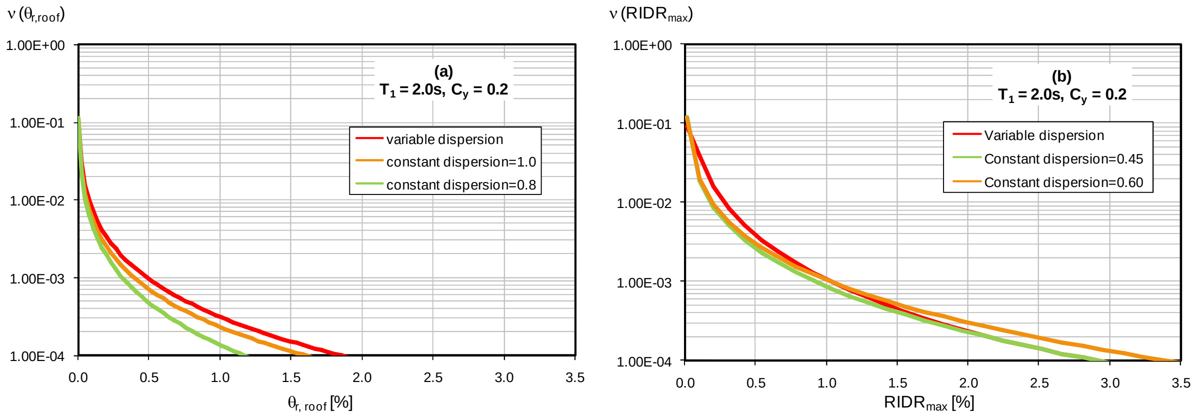

Figure 6 Comparison of residual drift hazard curves considering variable and constant variation of dispersion with changes in ground motion intensity: (a) $\nu(\theta_{r,roof})$; and (b) $\nu(RIDR_{\max})$

3.2 Comparison of residual and maximum deformation hazard curves

Following a similar procedure to the development of residual drift demand hazard curves, roof drift demand, $\nu(\theta_{roof})$, and maximum interstory drift demand, $\nu(IDR_{\max})$, hazard curves were also developed in parallel for the same 3-story and 18-story building models (Ruiz–Garcia and Miranda, 2005).. Thus, a direct comparison of both residual (permanent) and maximum (transient) drift demand hazard curves can be during the performance-based seismic assessment of existing structures. For example, Figure 7a shows a comparison between $\nu(IDR_{\max})$ and $\nu(RIDR_{\max})$ computed for the 3-story frame model while a similar comparison is shown in Figure 7b corresponding to the 18-story building model.

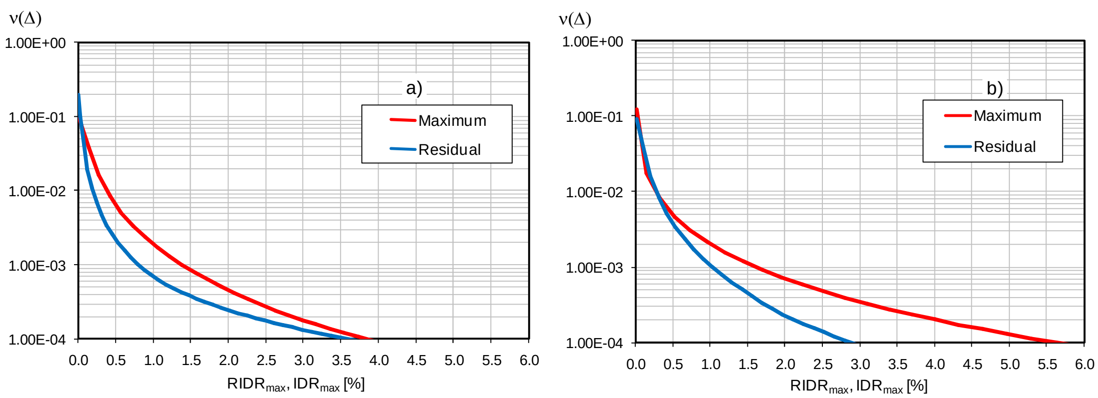

Figure 7 Comparison of $IDR_{\max}$ and $RIDR_{\max}$ hazard curves obtained from two generic building models: a) 3-story; and b) 18-story.

It can also be observed that the difference between $RIDR_{\max}$ and $IDR_{\max}$ depends on the MAF of exceedance. For example, for the short-period building, it can be observed that the magnitude of $RIDR_{\max}$ might be close to $IDR_{\max}$ for MAF of exceedance on the order of 0.0001. However, for the long-period building model, it can be seen that the difference between $RIDR_{\max}$ and $IDR_{\max}$ tends to increase as the MAF of exceeedance increases.

4. CONCLUSIONS

A rational procedure to allow incorporating the inherent epistemic uncertainty due to the record-to-record variability in the estimation of residual drift demands (i.e. roof residual drift demand, maximum residual drift demand at all stories, and residual drift demands at specific stories) of multi-story frames was presented.

- An inelastic intensity measure based on the maximum inelastic displacement of an equivalent elastoplastic SDOF system having the same initial lateral stiffness (i.e., fundamental period of vibration, T_1), of the building, $\Delta_i(T_1)$, seems to provide better efficiency than the traditional elastic IM based on $S_a(T_1)$ for probabilistic estimation of residual drift demands.
- A two-parameter lognormal probability distribution is adequate for characterizing the empirical probability distribution of residual drift demands. However, caution should be exercised when selecting the central tendency parameter.
- The variation of central tendency and dispersion of residual drift demands with changes in the ground motion intensity should be explicitly taken into account for developing residual drift demand hazard curves.
- Comparing both building-specific transient and residual drift hazard curves provides a better way of assessing the seismic performance of existing structures, since the difference between transient and residual drift demands depends on the mean annual exceedance frequency.

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