NUMERICAL CALCULATION OF FRAGILITY CURVES FOR PROBABILISTIC SEISMIC RISK ASSESSMENT

I. Zentner[1] , A. Nadjarian[2] N. Humbert[2] and E. Viallet[2]

[1] *Research Engineer, EDF Research & Development, Clamart, France*
[2] *Engineer, EDF SEPTEN, Engineering Division, Villeurbanne, France*
Email: Irmela.zentner@edf.fr

ABSTRACT :

In nuclear engineering practice, fragility curves are often determined by means of margin factors, using the scaling method This approach allows for determining fragility curves in a very convenient way but makes strong simplifying hypothesis. This is why it can be interesting to directly propagate uncertainties by means of Monte Carlo simulation. In this paper, we will discuss statistical estimation of the parameters of fragility curves and present results obtained for a reactor coolant system of nuclear power plant.

KEYWORDS: Fragility curve, Uncertainty, Probabilistic risk assessment, Log-normal, Monte Carlo, Bootstrap

1. INTRODUCTION

The seismic Probabilistic Risk Assessment (PRA) methodology has become the most commonly used approach for the evaluation of seismic risk in nuclear industry [Wakefield 2003]. In this framework, fragility curves express the conditional probability of failure of a structure or component for a given seismic input motion parameter A, such as peak ground acceleration (PGA), peak ground velocity or spectral acceleration (SA). In nuclear engineering practice, fragility curves are determined using safety factors with respect to design earthquake [Reed et al. 1994]. This approach allows for determining fragility curves based on design study but draws upon simplifying assumptions and expert judgment. In recent years, fragility curves have also become popular for characterizing seismic vulnerability of civil structures [Choi et al. 2004], [Güneyisi et al. 2007], [Rota et al. 2008]. In this framework, fragility curves are often determined with respect to damage levels going from minor over moderate to major damage. However, most authors use quite simple models and do not consider uncertainty other than due to ground motion.

We will present a complete probabilistic study of a nuclear power plant component and determine fragility curves by statistical estimation methods. We have performed non-linear dynamic response analyzes using artificially generated strong motion time histories. Uncertainties due to seismic loads as well as model uncertainties are taken into account and propagated using Monte Carlo simulation.

2. DEFINITION OF FRAGILITY CURVES

The fragility of a structure (or component) is determined with respect to "capacity". Capacity is defined as the limit seismic load before failure occurs. Therefore, if PGA has been chosen to characterize seismic ground motion level, then capacity is also expressed in terms of PGA. In what follows, and in order to simplify the notations, we will consider that PGA has been chosen to characterize seismic ground motion. The capacity of the structure, is generally supposed to be log-normally distributed [Reed et al. 94], [Wakefield et al. 2003].

2.1. Simple log-normal model

The currently applied approach consists in modeling capacity A as a random variable having log-normal distribution [Reed et al. 94], that is $A = A_m \varepsilon$ where A_m is median capacity and ε is a log-normally distributed random variable with unity median and logarithmic standard deviation β. Concurrently, the fragility curve represents the probability of failure for a given seismic ground motion level a.

It is clear that a component or structure fails if its seismic capacity is less or equal to given ground motion level. Then, the failure probability conditioned on ground motion parameter a is given by the cumulative distribution function of capacity A, yielding:

$$P_{f|a}(a) = \underbrace{\int_0^a \frac{1}{x\beta\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\log(x/A_m)^2}{\beta^2}\right)}}_{\text{log-normal distribution}} dx = \Phi\left(\frac{\ln(a/A_m)}{\beta}\right) \tag{2.1}$$

where $\Phi(\cdot)$ is the standard Gaussian cumulative distribution function. Thus, the fragility curve is entirely defined by two parameters which are median capacity A_m and logarithmic standard deviation β.

2.2. Log-normal model distinguishing random and epistemic uncertainties

The model described in paragraph ***2.1*** has been worked up distinguishing random uncertainties and epistemic uncertainties. The first kind of uncertainty is associated to inherent random phenomena whereas the latter uncertainty could be reduced by having a better model and/or collecting new data.
The two types of uncertainty are distinguished by introducing log-standard deviations β_R and β_U, respectively. In this context, capacity is expressed as [Reed et al. 94]:

$$A = (A_m \varepsilon_U)\varepsilon_R \tag{2.2}$$

where ε_U et ε_R are log-normally distributed random variables with median equal to one and respective log-standard deviations β_U and β_R. In this approach, ε_U characterizes uncertainty in the knowledge of the median value whereas ε_R refers to inherent randomness about the median.
Conditional probability of failure is then expressed as:

$$P'_{(f/a)}(a) = \Phi\left(\frac{\log(a/A_m) + \beta_U \Phi^{-1}(Q)}{\beta_R}\right) \tag{2.3}$$

with Q, the level of confidence that the conditional probability of failure $P'_{(f/a)}$ is less than $P_{(f/a)}$. Equation 2.3 defines a family of curves, corresponding each to a confidence level Q. The more sophisticated model 2.3 and the simple model described by expression 2.1 are linked by the relation

$$\beta \equiv \beta_C = \sqrt{\beta_R^2 + \beta_U^2} \tag{2.4}$$

Curve 2.1 is called composite fragility for $\beta = \beta_C$. It can be shown [Wakefield et al. 2003], that the composite curve corresponds to the mean curve, while expression 2.3 equals the median curve for $Q=0.5$ (or equivalently considering $\beta_U = 0.0$).

2.3. Use of fragility curves in probabilistic risk assessment studies

The first Seismic Probabilistic Risk Assessment (SPRA) studies have been carried out for nuclear power plants in the US in the late seventies and are now used worldwide in order to assess seismic safety of existing or projected nuclear power plants.

In summary, the key elements of a seismic probabilistic risk assessment study are seismic hazard analysis, seismic fragility evaluation for each component and substructure and, last but not least, system analysis and construction of logical fault tree model. These three elements allow for the proper risk quantification of the installation, that is the evaluation of failure probability due to all possible earthquake events.

The probabilistic seismic hazard analysis leads to an estimate of the probability of occurrence of different levels of earthquake ground motion at the studied site.
This means, that the entire range of possible earthquakes is considered as potential initiating event and not only design earthquake. A seismic hazard analysis results in the establishment of hazard curves *H(a)* giving the probability of annual exceedence of ground motion level a. In general, the output of hazard analysis is a family of curves, each corresponding to a confidence level and thus accounting for uncertainty in the estimation of seismic hazard. The failure probability due to a seismic event is obtained by "convolution" of seismic hazard curve with fragility curve, that is by calculating the total probability by integrating:

$$P_f = \int_0^{+\infty} P_{f/a}(a) \frac{d}{da}\left(1 - H(a)\right) = -\int_0^{+\infty} P_{f/a}(x) \frac{dH(a)}{da} da$$

When considering the safety of a structure composed of substructures and equipments, such as a nuclear power plant, the probability of failure of the entire installation is determined by means of fault trees using simple logical structures of AND and OR in order to combine different events likely to result in global failure.

3. PRACTICAL CONSTRUCTION OF FRAGILITY CURVES

3.1. Deriving fragility curves from design study

In nuclear engineering practice, a simplified method, called *response factor method* and dealing with safety factors, is used to determine fragility curves. The parameters of the fragility curve, median capacity and log-standard deviations are evaluated by means of P safety factors with respect to design and qualification testing of components and structures [Reed et al. 94]. In this approach, capacity reads more precisely:

$$A = \left(\prod_{i=1}^{P} F_i \right) a_{\text{design}} \quad , \tag{3.1}$$

where a_{design} is the PGA of design earthquake and F_i are the random margin factors. The latter are supposed to be log-normally distributed with median $\hat{F}_i$ and log-standard deviation β_i. Thus, we can write $A_m = \hat{F}\, a_{\text{design}}$, with $\hat{F} = \prod_i \hat{F}_i$, whereas log-standard deviation is given by the expression

$$\beta = \sqrt{\sum_i \beta_i^2} \,, \qquad \text{where} \quad \beta_i = \sqrt{\beta_R^2 + \beta_U^2} \,.$$

The F_i account for conservatism and uncertainty in structural and equipment response calculations and in capacity evaluation, the latter including effects of ductility. The structural response factor, for instance, is supposed to account for conservatism and uncertainty due to damping and frequency evaluation, mode combination, spectral shape and site effects, soil-structure interaction, etc.
It is clear that realistic evaluation of median margin factors as well as underlying uncertainties based on design and expert judgment is not a simple task. Furthermore, strong hypothesis are made, such as log-normally distributed safety factors and, in a more general manner, linear relationships between input and response quantities (design and median values).
This approach has not been carried out here, since our work is focused on numerical simulation of fragility curves.

3.2. Direct estimation of fragility parameters by means of numerical simulation

The principal steps for determining fragility curves by numerical simulation are the following:
- Determine site-specific seismic ground motion (artificial or recorded ground motion time histories),
- Construct numerical model accounting for soil-structure interaction,
- Definition of failure criteria,
- Uncertainty quantification, modeling and propagation

We have applied the maximum likelihood method in order to obtain an estimation of the unknown parameters from numerical experiments. The outcome of each simulation is supposed to be the realization of a binomial random variable *X*. If the failure criteria is fulfilled at simulation number *i*, then $x_i=1$. Otherwise (no failure), we have $x_i=0$. These events arrive with probability $P_{f/a}$ and, respectively, $1-P_{f/a}$. Adopting the log-normal model, the latter can be calculated by expression 2.1. Consequently, the likelihood function for this problem reads:

$$L = \prod_{i=1}^{N} \left[P_{f|a_i}\right]^{x_i} \left[1 - P_{f|a_i}\right]^{1-x_i} \quad . \tag{3.2}$$

The estimators of parameters A_m and β are solution of the following optimization problem:

$$(\hat{\beta}_C, \hat{A}_m) = \arg\min_{\beta, A_m} [-\ln(L)]. \tag{3.3}$$

This approach has been used by [Shinozuka et al. 2004] in order to evaluate fragility curves for bridges subjected to seismic load.
An alternative method would consist in deducing an estimation of parameters A_m and β directly from a sample of capacities. This approach makes it necessary to determine a certain number of limit loads, in terms of PGA. It is clear that such information is difficult to obtain when dealing with non-linear structural models and impossible when using realistic ground motion time-histories for which scaling is not admissible.
When determining a sample of capacities by simulation, scaling of time-histories (or power spectral densities in a stationary approach) is inevitable. Moreover, if the structural behavior is non-linear, repeated simulations have to be performed until failure is reached.
The maximum likelihood estimation used here deploys information provided by failure as well as non-failure, without knowing whether the respective PGA value corresponds to a limit load. Concurrently, one could directly estimate failure probabilities for a certain number of PGA values, but this requires a much greater number of simulations, especially for evaluating small probabilities, i.e. the tails of the distribution.
Other authors, see for example [Choi et al. 2004] and [Güneyisi et al. 2007], determine a sample of response quantities due to earthquake load, let's say stress. Then, by means of linear regression, stress is linearly related to PGA value. This allows for determining a median value of capacity as well as log-standard-deviation, the latter under the hypothesis that earthquake loads have been generated according to actual probability of occurrence. Here again, linear relations are assumed and a representative sample of earthquake loads is essential.

4. APPLICATION TO NUCLEAR POWER PLANT EQUIPMENT

4.1. Best estimate mechanical model

We consider a coupled model consisting of a supporting structure, which is the containment building, and a secondary system or equipment representing a reactor coolant system.
The containment building is represented by a stick model that has been identified from the respective 3D model, see figure 1. The reactor coolant system (RCS) is modeled principally by beam elements. It consists of a reactor vessel and four loops. Each loop contains a steam generator, primary pump and piping. In figure 2, we have represented a schematic view of one of the loops on the right and a top view of the four loops on the left hand side. The whole equipment is multi-supported by 36 supports. These supports are anchors located under the reactor coolant pumps, the steam generators and the reactor vessel. The model includes localized non linearities due to stops. These are four upper lateral supports located at the upper level of each steam generator and three lower lateral supports that guide the steam generator and limit its movement in accident conditions.
The response of the coupled model is calculated in two times [Descelier et al. 2003], [Duval et al. 1999]. First, response of the containment structure to earthquake is calculated. We consider relative displacements of the finite element (FE) model with fixed basis, subjected to ground acceleration. The model being linear, we have performed modal analysis considering modal damping for structure and soil.

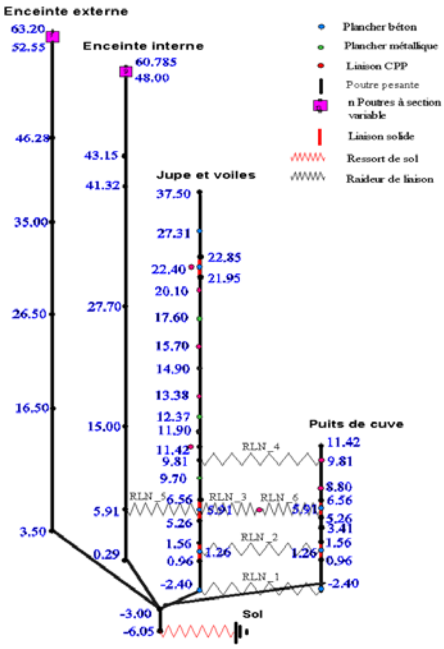

Figure 1 Stick model for containment building

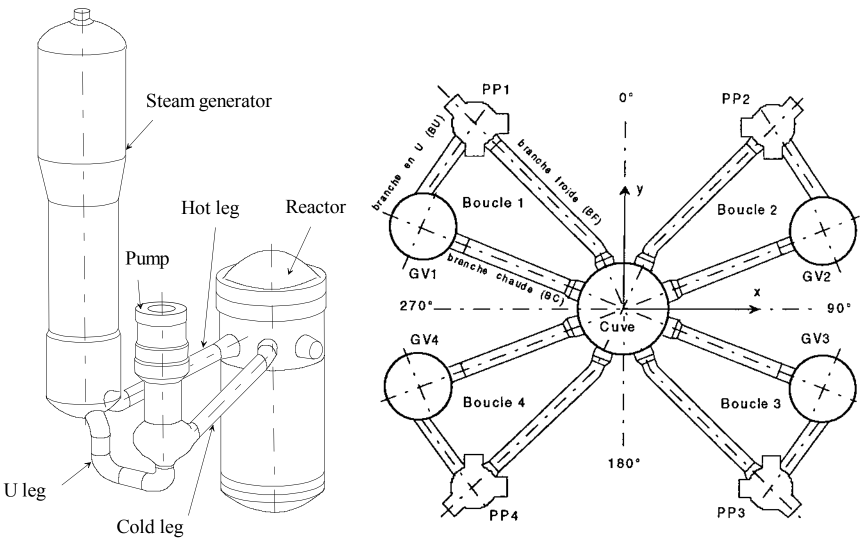

Figure 2 Schematic view of reactor coolant system

The calculated response time histories are then given Dirichlet boundary conditions for the supports of the reactor coolant system model. This allows for taking into account the differential displacements of the supports of RCS (reactor coolant pumps, steam generators and vessel). The nonlinearities of the RCS model are localized so that we have again reduced the model by projection on low-order modal basis determined for the model with fixed supports. The response is computed in the time domain where constraints on displacements (horizontal stops) are imposed at each time step by penalization.

4.2. Seismic ground motion

In the framework of time domain seismic analysis, seismic load is given by ground motion time histories.

We have generated 50 ground motion time histories [Mertens et al. 1997] according to site response spectrum for a rocky site. It has been verified that the 15% and 85% percentiles of the response spectra relative to the generated time histories correspond to the percentiles given by the attenuation law.

4.3. Uncertainty modeling and propagation

It has furthermore been verified that variability of the duration of strong motion period fits to variability observed for experienced earthquakes.
In what follows, the modeling of uncertainties is described more in detail; see also [Descelier et al. 2003].
Probability laws have been chosen with respect to Jayne's maximum entropy principle [Kapur & Kesavan 1992], using only available information such as mean, the upper and lower bound of the parameter values or coefficient of variation or.
Uncertainties with respect to the soil part concern the soil characteristics which influence on their term soil structure interaction impedances. For our study we have identified an equivalent spring model for the soil. The stiffness of the soil springs has been modeled by a random variable K having gamma distribution. This law follows from the the available information on mean, given support of the distribution. Furthermore, K and its inverse have to be of second order.
The uncertainty on structural properties for containment building and equipment is taken into account at the stage of the reduced model, after projection on a reduced basis of eigenmodes. Thence, the generalized matrices of mass, stiffness and damping are replaced by random matrices verifying a set of algebraic properties and whose mean is given by the "best-estimate" values [Soize 2005]. The dispersion of these matrices has been chosen such that coefficient of variation of the first eigenmode is equal to 1 %. This is little less than recommendations given in [Reed et al. 1994] (15% is recommended) but seems appropriate for quite realistic models as treated here.

Earthquake ground motion is modeled by a stochastic process. We suppose that the process is completely characterized by the sample of artificial time histories. Then, the different trajectories (time histories) have the same probability of occurrence and, when performing Monte Carlo simulation, we can proceed by random sampling from this data.

4.4. Failure criterion

For our study, we have chosen a rather simple failure criterion which is the limit stress in the piping system. The equivalent stress is calculated according to code specifications [RCC-M 2000]:

$$S_{eq} = B_1 \frac{PD_{ext}}{2t} + B_2 \frac{D_{ext}}{2I} M$$

where P is pressure and D_{ext} and t are the exterior diameter and thickness of the pipe, respectively. The resulting moment is noted by M, I is the moment of inertia and B_1 and B_2 are stress indices.
It is clear that the fragility curves calculated are conservative since ductility effects have not been taken into account.
But we have considered uncertainty on structural capacity. Thus, the admissible stress is modeled by a random variable with truncated log-normal distribution that takes values within the interval [0.9 *median; 1.1*median].

4.5. Numerical results

We have considered two configurations. For the first computations, we have introduced only uncertainties related to soil and earthquake event, considered here as random uncertainties. According to section **2.2**, see [Reed et al. 94], the estimations of the median and the β_R value are obtained for this case. Note, that in table 4.1, the median capacity has been normalized with respect to a reference ground motion level. Then, we have computed responses considering all of the sources of uncertainty introduced in § 4.3. This second configuration allows for determining an estimation of β_C.

Table 4.1

	A_m	β_R	β_C
Estimated value	2.40	0.14	0.42

For both cases, parameters have been estimated using maximum likelihood as described in § 3.2. The β_U is then obtained by virtue of relation 2.4, yielding $\hat{\beta}_U = 0.40$. Figure 3 shows the convergence of the estimations $\hat{A}_m$, and $\hat{\beta}_C$.

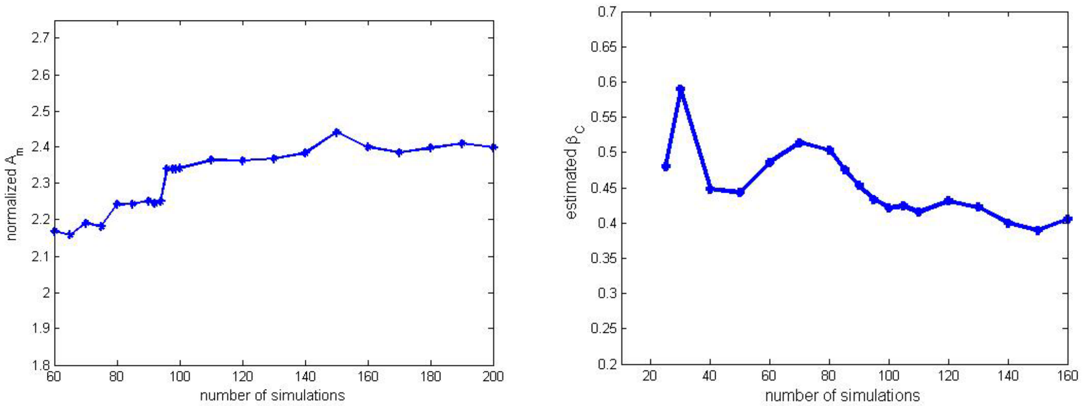

Figure 3 Convergence of estimations $\hat{A}_m$ and $\hat{\beta}_C$.

The family of fragility curves obtained is plotted in figure 4.

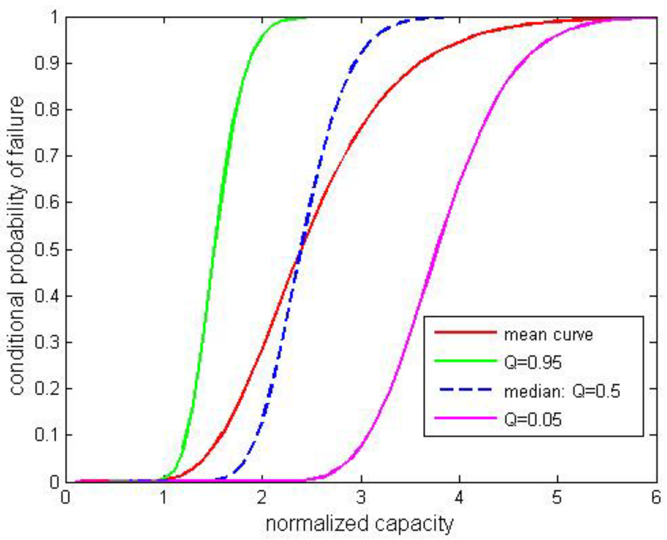

Figure 4 Family of fragility curves

In order to verify the hypothesis of log-normally distributed capacities we have simulated a sample of capacities A. For this, it has been necessary to "scale" the amplitude of accelerograms (and thus PGA value) until failure is observed. Figure 5 shows the normal probability plot for the sample of log-capacities, $\log(a)$.

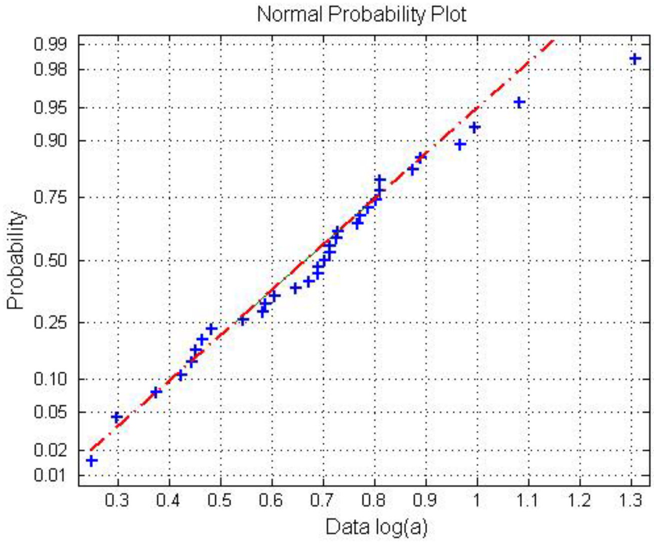

Figure 5 Normal Probability plot for a sample of log-capacities.

It can be seen that the data fits quite well to the reference line.

Secondly, we have performed resampling according to the bootstrap method [Efron et al. 1994] in order to evaluate the reliability of the estimations. Given the original sample of size *N*, The bootstrap method consists in the construction of N_b new samples, each of size *N*, by random sampling with replacement from the original dataset. This allows for determining mean values and confidence intervals for $\hat{A}_m, \hat{\beta}_R$ and $\hat{\beta}_C$.
We have estimated mean values and confidence intervals for N_b=100 bootstrap samples obtained from the original sample containing 200 values.

$$\overline{\hat{A}_m} = 2.41,\ \overline{\hat{\beta}_R} = 0.135 \text{ and } \overline{\hat{\beta}_C} = 0.421.$$

The corresponding $\hat{\beta}_U$ value is 0.399. These values are very close to the estimated values given by table 1. Furthermore, bootstrap standard deviation of the estimates reads:

$$\sigma_{\hat{A}_m} = 0.014;\ \sigma_{\hat{\beta}_R} = 0.024 \text{ and } \sigma_{\hat{\beta}_C} = 0.073.$$

This variability, due to estimation error, can be included in the model, equation (2.3), via the β values.

4. CONCLUSION AND PERSPECTIVES

We have determined fragility curves for nuclear power plant equipment by means of numerical simulation accounting for random excitation due to earthquake ground motion as well as structural and model uncertainties. The unknown parameters of the fragility curve, median and logarithmic standard deviation, have been estimated from the numerical experiments maximizing the corresponding likelihood function. This approach is very efficient and versatile since it is applicable to any kind of model, containing structural non-linearities or not. All the numerical computations have been carried out using *Code_Aster* open source FE-software.

We have chosen peak ground acceleration (PGA) in order to characterize ground motion level. Of course, any other indicator related to accelerograms could be chosen in the framework of the methodology presented here. It would be indeed interesting to evaluate the influence of the ground motion parameter on dispersion, characterized by the β parameter. However, peak ground acceleration (or spectral acceleration) remains the preferred parameter in seismic PRA since hazard curves are established with respect to this parameter.

REFERENCES

Cho S.G. & Joe, Y.H. (2005). Seismic fragility of nuclear power plant structures based on the recorded earthquake data in Korea. *Nuclear Eng. Design*, **235**, 1867-1874.

Choi E., DesRoches R. & Nielson B. (2004). Seismic fragility of typical bridges in moderate seismic zones. *Eng. Struct*., **26**, 2004, 187-199.

Code_Aster, general public licensed structural mechanics finite element software, http://www.code-aster.org.

Descelier, C., Soize C. & Cambier S. (2003). Nonparametric-parametric model for random uncertainties in nonlinear structural dynamics - Application to earthquake engineering. *Earthquake Engrg. and Struct. Dyn*., **33**, 315-327.

Devesa G., Guyonvarh, V. & Clouteau, D. (2005). Use of coupling and regulatory methods in soil structure interaction for nuclear power plants. *Proceedings of Eurodyn*, Paris, Millpress.

Duval C., Guyonvarh, V., Louche, V., Pigat J. & Waeckel, F., Méthodes d'estimation du comportement sismique d'un circuit primaire de réacteur à eau préssurisée. 5th National Colloque AFPS, 1999.

Efron, B. & Tibshirani, R.J. (1994). An Introduction to the Bootstrap, Monographs on Statistics and Applied Probability, Chapman &Hall, New York.

Elenas, A. & Meskouris, K. (2001). Correlation study between seismic acceleration parameters and damage indices of structures. *Eng. Struct*., **23:6**, 698-704.

Güneyisi, E.M. & Altay, G. (2008). Seismic fragility assessment of effectiveness of viscous dampers in R/C buildings under scenario earthquakes. *Structural Safety*, **in press**.

Kapur, J.N. & Kesavan, H.K., Entropy Optimization Principles with Applications, Academic Press, San Diego, 1992.

Mertens, P.G & Preumont, A. (1997). Improved generation and application of artificial time-histories and PSD functions. *Proceedings of SMiRT 14*, Lyon, France.

RCC-M, *Design and Construction Rules for mechanical components of PWR nuclear islands*, AFCEN, june 2000.

Reed, J.W. & Kennedy, R.P. (1994). Methodology for Developing Seismic Fragilities. Final Report TR-103959, EPRI.

Rota, M., Penna, A. & Strobbia, C.L.(2008). Processing Italian damage data to derive typological fragility curves. *Soil Dyn. & Earth. Engrg*., **28**, 933-947.

Shinozuka M., Feng, Q., Lee, J. & Naganuma T. (2000). Statistical analysis of fragility curves. *J. Eng. Mech. ASCE*, **126:12**, 1224-1231.

Viallet E. & Humbert, N. (2007). Considerations on the use of natural and artificial time histories for seismic transient non-linear analyses of structures and variability assessment. *Proceedings of SMiRT 19*, Toronto, Canada,

Wakefield, D., Ravindra, M., Merz, K. & Hardy, G. (2003). Seismic Probabilistic Risk Assessment Implementation Guide. Final Report 1002989, EPRI.

Zentner I. (2007). Méthodes probabilistes dans les analyses sismiques : Modélisation, propagation et hiérarchisation des incertitudes. *Colloque AFPS*, Chatenay Malabry.