# NON LINEAR ANALYSIS OF STRUCTURES ACCORDING TO NEW EUROPEAN DESIGN CODE

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**ABSTRACT :**
Structures designed in seismically active regions must comply with two basic demands: first, structure must be designed for loads during usage (ultimate limit state and serviceability limit state) and second, structure must be sound enough to avoid collapse during earthquake (ultimate limit state). Except from linear-elastic calculations, very often are used non-linear methods. In this article simple plain frame concrete structure will be analyzed using N2 method from Eurocode 8 (EN 1998-1:2004). N2 is simple non-linear method used for calculation of structures during earthquakes. It combines multi degree pushover analysis with spectrum analysis of equivalent single degree of freedom (SDOF) system. It is formulated in acceleration-displacement format, which is very suitable for visual overview of basic variables that account for seismic response of the structure. N2 method can be considered as combination of pushover analysis and spectrum analysis. Inelastic demand spectrum is obtained from elastic spectrum. Results obtained are accurate enough if structure has dominant first mode of oscillation. For now, it is used only for plane structures. This paper will give numerical example of N2 method. It is concluded that inelastic structural response is crucial in earthquake engineering. Modern methods, supported with usage of computers and strict design codes ensure better understanding of structural response during earthquakes and at the same time seismic resistant structures.

**KEYWORDS:**
seismic analysis, Eurocode, pushover, non linear analysis

## 1. INTRODUCTION

Structures designed in seismically active regions must comply with two basic demands: first, structure must be designed for loads during usage (ultimate limit state and serviceability limit state) and secondly, structure must be sound enough to avoid collapse during earthquake (ultimate limit state). For purpose of the seismic calculation, we can use both linear elastic structural analysis and non linear analysis. In European norm EN 1998-1:2004 calculation methods are given in part 4.3.3. of [1]. These two methods are as follows:

a) Lateral force method of analysis - this type of analysis may be applied to structures whose response is not significantly affected by contributions from modes of vibration higher than the fundamental mode in each principal direction.

b) Modal response spectrum analysis which can be used for all structures whose respone is/or can be significantly affected by contributions from modes of vibration higher than the fundamental mode in each principal direction.

Need for the non linear seismic analysis dates a long time ago. It was not used primarily because of insufficient computer power, software limitations and insufficient research in this field. As software and computation power increases rapidly, so this kind of analysis is becoming more and more popular. New generation of procedures for design of the new and rehabilitation of the damaged structures is now available (performance based engineering concept). It is now evident that during design phase more attention must be given to damage control. This can not be efficiently incorporated if non linear methods are not used. Except of linear methods mentioned previously, non linear are as follows:

c) Non-linear static (pushover) analysis mentioned is eurocode part 4.3.3.4.2.
d) Non linear dinamic analysis (time history)

In this article simple plain frame concrete structure will be analyzed using N2 method from Eurocode 8 (EN 1998-1:2004). N2 is simple non-linear method used for calculation of structures during earthquakes. It combines multi degree pushover analysis with spectrum analysis of equivalent single degree of freedom (SDOF) system. It is formulated in acceleration-displacement format, which is very suitable for visual overview of basic variables that account for seismic response of the structure. N2 method can be considered as combination of pushover analysis and spectrum analysis. Inelastic demanded spectrum is obtained from elastic spectrum. Results obtained are accurate enough if structure has dominant first mode of oscillation. For now, it is used only for plane structures. Results given here are based on calculations and research from [2]. Seismic load (demand) in N2 method is defined in the shape of elastic acceleration spectrum (figure 1). For better visualization seismic demand in N2 method is defined as elastic spectrum in acceleration-displacement format (figure 1 – right picture).

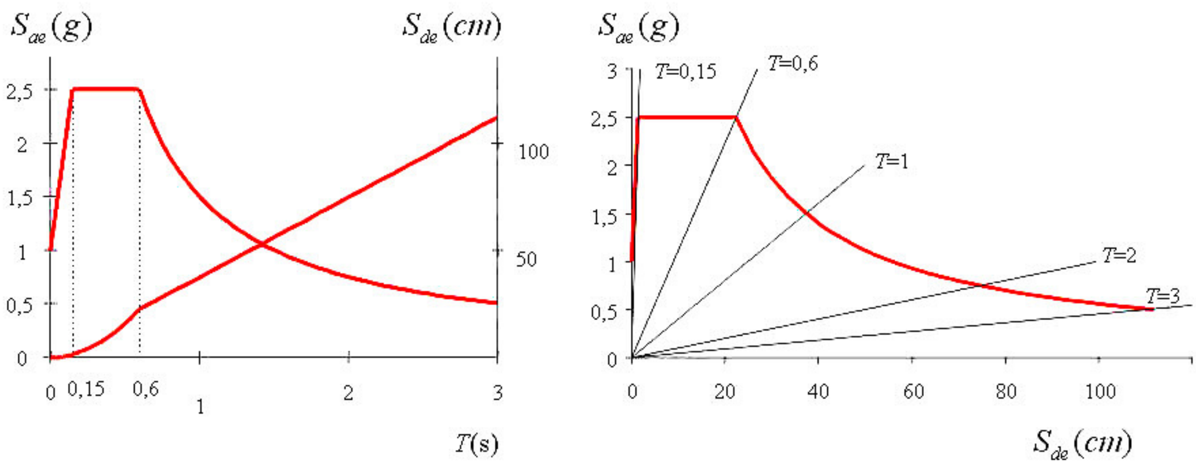



Figure 1 Acceleration spectrum (N2 method)

As this method is non linear, inelastic spectrum must be defined. Only two factors are needed: ductility factor and reduction factor. This kind of inelastic spectrum in acceleration-displacement format is called demand spectrum. Typical demand spectrum is shown in figure 2.

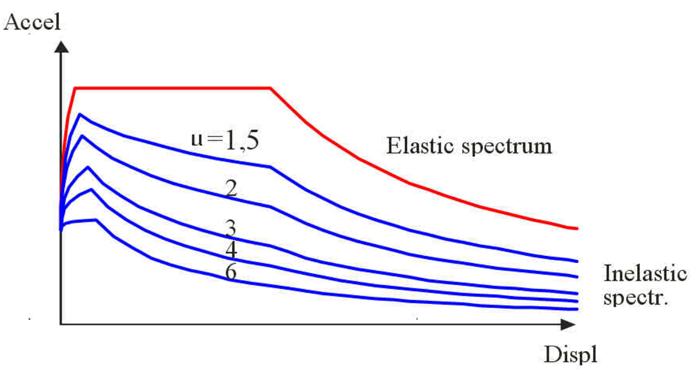
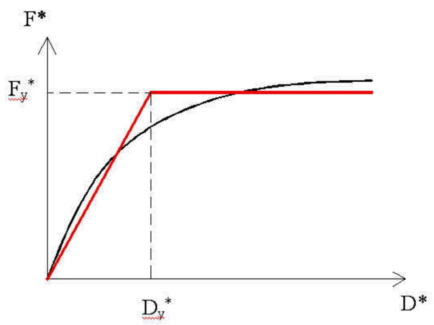



Figure 2 Inelastic spectrum

Structure is modeled as plain frame model with multiple degrees of freedom (MDOF) model. With pushover method characteristic non linear force-displacement relation for MDOF can be calculated (usually base shear and displacement in highest point are used). Using transformation factor $\Gamma$ transfer to equivalent single degree of freedom (SDOF) is made. Non linear force – displacement relation is simplified using ideal elastic – plastic relation as shown in figure 3.




$$D^* = \frac{D_t}{\Gamma}$$

$$F^* = \frac{V}{\Gamma}$$

Figure 3 Idealization of force – displacement relation

As the final result capacity diagram in acceleration – displacement format is obtained (see figure 4). Demand spectrum and capacity diagram are always on the same figure. Intersection of radial line which corresponds to elastic period $T^*$ of idealized bilinear system with elastic demand spectrum ($\mu$=1) defines demand elastic displacement $S_{de}$. Inelastic demand, related to acceleration $S_{ay}$ and displacement $S_d$ corresponds to intersection of capacity diagram with demand spectrum (with demanded ductility $\mu$). For medium and short range periods structures ($T^* \geq T_C$) rule of equal displacement can be applied (demand inelastic displacement $S_d$ equals to demand elastic displacement $S_{de}$).

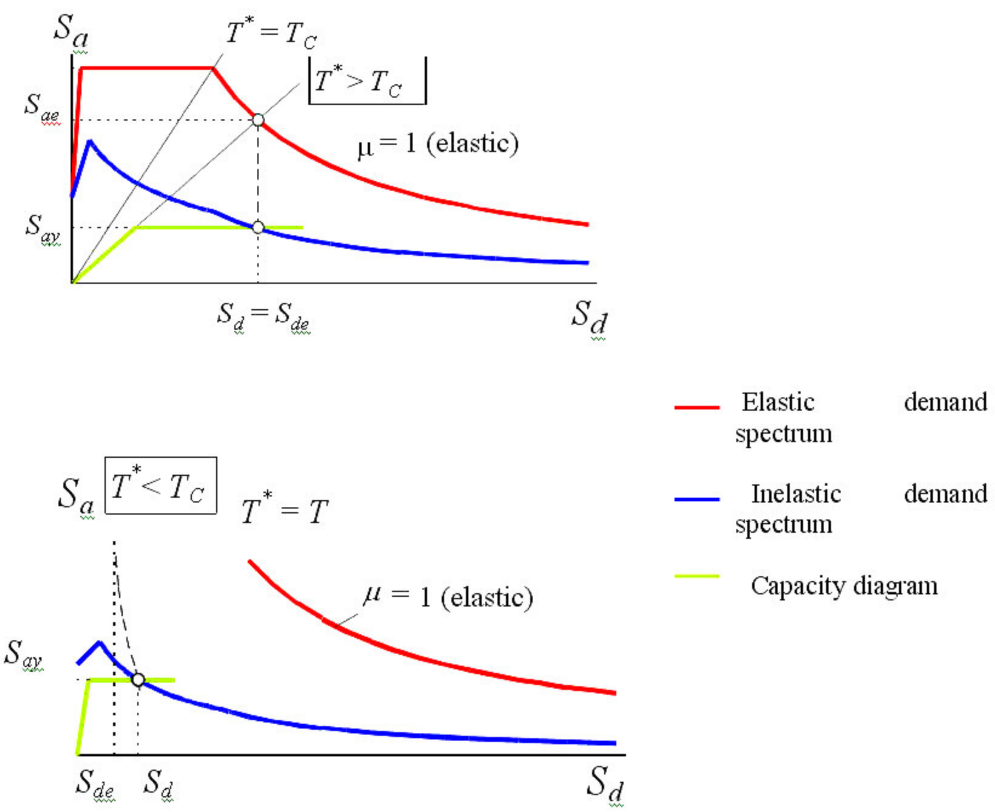



Figure 4 Elastic and demand spectrum in relation with capacity diagram
($T^* \geq T_C$ picture above and for $T^* < T_C$ picture below)

## 2. N2 METHOD

For the purpose of better understanding of N2 method will be given in steps.

### *2.1. Step 1*

Beside the data needed for usual elastic analysis, data concerning non linear relation force – displacement must be given. Bilinear or tri-linear diagram are often used. Seismic demand is defined in the shape of elastic spectrum. Spectral acceleration is given in relation to period ($T$).

### *2.2. Step 2*

For elastic system with single degree of freedom following relation applies:

$$S_{de} = \omega^2 S_{ae} = \frac{T^2}{4\pi^2} S_{ae} \tag{2.1}$$

where $S_{ae}$ i $S_{de}$ are values from elastic spectrum (acceleration and displacement) for period $T$ and corresponding damping. For nonlinear SDOF with bilinear relationship spectral acceleration $S_a$ and spectral displacement $S_d$ can be calculated as in Eqns. 2.2 and 2.3:

$$S_a = \frac{S_{ae}}{R_\mu} \tag{2.2}$$

$$S_d = \frac{\mu}{R_\mu} S_{de} = \frac{\mu}{R_\mu}\frac{T^2}{4\pi^2} S_{ae} = \mu \frac{T^2}{4\pi^2} S_a \tag{2.3}$$

where $\mu$ is ductility factor (defined as ratio between maximal displacement and yield displacement) and reduction factor $R_\mu$. For reduction factor several propositions are given as in Eqns. 2.4 and 2.5.

$$R_\mu = (\mu - 1)\frac{T}{T_C} + 1, \;\; T < T_C \tag{2.4}$$

$$R_\mu = \mu, \;\; T \geq T_C \tag{2.5}$$

where $T_C$ is characteristic period defined as intermediate value between short and medium periods (between constant acceleration and constant velocity). Starting from elastic spectrum and using equations defined previously demand spectrum can be calculated (see figure 5).

### *2.3. Step 3*

Pushover is conducted in a way that structure is subjected to monotone lateral loads, which correspond to inertial forces during seismic actions. By gradually increasing lateral forces, constructive elements are failing consecutively and stiffness of the structure decreases. Due to limitation of this article equations will be omitted. All equations can be found in [2]. Base shear will correspond to force and roof displacement will correspond to displacement.

### *2.4. Step 4*

Seismic demand must be obtained form spectrum which is usually made for SDOF system. That implies that structure must be modeled as equivalent SDOF system and than this two model must be compared. For the purpose of obtaining SDOF system various procedures are available. One of them will be given in short in this chapter. Start point is the equation of shear building (only translations are taken into account as in Eqn. 2.6).

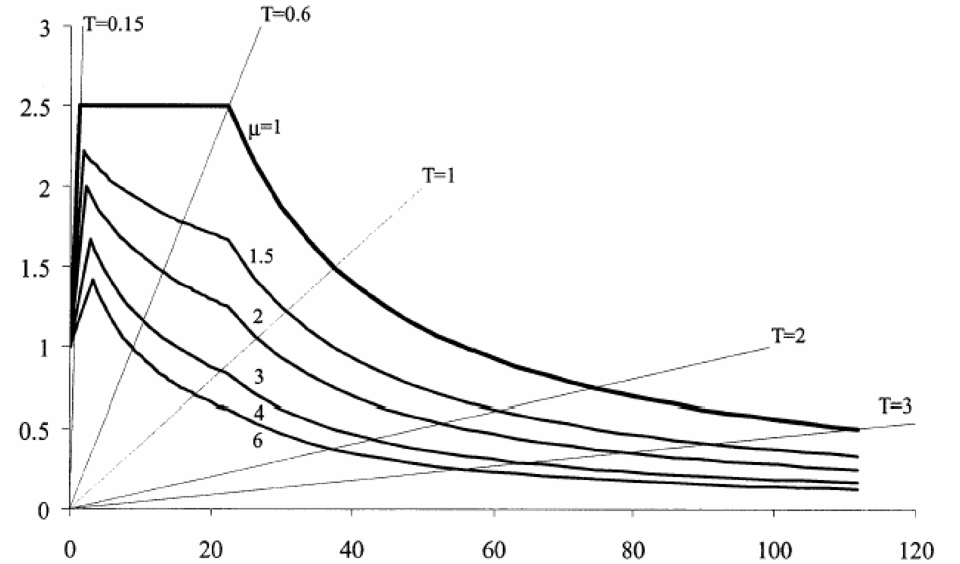


Figure 5 Demand spectrums for constant ductility values

$$m\,u'' + R = -m\;1\alpha \tag{2.6}$$

In Eqn. 2.6. $u$ and $R$ are vectors (displacements and internal forces), 1 is unit vector while $\alpha$. Damping is not included in this Eqn. but it will be included in design spectrum. $V$ is shear force (see Eqn. 2.8). Constant $\Gamma$ can be obtained as in Eqn. 2.9. Elastic period of SDOF system ($T^*$) can be calculated as in Eqn. 2.10., where $F_y^*$ and $D_y^*$ are yield strength and yield displacement, respectively. Eqn. 2.11 represents capacity diagram which is given as force and equivalent mass ratio.

$$F^* = \frac{V}{\Gamma} \tag{2.7}$$

$$V = \sum P_i = \mathbf{\Phi}^T \mathbf{m1}\,p = p\sum m_i\,\Phi_i = p\,m^* \tag{2.8}$$

$$\Gamma = \frac{\mathbf{\Phi}^T\mathbf{m1}}{\mathbf{\Phi}^T\mathbf{m\Phi}} = \frac{\sum m_i\Phi_i}{\sum m_i\Phi_i^2} = \frac{m^*}{\sum m_i\Phi_i^2} \tag{2.9}$$

$$T^* = \frac{2\pi}{\omega^*} = 2\pi\sqrt{\frac{m^* D_y^*}{F_y^*}} \tag{2.10}$$

$$S_a = \frac{F^*}{m^*} \tag{2.11}$$

### *2.5. Step 5*

Seismic demand for equivalent SDOF system can be obtained graphically as in figure 6 for structures with short and medium periods. Figure 6 represents demand and capacity spectrum. For structures with short periods figure 4 implies. Intersection of radial line which corresponds to elastic period $T^*$ of idealized bilinear system, with elastic demand, defines demand acceleration (strength) $S_{ae}$ for elastic behavior and corresponding elastic displacement $S_{de}$. Acceleration at yield strength represents both demand acceleration and capacity of inelastic system.

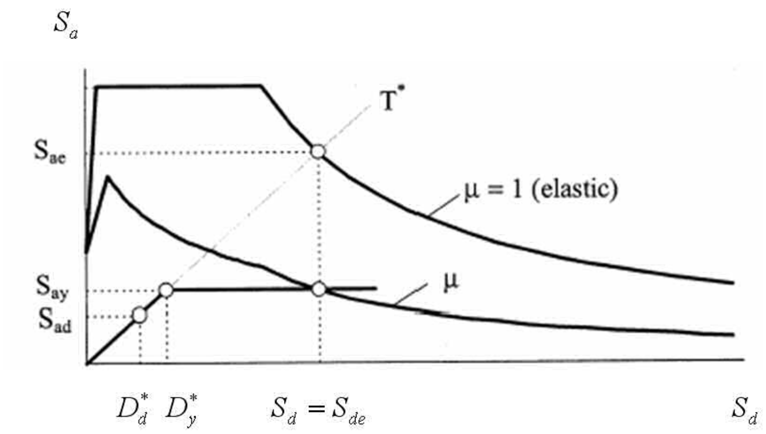



Figure 6 Demand and capacity spectrum

### *2.6. Steps 6 and 7*

Demand displacement of SDOF model $S_d$ is transformed into global demanded maximal displacement $D_t$ using transform equations. That maximal displacement $D_t$ corresponds to target displacement (part B of [2]). Local seismic demand can also be obtained by means of pushover analysis. Due to monotonically increase of lateral loads, structure is "pushed" until target displacement is not reached.

### *2.7. Step 8*

In the last step, predicted behavior of the structure can be estimated by comparison of local and global seismic demands from chapter 7 with the capacities of different levels. Global behavior can also be estimated by comparison of displacement capacity and displacements according to seismic demand.

## 3. NUMERICAL EXAMPLE

As example, response of four story reinforced concrete plane frame will be given. Structure is subjected to three different earthquakes. Building is also tested in European Laboratory for Structural Assessment (ELSA). Results of experiment are used to verify mathematical model. Structure is designed according to European norms as structure with peak ground acceleration of 0,3g. Masses of floors (from first floor) are 87, 86, 86 and 83 tons. Resulting shear factor has a value of 0,15. Additional details are given in [2]. Figure 7 represents bearing structure. In figure 8 same curve define two relations: $V - D_t$ for MDOF system and $F^* - D^*$ ratio of equivalent SDOF system. In must be noted that scales are different, however ($\Gamma = 1{,}34$). Bilinear representation of the pushover curve is represented in figure 8 (left part of the picture, see thick line) and values of strength and displacement at yield point can be determined: ($F_y^* = 830 kN$ and $D_y^* = 6{,}1\, cm$). Elastic period can be calculated as in Eqn. 3.1:

$$T^* = 2\pi \sqrt{\frac{m^* D_y^*}{F_y^*}} = 0{,}79 s \tag{3.1}$$

Capacity diagram (figure 8, left part) is determined by Eqn. 3.2:

$$S_{ay} = \frac{F_y^*}{m^*} = \frac{830}{217} = 3{,}82 m s^{-2} = 0{,}39 g \tag{3.2}$$

Capacity diagram and demanded spectrum are compared in figure 8 (right part of the picture).

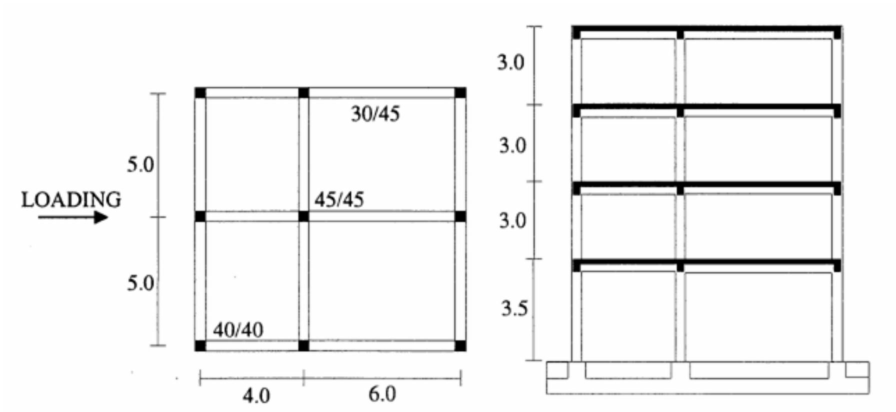



Figure 7 Overview of the structure

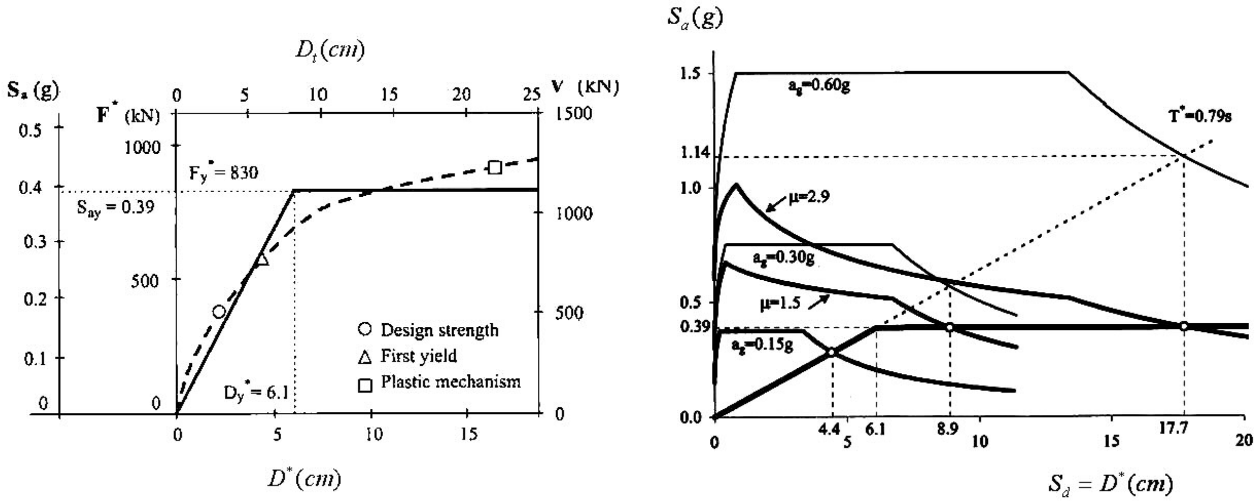



Figure 8 Pushover curve and capacity diagram (left part of the picture); demand spectrum for 3 different ground accelerations and capacity diagram (right part of the picture)

Pushover analysis preformed on MDOF model, with maximal displacement $D_t$ (roof) provides displacements for whole structure, local seismic demands (in term of relative floor displacements and joint rotations) as given in figure 9. Maximal rotation in figure 9 is 2,2%. Other rotations are proportional to maximal. Resulting envelopes are obtained by pushover from left to right and vice versa. Experimental results from ELSA are similar to this model.

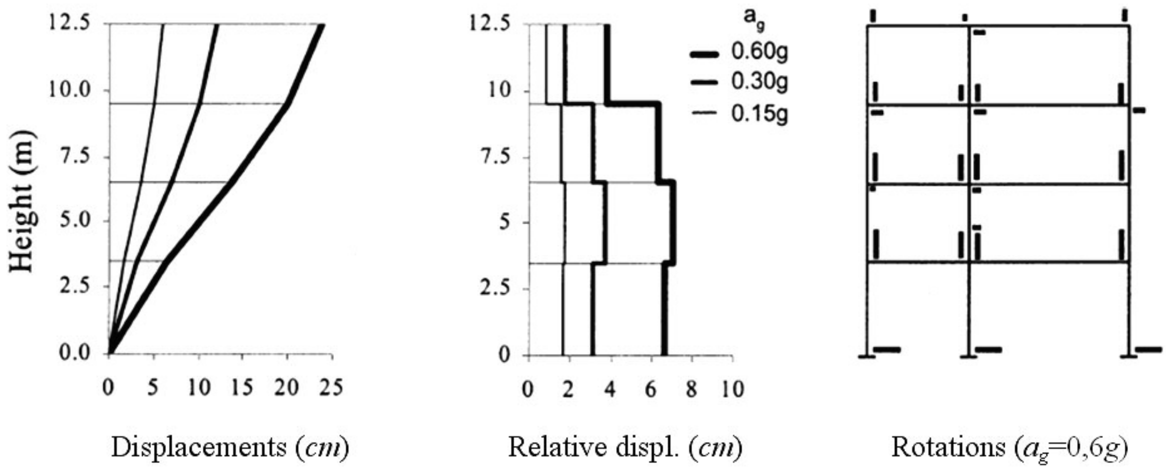



Figure 9 Displacements, relative displacements and rotations of frame

## 4. CONCLUSION

In this paper overview of N2 method is given. This is simple non-linear method used for calculation of structures during earthquakes. It combines multi degree pushover analysis with spectrum analysis of equivalent single degree of freedom (SDOF) system. It is formulated in acceleration-displacement format, which is very suitable for visual overview of basic variables that account for seismic response of the structure. The inelastic structural response is crucial in earthquake engineering as all concrete buildings behave inelastic during seismic exitation. Modern methods like N2, supported with usage of computers and strict design codes will ensure better understanding of structural response during earthquakes and at the same time seismic resistant structures.

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