STOCHASTIC EARTHQUAKR RESPONSE OF COLUMN OVER DESIGNED STRUCTURES

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ABSTRACT:
The column over-design factor is generally used in structural design to prevent plastification of columns. This study performs the stochastic dynamic analysis to evaluate the effect of COF on the mean and deviation of maximum responses of structures subjected to earthquake load. The distributions of peak response of story drift, beam rotation angle, and ductility ratio along the height of the structure are investigated. By conducting random vibration analysis of structures with infinitely strong columns, the COF requirements ensuring elastic column behavior are also evaluated. It is shown that a COF much larger than that addressed by present code is needed for ensuring completely elastic behavior of columns.

KEYWORDS:
Stochastic response analysis, *column over-design factor*, *maximum response*, *expected value*, *response spectrum.*

1. INTRODUCTION

In seismic design of framed structures, it is considered desirable to have plastic hinges form in beams rather than in columns during an earthquake attack. That is because the beam failure only causes localized effect but the column failure would lead to the collapse of the entire building, and in a weak-column structure, plastic deformation is concentrated in a story and relatively large ductility ratio is consequently required. For ensuring a beam-hinging failure pattern, the sum of the columns at a joint is required to possess greater moment capacity than that of the adjoining beams, also well known as the weak-beam-strong-column philosophy. For designing a weak-beam-strong-column structure, the column over-design factor (COF) has been stipulated by various structural seismic codes, such as the minimum COF requirement of 1.2 by ACI (1999), the minimum COF of 1.5 by cold-formed square tube structure specification of Japan (BCJ,2004), etc. Although structural provisions have provided different minimum requirements of column over-design factor for various kinds of structures, the research on column over-design factor has recently been of primary interest, as witnessed by the numerous published papers related to the development of COF evaluation method, minimum COF requirements etc. The main reason could be that the minimum COF values described in provisions can't meet the performance requirements of structures subjected to seismic actions, as represented by large quantity of failure of columns observed in recent major earthquakes of Mexico City (1985), Loma Prieta (1989), Northridge (1994) and Kobe (1995). How the column over-design factor affect the structural seismic behavior is still not clear, and further researches are needed.

Some representative recent studies on COF can be found in Kuwamura et al. (1989), Nakashima and Sawaizumi (1999), Dooly and Bracci (2001), Ogawa (1983), Hibino and Ichinose (2005), Kawano et al. (1998). Most of these studies, including experiment examination and numerical analysis, were performed using one or small quantity of specific earthquake ground motions as input. However, the ground motions are of great randomness, and the properties of earthquakes still cannot be predicted accurately. The obtained results by this way are consequently hard to represent the statistical properties of the potential response to the next unknown ground motion. Zhao et al. (2002) proposed a COF evaluation method with the uncertainties of member strength and earthquake load in consideration, but the dynamic properties of structures are not considered. For a better understanding of the seismic behavior of structures, the statistical properties of dynamic response are desired.

Basically, two approaches are available for computing the statistical properties of structural response. The one is to conduct time history analysis using a large quantity of ground motion records, well known as the Monte Carlo simulation. The alternative way is to conduct stochastic dynamic analysis based on the mean earthquake response spectrum to obtain the statistical indices directly. Comparably, the stochastic response analysis is more efficient. Usually, some linearization techniques are involved in stochastic response analysis, which makes it an approximation method. Despite of this, the stochastic response analysis has been well developed and popularly used because it provides helpful information about structural seismic behaviors in statistical meaning.

This paper conducts the stochastic response analysis to evaluate the effect of COF on the statistical properties of maximum earthquake responses. The fishbone model is incorporated to account for the interaction between beams and columns. The distribution of the mean and standard deviation of the maximum story drift, beam rotation angle, and ductility ratio along the height of the structure are investigated, and the relationship between the COF and those responses are analyzed. By conducting random vibration analysis of structures with infinitely strong columns, the COF requirements ensuring elastic column behavior are also evaluated.

2. STOCHASTIC RESPONSE ANALYSIS METHOD

2.1 Random Vibration Analysis

Assume the ground acceleration is idealized as a segment of finite duration of a stationary Gaussian process having mean of zero. Using the random mode decomposition method combined with the mean-value response spectrum, the deviation of the stochastic response is given as (Zhao et al. 1999):

$$\sigma_r^2 = \sum_i \sum_j B_i B_j \rho_{0,ij} \sqrt{\lambda_{0,ii}\lambda_{0,jj}} \tag{1}$$

where B_i is the participation factor of the ith mode obtained by dynamic analysis of structures. $\rho_{0,ij}$ is the correlation coefficient between the ith and jth modes. The correlation coefficient is generally expressed as the function of frequencies and damping ratios, and the following approximation equation is commonly used:

$$\rho_{0ij} = \frac{2\sqrt{\xi_i\xi_j}\left[\left(\omega_i+\omega_j\right)^2\left(\xi_i+\xi_j\right)+\left(\omega_i^{\,2}+\omega_j^{\,2}\right)\left(\xi_i-\xi_j\right)\right]}{4\left(\omega_i-\omega_j\right)^2+\left(\omega_i+\omega_j\right)^2\left(\xi_i+\xi_j\right)^2} \tag{2}$$

where ω_i and ξ_i are the frequency and damping ratio, respectively, of the ith vibration mode. $\lambda_{0,ii}$ is the spectral moment of ith mode, which can be obtained using the following equation:

$$\lambda_{0i} = \bar{Z}_a^{\,2}(\omega_i,\xi_i) / p_i^{\,2}\omega_i^{\,4} \tag{3}$$

with $\bar{Z}a$ being the response spectrum and p_i being the peak value factor corresponding to the ith mode.

Using the result of the standard deviation of earthquake response σ_r obtained from random vibration analysis, the mean value and standard deviation of the maximum earthquake response are obtainable by:

$$\mu_{R_m} = \sigma_r\left[\sqrt{2\ln\nu D} + \frac{0.5772}{\sqrt{2\ln\nu D}}\right] \tag{4}$$

$$\sigma_{Rm} = \frac{\pi\sigma_r}{\sqrt{2\ln\nu D}} \tag{5}$$

where μ_{Rm} and σ_{Rm} are the expected mean and standard deviation, respectively, of the maximum response R_m, ν is the mean cross ratio, and D is the duration of ground motion.

For hysteretic MDF frame structures, the non-linear random vibration analysis is generally conducted using equivalent linearization (Wen, 1980), in which the inelastic deformations of structures are estimated from an equivalent elastic system with a lateral stiffness smaller than that of the initial inelastic system and with a damping ratio larger than that of the inelastic system. Basically, the equivalent stiffness and damping ratio are computed as functions of the ductility ratio γ that is defined as the ratio of the maximum deformation to the yield deformation. The equivalent stiffness and damping ratio used in this study are given by the following two equations:

$$K_e(\gamma) = \frac{K_0}{\gamma}\left[(1-\alpha)(1+\ln\gamma)+\alpha\gamma\right] \tag{6}$$

$$\xi_e(\gamma) = \xi_0 + 0.15\left[1-\frac{1}{\gamma}\sqrt{1+\alpha(\gamma-1)}\right] \tag{7}$$

where K_e is the equivalent stiffness, ξ_e is the equivalent damping ratio, α is the postyield-to-initial stiffness ratio, K_0 is the initial stiffness and ξ_0 is the damping ratio of the original system.

The equivalent stiffness and equivalent damping given above were developed originally for single-degree-of-freedom (SDOF) system. To obtain the damping ratios of vibration modes, the equivalent damping ratios need to be transformed into equivalent Rayleigh damping coefficients firstly, and then the equivalent damping ratios for vibrations modes can be determined by mode decomposition method.

Table 1. Moment strength and stiffness of structural members

3 story frame				
Story	M_c (kN·m)	EI_c (10^5 kN·m^2)	M_b (kN·m)	EI_b (10^5 kN·m^2)
3	748	1.47	808	1.47
2	1202	2.37	1885	2.37
1	1512	2.98	2331	2.98
6 story frame				
Story	M_c (kN·m)	EI_c (10^5 kN·m^2)	M_b (kN·m)	EI_b (10^5 kN·m^2)
6	1028	2.33	1116	2.33
5	1650	3.74	2636	3.74
4	2142	4.86	3774	4.86
3	2529	5.74	4652	5.74
2	2821	6.40	5254	6.40
1	3024	6.86	5083	6.86
9 story frame				
Story	M_c (kN·m)	EI_c (10^5 kN·m^2)	M_b (kN·m)	EI_b (10^5 kN·m^2)
9	1245	2.96	1353	2.96
8	1983	4.72	3174	4.72
7	2588	6.17	4552	6.17
6	3099	7.38	5680	7.38
5	3528	8.41	6624	8.41
4	3883	9.25	7407	9.25
3	4167	9.93	8032	9.93
2	4384	10.44	8410	10.44
1	4536	10.80	7747	10.80

2.2 Modeling of Structures

The nonlinear stochastic response analyses are conducted using two-dimensional frames with number of stories, n, equal to 3, 6 and 9 and with the constant floor height of 4 m and span length of 8 m, as shown in Fig. 1. The frame is designed according to the structural specifications of buildings of Japan (BCJ, 2007). The main characteristics of the frames considered are as follows.

(1). The lumped mass of 840 kN is used for each floor level.
(2). Geometrical inertia moments of beam sections are two times of those of column sections.
(3). The shear forces acting on each floor obey an Ai distribution with the
Ai coefficient defined by:

$$A_i = 1 + \left(\frac{1}{\sqrt{a_i}} - a_i\right)\frac{2T}{1+3T} \tag{8}$$

where T is the fundamental natural period computed as the multiplication of the structure height with 0.03, and a_i is the ratio of the weight carried by ith floor to the total weight of structure.

(4). Frames are designed so that the roof drift angle of 1/200 is attained under the base shear coefficient C_0 = 0.2. Under the base shear coefficient C_0 = 0.3, all the beams reach the plastic moment strengths.

(5). The distributions of the geometrical inertia moments of beams and columns along the height of structure are the same as the distribution of the shear forces acting on each floor level.

The moment-rotation relationship is assumed to be bilinear, both the initial damping ratio and the postyield-to-initial stiffness ratio are assumed to be 0.05. The moment strengths and stiffness of beam and column of frames with COF=1.0 are given in Table 1. The COF of frame can be changed by multiplying a factor to the column strength.

2.3 Response Spectrum

The acceleration response spectrum recommended by AIJ (1993) is used as the earthquake input. It is given Eq. (7), where f_A, f_V, d, G_A, G_V, and T_C are parameters describing the site foundation and assumed to be 2.5, 2.0, 0.5, 1, 1, and 0.55, respectively, according to the recommendations by AIJ. A_0 is the maximum acceleration of standard ground base. Figure 3 illustrates the acceleration response spectrum for A_0 = 500 gal and damp ratio h = 0.05. The maximum velocity V_0 equals $A_0/15$.

$$S_A(T,h)=\begin{cases}(1+\dfrac{f_A-1}{d}\dfrac{T}{T_c})F_hG_AR_AA_0 & 0\le T\le dT_c\\ F_hf_AG_AR_AA_0 & dT_c\le T\le T_c\\ \dfrac{2\pi F_hG_AR_AA_0V_0}{T} & T_c\le T\end{cases}\tag{7}$$

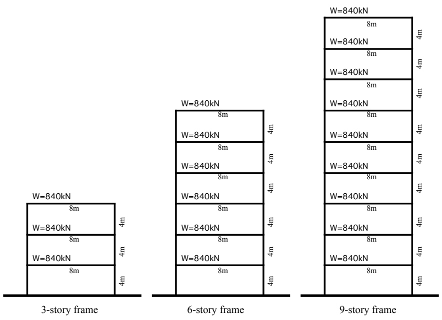


Figure 1. Example frames

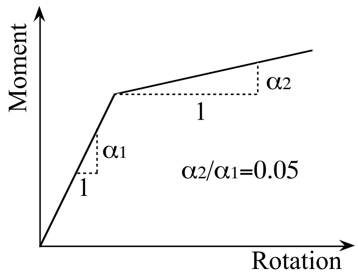


Figure 2. Moment-rotation relationship

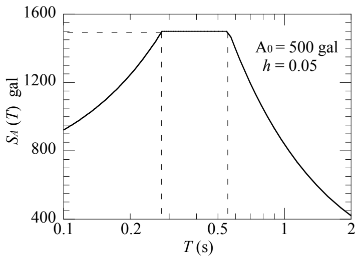


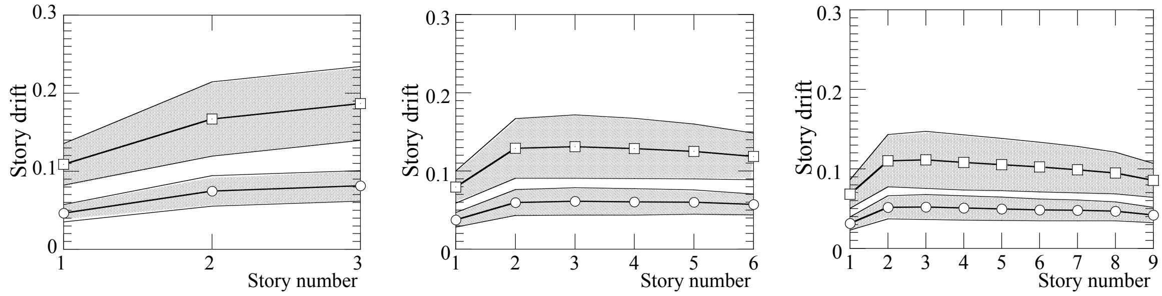
Figure 3. Response spectrum

3. MAXIMUM RESPONSE OF STRUCTURE INVOLVING COLUMN YIELDING

The estimation presented in this study consists of two parts: maximum response of structure involving column yielding and maximum response of structure with elastic column behavior. The reason for such a classification is based on the seismic design criteria that the failure of columns should be avoided as possible. In this section, the investigation on the maximum response of structure involving column yielding is presented. The estimation focuses on the distribution of displacements and ductility ratios of beams and columns, and the effect of COF and maximum ground acceleration on the maximum structural responses.

3.1 Distribution of Story Drift

Figure 4 shows the distributions of the maximum story drift along the height of frames. The story drift distribution is generally uniform for the frames considered in this study. Relatively, the distribution of story drift fluctuates more significantly for higher ground motion amplitude. For the 3-story frame, the mean of the maximum story drift of top floor is larger than those of other floors. For 6-story and 9-story frame, the mean of maximum story drift distribute uniformly on each floor level except the first floor where the mean of maximum story drift is obviously smaller than that of other floors. The shadow shows the fluctuation range of each maximum response from (μ-σ) to (μ+σ). One can see that the standard deviations of story drifts have similar distributions with the averages, and they are about 23-27% of the averages.

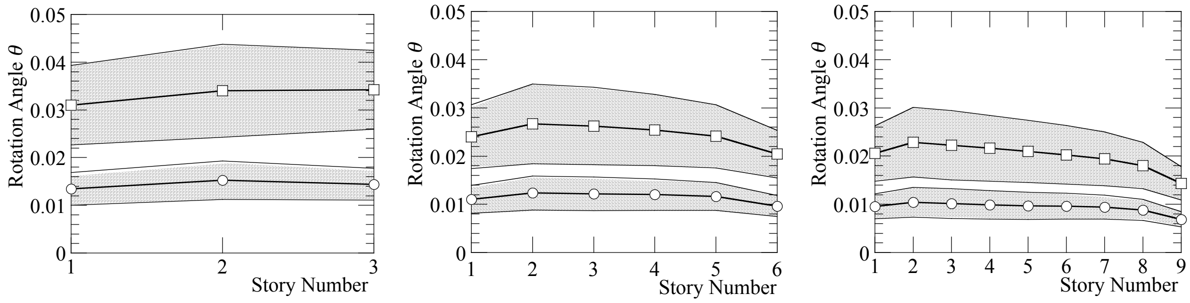


(a) (b) (c)

Figure 4. The mean and standard deviation of maximum story drift, (a) for 3-story frame, (b) for 6-story frame and (c) for 9-story frame

3.2 Distribution of Beam Rotation Angle

The distribution of beam rotation angle θ is shown in Figures 5(a), 5(b) and 5(c) for 3-story, 6-story and 9-story frame, respectively. It is relatively uniform for small ground motion amplitude. The uniform distribution of the beam rotation angles and story drifts in section (1) are considered as the results of stiffness and strength design that are in proportion to the earthquake load applied on floor levels. When the ground motion amplitude increases, the deformation will concentrate partially in some beams. The coefficient of variation for the maximum beam rotation angle is obtained to be 0.23-0.32.



(a) (b) (c)

Figure 5. The mean and standard deviation of floor rotation angle (a) for 3-story frame, (b) for 6-story frame and (c) for 9-story frame.

3.3 Distribution of the Ratio of Beam Rotation Angle to Story Drift Angle

In order to investigate how the deformations of beams contribute to the deformation of columns, the ratios of the mean beam rotation angle to mean story drift angle changed with respect to COF are shown in Fig. 6, in which Figs. 6(a), 6(b), and 6(c) show the results of 3-story, 6-story, and 9-story frame, respectively, for A_0 = 600 gal, Figs. 6(d), 6(e), and 6(f) show the similar results for A_0 = 1200 gal. One can see that the ground motion amplitude has no effect the ratio of beam rotation angle to story drift angle, since almost the same results were obtained for the two ground motion amplitudes investigated. The ratio of beam rotation angle to story drift angle is affected more significantly by the COF and distributes non-uniformly in each floor level. For 3-story frame, the ratios for all the three floor levels are given in the corresponding figures. The results show that the ratios for lower floors are higher than those of upper floors. The similar trend can be observed in the results of 6-story and 9-story frames. That means the earthquake energies are more dissipated by deformation of beams in lower stories. The increase of COF leads to an increase in the ratio of beam rotation angle to story drift angle. The increase of COF means relatively stronger columns, and beams experience more deformations, and the beam failure patterns are more prone to form.

3.4 Distribution of Ductility Ratios of Columns and Beams

Figure 7 shows the ductility ratios of columns and beams varied with COF of frame. In Figure 7, the curves are named by combining the story number and a capital letter indicating beam ("B") or column ("C"). Generally, with the increase of COF, the ductility ratios of column decrease, and the ductility ratios of beams increase. The large ground motion amplitude produces naturally large ductility ratios of beams and columns. A relatively high COF range causes a ductility ratio of beam larger than that of column.

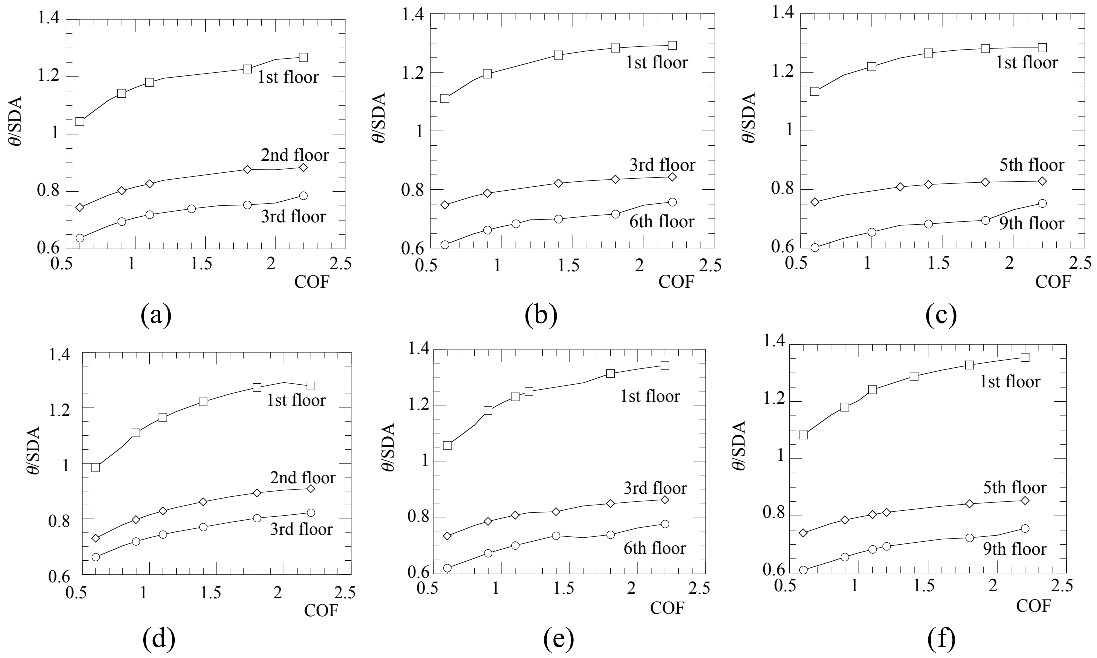


Figure 6. The ratio of rotation angle to story drift angle: (a) 3-story frame, (b) 6-story frame and (c) 9-story frame with $A_0 = 600$ gal; (d) 3-story frame, (e) 6-story frame and (f) 9-story frame with $A_0 = 1200$ gal.

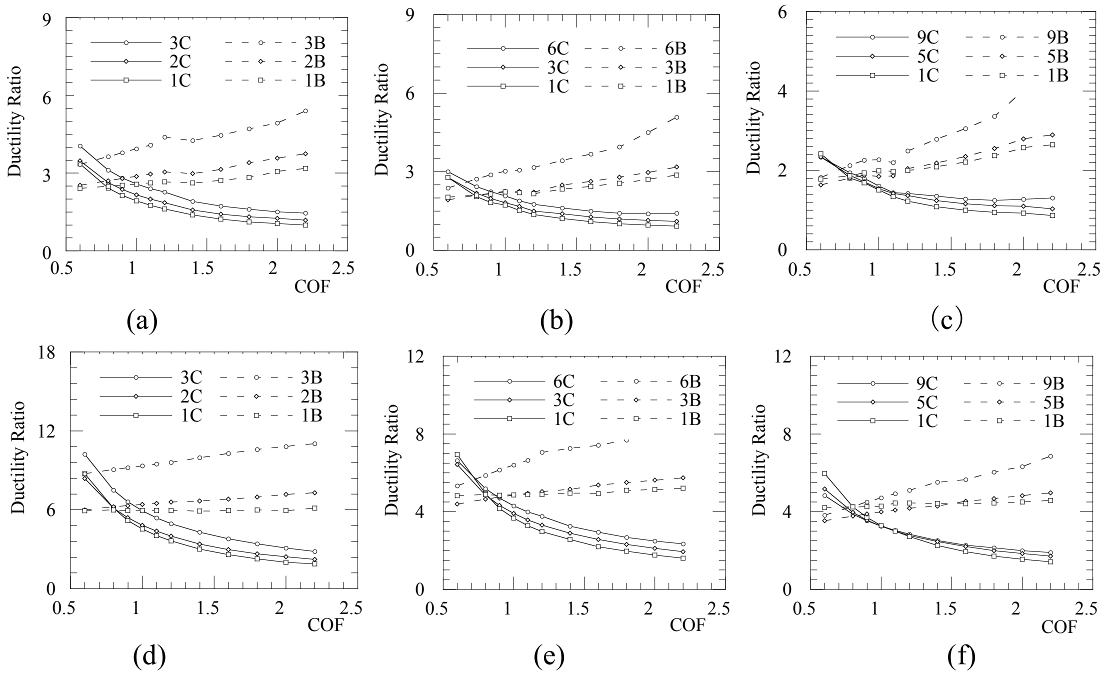


Figure 7. The effect of column over-design factor on the ductility ratio of columns and beams of frames: (a) 3-story frame, (b) 6-story frame and (c) 9-story frame for $A_0 = 600$ gal; (d) 3-story frame, (e) 6-story frame and (f) 9-story frame for $A_0 = 1200$ gal.

4. MAXIMUM RESPONSE OF STRUCTURE ENSURING ELASTIC BEHAIOR OF COLUMNS

In order to obtain the necessary COF value required to avoid column hinging, the columns of frame structure are modeled with infinite moment strength to identify all of the potentially over-stressed columns. This is considered practical from both an original design perspective and an existing frame analysis since in both cases we strive to prohibit the columns from reaching their full strength and therefore rotating freely in a plastic state. The plastic hinge in column can be only prohibited in the case that the strength of the column is larger than the elastic moment under the state of beam hinging. The COF requirement for avoiding plastic hinging in columns is then defined as the ratio of the maximum moment exerted to the elastic column to the plastic moment of the adjoining beam (Kuwamura, 1989).

In this investigation, the parameters of G_A, G_V and T_C for response spectrum are assumed to be 1, 1, and 0.33, respectively, which are corresponding to the standard ground.

4.1 Distribution of COF Requirements along the Height of Structures

Due to the randomness of earthquake input, the structural responses are also in dispersion but obey some specific probabilistic distributions. Through random vibration analysis, the mean value, standard deviation, and also the cumulative probability distribution of the maximum structural earthquake responses are obtainable. The mean describes the fluctuation center of response, and the standard deviation measures the dispersion of response. Also, the probabilities of structural responses exceeding some specific values (P_e) can be obtained directly from the distribution function. The mean of column moment response is obtained to be 377.5 kN•m and the COV is 0.199. One can see that the mean can only produce reliability of 51%, and in order to assure an exceeding probability (P_e) as low as 10%, the column moment will be up to 460 kN•m which is 1.22 times of the mean. As mentioned above, the COF requirement for ensuring elastic columns is defined as the ratios of the moments experienced by columns to the plastic moment strengths of beams. The plastic beam strengths are invariable for a specific structure. The COF requirement depends on what level of column response is used. Clearly, it is not adequate to define the COF requirement using the mean of moment responses of columns. The COF used in structural design should provide a low exceeding probability of column moment response.

Figure 8 shows the distribution of COF requirements along the height of frames. The COF requirements are determined determined by specific exceeding probability, which is assigned to be 1%. One can see that for low amplitude of earthquake motion the distribution of COF is generally uniform, and increase in amplitude of acceleration causes fluctuation in the distribution.

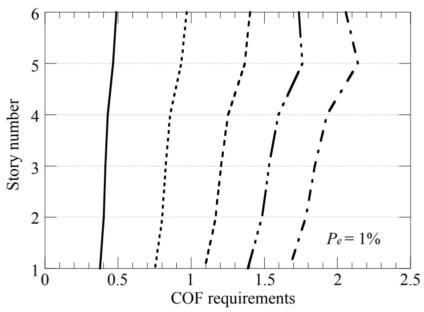


Figure 8 COF requirement with P_e=1%

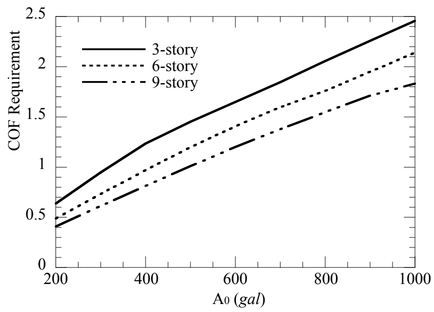


Figure 9 COF-A_0 curves for P_e = 1%

4.2 Relationship between COF Requirement and Maximum Ground Acceleration

From Fig. 8, one can see that the COF requirements for assuring elastic-column response generally increase with the increase of maximum acceleration of ground motion. This section aims to provide a quantitative relationship between the COF requirements and the maximum acceleration. In all the cases investigated, the maximum COF appears at the top two floors. Figure 9 shows the COF-A_0 curves for all three frames with P_e = 1%. The approximately linear relationship between the COF requirement and the maximum acceleration of ground motion can be observed. The comparison between the results of different frames shows that the COF requirement for relatively high structure is smaller than that for low structure. This could be explained by the mean response spectrum used in this study. As shown before, the parameters d and T_c of response spectrum are assigned to 0.5 and 0.33, respectively, which means the period range where response spectrum value is at peak is from 0.165 to 0.33. Generally, this period range is close to the fundamental period of low-rise buildings. Over this range, the response spectrum decreases with the increase of period of building. The fundamental periods for 3-story frame, 6-story frame, and 9-story frame under consideration are 1.09s, 1.48s, and 1.78s, respectively. Therefore, the low-rise buildings experience large deformations, and as a result, the relatively larger COF for 3-story frame and relatively smaller COF for 9-story frame are obtained in this investigation.

5. CONCLUSIONS

The mean value and deviation of structural response are estimated through random vibration analysis method with equivalent linearization technique incorporated. The effect of the column over-design factor on the responses is evaluated, and the necessary COF requirements for ensuring beam failure mechanism of frame structures are also evaluated. It is found that:

(1) Generally, both the maximum story drift and the maximum beam rotation angle distribute uniformly along the height of structure considered in this study. The coefficient of variation for story drift ranges from 0.23 to 0.27, and that for beam rotation angle is from 0.23 to 0.32.

(2) The ratio of beam rotation angle to story drift angle increases with the increase of COF of structure, which implies earthquake energy are more dissipated by deformation of beams in structures with relatively high COFs.

(3) The ductility ratios of columns decrease with the increase of COF, accompany with the increase in the ductility ratios of beams. An adequate COF is effective to induce a beam failure mechanism in structures.

(4) The COF requirements for ensuring beam failure mechanism with exceeding probability of column moment response being 1% were determined. The COF requirements increase approximately linearly with the increase of amplitude of ground motions. For relatively low frame structure, relatively larger COF requirements are obtained, because the natural period of low building is closer to the predominant period of the response spectrum in use.

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