

## ESTIMATING INELASTIC BI-DIRECTIONAL SEISMIC RESPONSE OF MULTI-STORY ASYMMETRIC BUILDINGS USING A SET OF COMBINATION RULES

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### ABSTRACT :

Time history response analysis on 3D parametric models of multi-story asymmetric-plan buildings with nonlinear behavior considering both Uni- and Bi-Directional earthquake actions are carried out. Two different 5 story building models formed by six, and seven resisting planes (frames) connected at each floor by a flat slab (rigid diaphragm) and fixed at the base. The plan has an aspect ratio 2:1, and in the center of mass (CM) three degrees of freedom (two horizontal displacements and the in-plan rotation) and lumped masses were considered. Structures with different characteristics were defined based on the typical elastic parameters, and the seismic response modification factor  $R^*$ .

The artificial earthquake action was applied in Uni- and Bi-Directional form, varying the angle of incidence  $\alpha$  in  $15^\circ$  increments to find the critical angle where the overall and local responses are maximized. For the inelastic model it was considered that the yielding takes place in the end joints of each element and is generated by interaction of bending moments and axial forces.

Overall and local maximum responses were studied. The ratios between the maximum responses considering Bi-Directional earthquake action and inelastic behavior, and the maximum response estimated by some combination rules (SRSS and  $100/\beta$ , with  $\beta = 40, 60$ ) considering elastic response and Uni-Directional earthquake actions in two perpendicular directions ( $\alpha = 0^\circ; 90^\circ$ ), were evaluated. The results show that this procedure provides reasonable estimates of the inelastic maximum responses. In addition, the translational and torsional displacements of the CM of each floor, as well as the vertical translational displacements ( $\delta$ ) and bi-axial rotations ( $\phi$  and  $\varphi$ ), were seen to be similar to the values obtained from elastic analysis amplified by the magnitude of the response modification factor  $R^*$  used in the design.

### KEYWORDS:

Three-Dimensional parametric models, multi-story asymmetric-plan buildings, nonlinear structural behavior, Uni- and Bi-Directional earthquake, angle of seismic incidence, Combination rules.

### 1. INTRODUCTION

In general, the simultaneousness of the orthogonal seismic actions on three-dimensional structural systems has not been considered in explicit form in the code recommendations for earthquake resistant design of buildings (INN, 1996). In fact, present codes recommend to make two independent analyses using the RSA method and defining the earthquake action based on a single design spectrum. The earthquake action is then applied as a single component (in Uni-Directional form) along at least two perpendicular directions arbitrarily defined by the designer.

Most of the buildings designed by normal engineering offices have several resisting planes that are usually oriented along two orthogonal directions, and thus they are selected as the two analysis directions for the building. With this choice, most of the resisting planes are oriented either parallel or perpendicular to the main analysis directions. The design of the resistant elements of the building is made using the responses obtained

after applying the Uni-Directional seismic loads acting separately in each one of the two “analysis directions” of the plan. In some cases the results of both seismic analyses are combined by means of some empirical rule (e.g. 100/30, SRSS, etc.) in order to obtain an estimation of the response that would be obtained when considering the bi-directional effect of the seismic action; these rules generally lack theoretical basis (Fernandez-Davila and Cruz, 2006). Menun and Der Kiureghian (1998, 2000) have presented elastic combination rules for multiple components of ground motions and interacting seismic responses (e.g.,  $N-M_y-M_z$  in columns) that have the theoretical basis that the other rules lack. These rules have not yet found their way into the seismic design codes. As a consequence, it is expected that the results obtained in the analyses that consider a single horizontal earthquake component, like for example that the structural elements in the resisting planes that are perpendicular to the earthquake action remain elastic (Cruz et al., 1994), could not be representative of the actual behavior of a structure when subjected to a real earthquake. A literature survey has confirmed that systematic studies to test different alternatives to consider in a simple manner the effect of the bi-directionality in the analysis and design of real structures that go into the non-linear behavior range by comparing it to the actual responses of systems under the action of the real earthquake ground motions do not in fact exist (Fernandez-Davila, 2007).

The scope of the study presented here is restricted to real five-story RC frame structures, discarding the use of wall elements due to the difficulty to model the actual nonlinear shear force-deformation relationship. The objective is to try to provide some insight into the influence of the angle of incidence when bi-directional excitation is applied (maximum response), and also into the combination rules that allow to estimate the actual inelastic response.

## **2. BASIC STRUCTURAL MODEL**

The 3D analysis model was created for a five-story RC moment-frame subjected to a ground motion represented by the two horizontal components of the ground acceleration. The elements were considered as “3D beam-columns”, and flexural and shear deformations were considered. The model considered concentrated masses in each of the five stories. The floor diaphragms were considered to be rigid with three DOF at the CM of each story, two orthogonal horizontal displacements and the rotation around the vertical axis. The axial deformations of the columns were also taken into account. The buildings have the same floor plan throughout the height (Fig. 1). In the long direction, there are three identical resisting planes (frames in X direction). In the short direction, there are three resisting planes (frames in Y direction) with different stiffness: two resisting planes are at the edges of the plan (frames 1 and 3), and one is at the CM (frame 2). The structural elements (beams and columns) of each frame are the same throughout the height, and have a beam-column stiffness ratio  $\rho=0.125$ . The building has an inter-story height equal to 3m and the plan dimensions are 20m by 10m. The flat slab on each story has 15cm of thickness, and beams and columns have uniform rectangular cross sections. The analysis was carried out using the computer program, ANSR-1 as implemented by Mondkar (1975a, 1975b) and Rihai (1978). The non-linear behavior was concentrated at the end nodes of the elements, and the force-deformation curve for the material behavior was constructed from a preliminary RC design considering a bi-linear type curve with a stiffness in the second branch of less than 5% of stiffness in the elastic branch. Degradation of stiffness and strength in the loading/unloading cycles was not considered. Viscous damping was included in the model considering a Rayleigh type damping matrix defined so that for the first and last elastic vibration modes the damping ratio is 5%.

## **3. ANALYZED CASES**

The element properties were varied in order to obtain different overall behavior characteristics of the structure. To study structures with different characteristics several different models were defined based on the following parameters: the degree of torsional coupling ( $\Omega_\theta$ ), the uncoupled fundamental vibration period ( $T_Y$ ), the torsional stiffnesses ratio ( $\gamma_X$ ), the uncoupled lateral frequencies ratio ( $\omega_X/\omega_Y$ ), the normalized static eccentricity ( $e_X/r$ ), and the seismic response modification factor  $R^*$ . One symmetric and three asymmetric cases were studied, corresponding to semi-rigid buildings that had: different lateral stiffness in both the longitudinal and the

transverse direction, different lateral stiffness in both the longitudinal and the torsional direction, and different normalized static eccentricity. Table 1 shows the values used for these parameters.

To make sure that the models correspond to real buildings it was required that they satisfy the requirements of the Chilean code regarding minimum design base shear and maximum relative inter-story displacements (INN, 1996). Other design requirements, such as accidental torsion, were not taken into account. The parameters used for the earthquake definition were: Soil type II, Seismic Zone 3 ( $A_0=0.4 \cdot g$ ), and basic response modification factor  $R_0 = 11$ . Each model was analyzed independently in the X and Y directions considering linear elastic behavior, using the design spectrum of the code including the response modification factor  $R^*$ . Table 1 shows that the values adopted by  $R^*_X$  and  $R^*_Y$  for each case are consistent with the corresponding average values in the Chilean code. The maximum responses were estimated from the Uni-Directional maximum responses (computed by RSA method using the CQC combination rule) for each analysis direction.

#### **4. GRAVITY LOADS**

When considering structural components that behave non-linearly, it is not possible to separate the effects of the gravitational loads from the effects of the seismic loads. Therefore, all the loads that will be present in the structure during the occurrence of an important earthquake have been considered to act simultaneously. The effect of dead and live loads are considered, taking into account the self weight of the elements (beams, columns, and slabs), and dead and live loads applied as a uniformly distributed load on the slab ( $\omega_s \cong 0.7 \text{ T/m}^2$ ). Changes in the plan distribution of mass were ignored, so that the eccentricity of the structure comes only from an unequal distribution of the frames lateral stiffness.

#### **5. EARTHQUAKE ACTIONS**

The two horizontal components of a set of twenty artificial earthquakes records obtained from a set of actual strong-records obtained in the earthquake of March 3, 1985 in Central Chile were considered (10 sites). The characteristics of the artificial records match those of the real accelerations records as described and quantified using the concepts of Arias's Intensity, Coefficient of Correlation (over the duration of the record), Evolution in time of the Correlation (10 seconds moving window), the envelopes of the correlations in time, and the average of their maximum values. For the two horizontal components (principal and secondary) the average response spectra in terms of pseudo-accelerations were obtained for elastic response and 5% damping ratio after the principal components were scaled to a peak acceleration of  $0.4 \cdot g$  (Fernandez-Davila, 2007).

#### **6. NON-LINEAR BEHAVIOR MODEL**

The non-linear analysis model of the structure was carried out with the program ANSR-1 (Rihai, 1978). The step-by-step solution strategy used the Newton-Raphson iteration scheme. The non-linear behavior of each element was assumed to occur at its end nodes only and the force-deformation behavior curve to be bi-linear with a loss of 95% of stiffness in the second branch, approximately. The interaction surface for bending moments  $M_y$  and  $M_z$  and axial force  $N$  defined in the program was built with two uni-axial curves,  $M_u$ - $N$  and  $M_v$ - $N$ .

#### **7. ELEMENT SECTION DESIGN STRENGTHS**

The response modification factor  $R^*$  as defined in the Chilean code (INN, 1996) was used to define the design spectrum. The first columns in Tables 2 and 3 show the  $R^*$  values used for each of the cases considered the four cases. For the beams, the design moment  $M_d$  is selected as the maximum bending moment among all the elements of each resisting plane. The values are computed using the ACI's load combination factors ( $f_1$  and  $f_2$ ) and the results of the analysis of the structure subjected to gravitational loads ( $M_g$ ) and the results of the analysis

for earthquake loads when only elastic behavior is considered ( $M_{sel}$ ), divided by the response modification factor  $R^*$ ; this is,  $M_d = f_1 * M_g \pm f_2 * M_{sel} / R^*$ .

For the columns, M-N interaction curves were determined from maximum at the elements ends. The shapes of the interaction curves were defined based on the interaction curves commonly used in RC column design, considering symmetrical reinforcement in each direction and a steel reinforcement ratio equal to  $\rho_s$  of the gross section. For the material properties, the concrete was selected as H30 ( $f_c = 25$  MPa) and the steel as A63-42H ( $f_y = 420$  MPa). The maximum of the time history responses of axial force  $N_{sel}$  and bending moment  $M_{sel}$  are determined considering elastic behavior. To evaluate the most unfavorable combination of  $N$  and  $M_y$ , and  $M_z$  for each column, and considering that the seismic response value can be positive or negative, the following possibilities were evaluated using the ACI's load factors:  $M_{y1} = f_1 * M_{yg} + f_2 * M_{ysel} / R^*$ ,  $M_{y2} = f_1 * M_{yg} - f_2 * M_{ysel} / R^*$ ,  $M_{z1} = f_1 * M_{zg} + f_2 * M_{zsel} / R^*$ ,  $M_{z2} = f_1 * M_{zg} - f_2 * M_{zsel} / R^*$ ,  $N_1 = f_1 * N_g + f_2 * N_{sel} / R^*$ , and  $N_2 = f_1 * N_g - f_2 * N_{sel} / R^*$ . The design strength values were defined as an envelope to these values. The surface is further amplified by 25% to represent the use of the "strong column - weak girder" design concept. To simplify the design process a constant value of the strength reduction factor  $\phi = 0.85$  is considered.

## 8. RESULTS OBTAINED

Results for each case were obtained from: i) RSA applying independently each of the earthquake horizontal components (Uni-Directional analysis) in both directions using the average elastic response spectra; ii) Time history of the response (THR) considering non-linear behavior and applying simultaneously the two earthquake components (Bi-Directional analysis) of the twenty artificial records, varying the incidence angle  $\alpha$  of the earthquake action in  $15^\circ$  increments starting at the X-axis resulting in different cases (Fig. 2).

The elastic responses obtained from RSA (as described above) in each one of the two "analysis directions" are combined by means of empirical rules to obtain an estimation of the response considering the bi-directional effect of the ground motion. The combination rules used are: SRSS, 100/40, and 100/60 (Fernandez-Davila and Cruz, 2006). Due to space limitations the overall responses as the maximum diaphragm displacements and the axial displacements of one of the columns located in the flexible and the stiff sides, both in the top story are discussed.

The average over the set of twenty artificial earthquakes of the results for the models designed with different  $R^*$  values are shown for each of the earthquake incidence angles and two of the models considered, a symmetric case and a non-symmetric case. Fig. 3 shows the maximum top story diaphragm displacements and Fig. 4 shows the maximum axial displacements of two columns of the fifth floor. It is observed that the symmetric behavior is lost when incursions in the non-linear range occur and that the magnitude of the maximum displacements for bi-directional action depends on the angle of incidence of the earthquake. For the cases studied, it has been observed that all the responses have very similar behavior to the one shown in these figures.

In Tables 2 and 3 the estimates using different combination rules (SRSS and 100/ $\beta$ , with  $\beta = 40, 60$ ) to estimate the Bi-directional response from the uni-directional response are computed ( $\delta_{est}$ ) and compared to the results from the non-linear THR for the axial displacements of Columns  $P_1$  and  $P_5$  located at the flexible and stiff side of the fifth story plan, respectively. In the tables  $\delta$  represents the maximum of the average (over the set of 20 ground motions) nonlinear response for the different values of  $R^*$  and the angle of incidence where it occurs is also shown ( $\alpha$ ). The estimate of the bilinear response is made based on the responses obtained from elastic RSA in X and Y direction ( $\delta_{Ex,y}$ ) using the average elastic response spectrum of the set of ground motions. The displacements due to the gravity loads ( $\delta_{D+L}$ ) are also shown as they are required since the combination rule only applies to the part of the response induced by the earthquake action. The ratio of the nonlinear response due to earthquake action ( $\delta_{inel}$ ) and the estimated response ( $\delta_{est}$ ) are shown in the last column of the table. The ratios of the estimates obtained from the combination rules are seen to be of the same order of magnitude as the  $R^*$  factors used in the design. The ratios for the positive maximum values are rather close to the corresponding  $R^*$

values, while the ratios for the negative maximum values are much smaller.

## **9. CONCLUSIONS**

The results have shown that it is possible to estimate the maximum values of the response under Bi-Directional excitation and considering nonlinear behavior using the appropriate combination rules and the elastic responses for Uni-Directional excitation ( $\alpha = 0^\circ; 90^\circ$ ) represented by the average response spectrum. The amplification of the elastic response is similar in magnitude to the response reduction factor  $R^*$ .

In general, both the global and the local maximum responses obtained using Bi-Directional excitation depend rather strongly on the angle of incidence  $\alpha$  of the earthquake action. The symmetric structure (Case 1) shows asymmetric behavior when its elements enter the non-linear behavior range.

## **10. ACKNOWLEDGMENTS**

The financial support provided by the Escuela de Ingenieria of the P. Universidad Católica de Chile and the MECESUP program of the Chilean Ministry of Education, for the development of this investigation is gratefully acknowledged.

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Table 1: Values of the parameters that define the model.

Case	Id	$\Omega_0$	$T_Y$ (s)	$\gamma_X$	$\omega_X/\omega_Y$	$e_X/r$	$R^*_X$	$R^*_Y$
1	23211	0.50	0.50	0.25	0.75	0.000	8.480	7.932
2	23212	0.50	0.50	0.25	0.75	0.125	8.475	7.920
3	23213	0.50	0.50	0.25	0.75	0.250	8.463	7.876
4	23214	0.50	0.50	0.25	0.75	0.375	8.453	7.791

Table 2: Axial Displacements of columns  $P_5$  (stiff side) and  $P_1$  (flexible side) for Case 1 (symmetrical plan).

P <sub>5</sub>	Nonlinear (D+L+E)		Elastic		Combination Rules						δ <sub>inel</sub> / δ <sub>est</sub> (*)
R*	δ	α (°)	δ <sub>D+L</sub>	δ <sub>Ex,y</sub>	SRSS	100/40	40/100	100/60	60/100	δ <sub>est</sub>	
10.07	2.58 -1.34	270 90	-0.133	0.098	0.188	0.163	0.200	0.195	0.220	0.220	12.37 -5.48
7.93	1.80 -1.24	255 90		0.160							8.81 -5.03
5.56	1.20 -1.15	285 90									6.05 -4.65
P <sub>1</sub>	Nonlinear (D+L+E)		Elastic		Combination Rules						δ <sub>inel</sub> / δ <sub>est</sub> (*)
R*	δ	α (°)	δ <sub>D+L</sub>	δ <sub>Ex,y</sub>	SRSS	100/40	40/100	100/60	60/100	δ <sub>est</sub>	
10.07	2.11 -1.52	270 90	-0.133	0.098	0.188	0.163	0.200	0.195	0.220	0.220	10.22 -6.30
7.93	1.55 -1.39	285 90		0.160							7.65 -5.72
5.56	1.11 -1.27	285 90									5.68 -5.17

(\*)  $\delta_{inel} = \delta - \delta_{D+L}$

Table 3: Axial Displacements of columns  $P_5$  (stiff side) and  $P_1$  (flexible side) for Case 2 (asymmetrical plan).

P <sub>5</sub>	Nonlinear (D+L+E)		Elastic		Combination Rules						δ <sub>inel</sub> / δ <sub>est</sub> (*)
R*	δ	α (°)	δ <sub>D+L</sub>	δ <sub>Ex,v</sub>	SRSS	100/40	40/100	100/60	60/100	δ <sub>est</sub>	
10.07	2.60 -1.26	270 90	-0.123	0.088	0.213	0.166	0.229	0.205	0.247	0.247	11.01
7.93	1.81 -1.23	270 90		0.194							-4.59
5.56	1.17 -1.14	285 105									-4.50
P <sub>1</sub>	Nonlinear (D+L+E)		Elastic		Combination Rules						δ <sub>inel</sub> / δ <sub>est</sub> (*)
R*	δ	α (°)	δ <sub>D+L</sub>	δ <sub>Ex,v</sub>	SRSS	100/40	40/100	100/60	60/100	δ <sub>est</sub>	
10.07	2.56 -1.33	270 270	-0.146	0.105	0.158	0.153	0.160	0.176	0.181	0.181	14.96
7.93	1.85 -1.28	270 90		0.118							-6.54
5.56	1.20 -1.19	285 105									11.01 -6.25

(\*)  $\delta_{inel} = \delta - \delta_{D+L}$

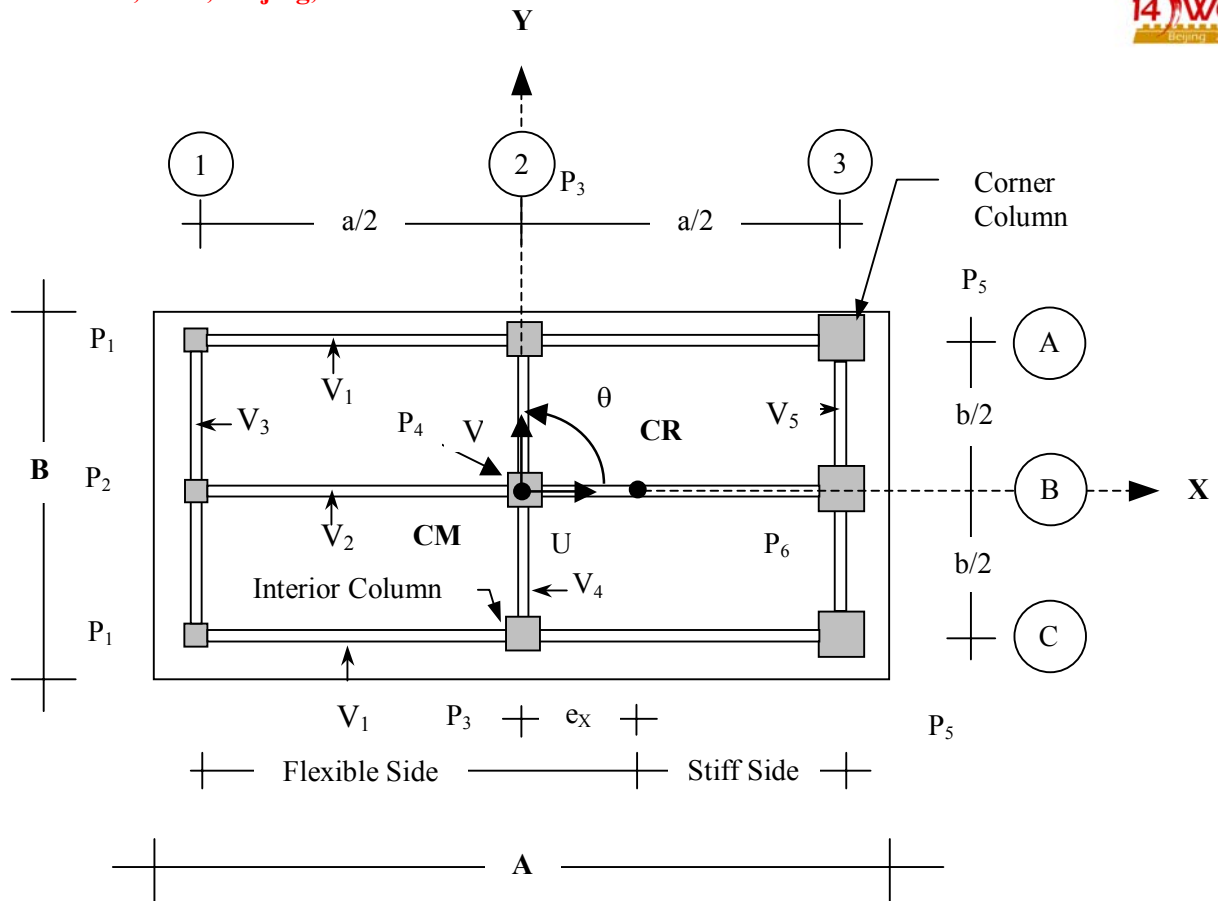


Figure 1: Typical Plan of the Three-Dimensional Structural Model.

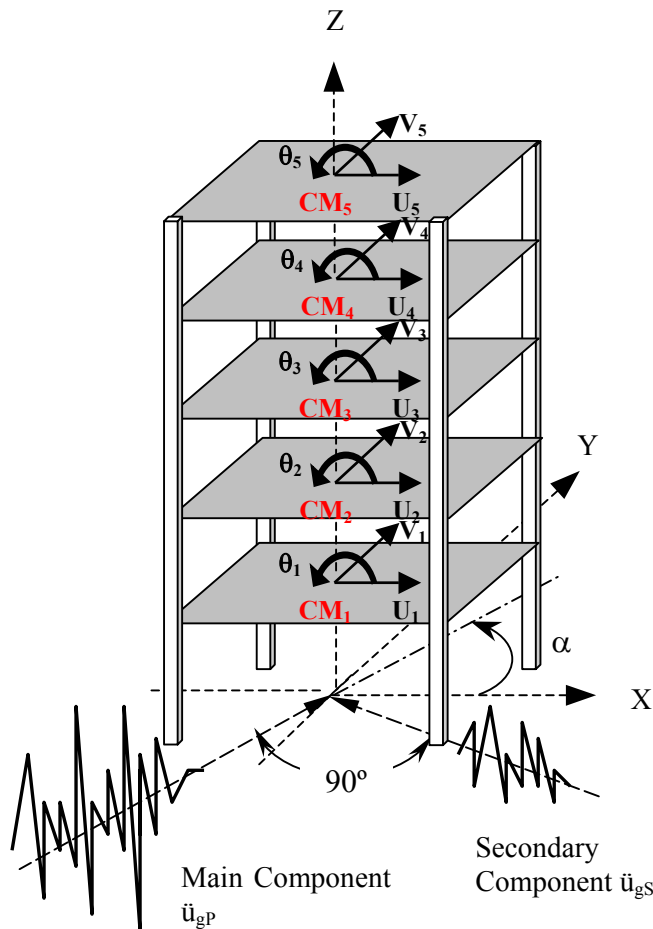


Figure 2: Schematic representation of the Three-Dimensional Building subjected to Bi-Directional Ground Motion

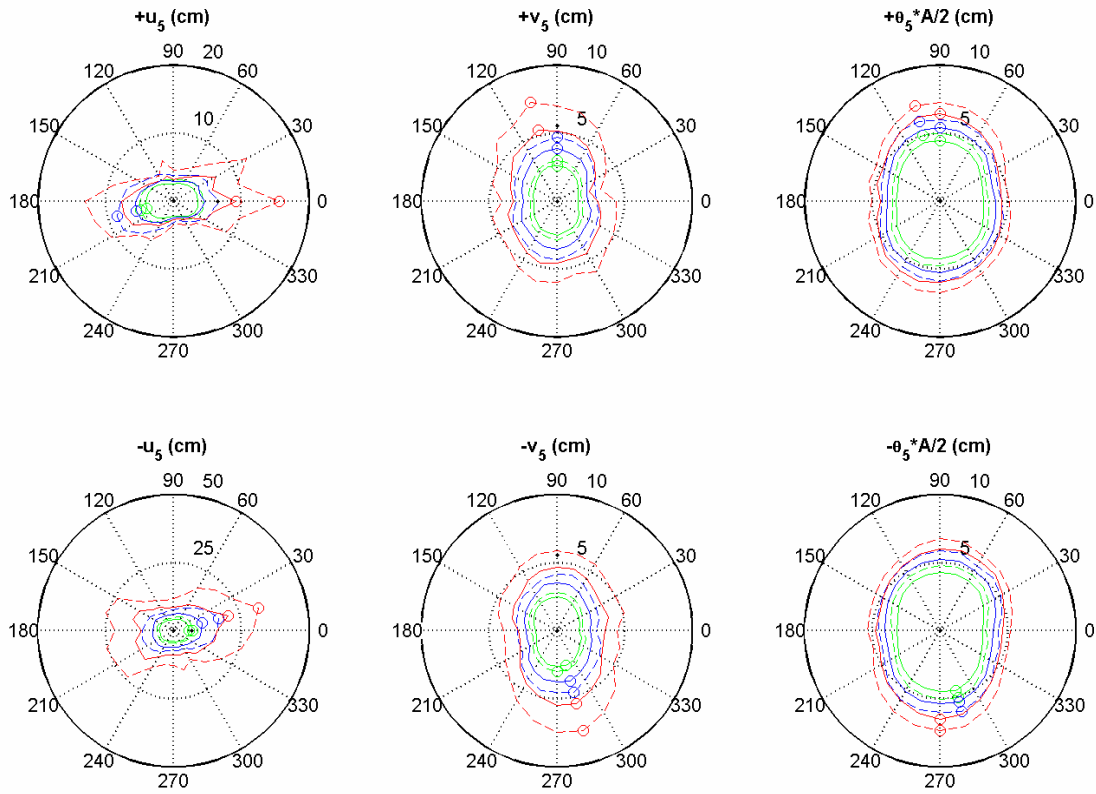


Figure 3: In-plan evolution of the maximum diaphragm displacements of the fifth floor of the case 1 for different  $R^*$  values.

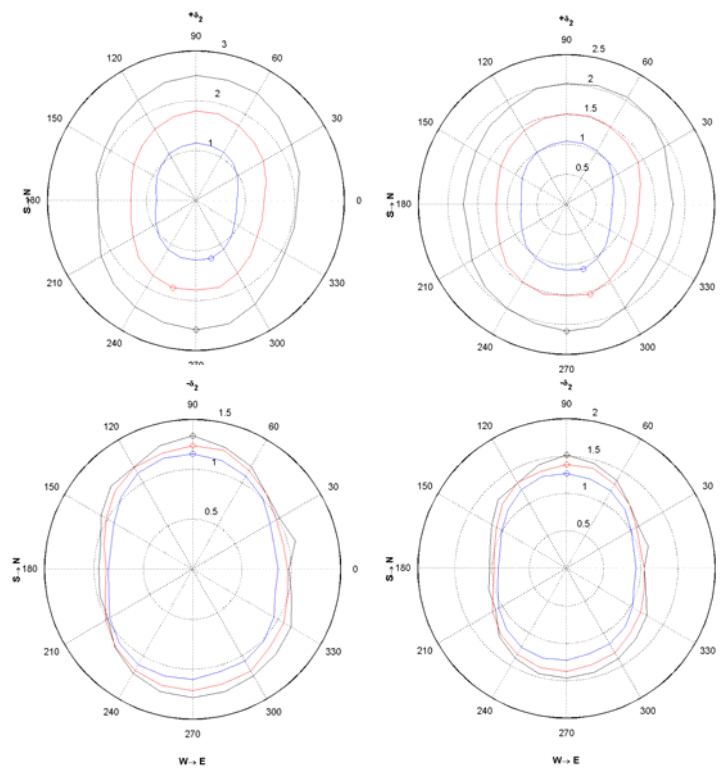


Figure 4: In-plan evolution of the maximum axial displacements of the fifth floor of the case 1 for different  $R^*$  values.