

# VIBRATION ABSORPTION AND ISOLATION IN DYNAMICALLY LOADED FOUNDATIONS

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## **ABSTRACT :**

Machine foundations are known as dynamically loaded foundations built with reinforced concrete, in general. The design of a machine foundation is more complex than of a foundation which supports only static loads. In machine foundations, the designer must consider, in addition to the static loads, the dynamic forces caused by the working of the machine and seismic effects. The occurence of resonance and the consequent effect on increase of vibration amplitudes is one of the most common sources of trouble in machine foundations. The possible methods of vibration isolation in existing machine foundations are counter-balancing the exciting loads, stabilization of soils, use of structural measures, isolation by trench barriers, isolation in buildings. This study includes basic principles of vibration absorption and isolation, introducing of common vibration absorbers, the design procedure for foundation on absorbers, design principles of vibration isolation with wave barriers, methods of reducing vibration amplitudes in existing machine foundations.

KEYWORDS: Vibration, absorption, isolation, machine foundations, dynamic loads

## **1. INTRODUCTION**

The design of a machine foundation is more complex than of a foundation which supports only static loads. If a machine is rigidly bolted to the floor, the vibratory movement of the machine itself may be reduced, but the vibration transmitted to the floor will be large. This may produce harmful effects even at large distances. On the other hand, if a flexible support is provided under the machine or its foundation, the vibration transmitted to the floor will be considerably reduced, but this may cause significant motion to the machine itself during normal operation or during the starting and stopping stages. Some compromise has, therefore, to be reached between the two requirements. This is achieved in design practice by selecting a suitable natural frequency for the machine foundation. For machines running at a steady speed the degree of isolation is determined by the ratio ? (defined as the ratio of the operating frequency of the machine  $f_m$  to the natural frequency of faoundation  $f_n$ ). By choosing a suitable natural frequency, therefore, it is possible to obtain the required degree of isolation which obviously depends on the environmental conditions at site. From the point of wiev of isolation, two types of vibration problems are encountered in industrial practice; active isolation and passive isolation. In the active type, the isolation is required against vibration caused by the machine itself. The foundation for such such a machine should be so designed as to reduce the transmitted vibration to the permissible level prescribed. In the passive type of vibration isolation, the foundation for a delicate machinery is designed in such a way that the aplitude of its motion due to floor vibration is reduced to an acceptable limit. It is now realized that to provide effective isolation, the machine or its foundation should be mounted on a suitable isolating medium properly designed on the basis of the theory of transmissibility. The occurence of resonance and the consequent effect on increase of vibration amplitudes is one of the most common sources of trouble in machine foundations. The possible methods of vibration isolation in existing machine foundations are; counter-balancing the exciting loads, stabilization of soils, use of structural measures, isolation by



trench barriers, isolation in buildings. The choice of structural measures depends on the nature of vibration and the ratio of natural frequency to the operating frequency. Following are the possible structural measures that can be adopted in appropriate cases; increasing base area or mass of foundation, use of slabs attached to foundation, use of auxiliary spring-mass systems (Demir,1992). If probable disturbance of vibrations to near structures or equipment is understood during design, absorbers must be used in the design to avoid harmful effect of vibrations. If there is a situation where absorbers are inadequate to obtain desired amplitudes, the method of vibration isolation by trench barriers is used. Providing a barrier in the vicinity of the source is defined as active isolation and a barrier remote from vibration source to protect a structure or equipment is defined as passive isolation. There are situations both open trenches and in-filled trenches by concrete or bentnite–soil mixture used. This study includes basic principles of vibration absorbers, design principles of vibration isolation on absorbers, design principles of vibration isolation and isolation, introducing of common vibration absorbers, the design procedure for foundation on absorbers, design principles of vibration isolation amplitudes in existing machine foundations.

### 2. DEVELOPMENT OF ANALYTICAL MODELS FOR DYNAMIC SYSTEMS

A detailed dynamic analysis of a structural system as it physically appears in real life is rarely attempted. The usual practice is to choose an idealized model consisting of springs and lumped masses which will closely perform in the same way as the actual structure. It is only necessary that a proper selection of the system parameters be made such that equivalence of the idealized spring, damping element, and lumped mass in the model results in equivalent displacements at analogous points of significance in the prototype structure. In addition, the idealized model should behave, time-wise, in exactly the same manner as the actual prototype structure. Model of machine supported on an inertia block and vibration isolated from the foundation is shown in Fig. 1 (Arya, 1979).



Figure 1 Machine supported on inertia-block and vibration isolated from the foundation

In special cases and due to environmental conditions, it may be necessary to limit the vibration amplitude at the foundation base to much lower values than those usually allowed. This requirement



may not be practical to avhieve even by proper selection of mass or base area of the foundation. In such cases, use of an inertia block and spring absorbers is recommended. In normal behavior, three forms of excitation are possible. Excitation in the vertical direction is independent of the other forms of oscillation. Excitation in the horizontal direction is generally coupled with the rocking mode; however, for a machine which is located at relatively low height (h is 4/2b) then investigation of the horizontal and rocking excitation independent modes is sufficient. The parameters  $k_{x1}$  and  $k_{x1}$  are properties of the spring absorbers. Parameter m<sub>i</sub> is the combined mass of the machine and the inertia block together. The parameters  $k_{22}$ ,  $k_{22}$ ,  $k_{22}$  and  $C_{x2}$ ,  $C_{22}$ ,  $C_{22}$  are spring constants and damping coefficients, respectively, of the soil in the three modes considered and should be determined using the elastic half-space theory. Parameters m<sub>2</sub> and I<sub>2</sub> are the mass and mass moment of inertia, respectively, of the foundation. The solution of the differential equations (2.1) and (2.2) can readily be found for the natural frequencies, mode shapes, transmissibility factors, and the vibration response. Often, the fundamental frequency and the transmissibility factor are the principal result of the analysis. The set of differential equations (2.3) is in simultaneous form, and a manual solution is tedious to perform. This system of simultaneous equations is rarely solved by hand unless a thorough investigation of the system is required, and then the solution is obtained with the help of a computer program.

$$m_1 \ddot{z_1} + k_{z1}(z_1 - z_2) = F_z(t)$$
,  $m_2 \ddot{z_2} + C_{z2} z_2 + k_{z1}(z_2 - z_1) + k_{z2} z_2 = 0$  (2.1)

$$m_1 \ddot{x_1} + k_{x1}(x_1 - x_2) = F_x(t) \quad , \quad m_2 \ddot{x_2} + C_{x2} x_2 + k_{x1}(x_2 - x_1) + k_{x2} x_2 = 0$$
(2.2)

$$m_{1}x^{"}_{1} + k_{x1}(x_{1} - x_{2} - ?_{2}h) = F_{x}(t) , \qquad m_{2}x^{"}_{2} + C_{x2}x^{"}_{2} + k_{x2}x_{2} - k_{x1}(x_{1} - x_{2} - ?_{2}h) = 0$$

$$I_{2}?^{"}_{2} + C_{?2}?^{"}_{2} + m_{1}x^{"}_{1}h + k_{?2}?_{2} = F_{x}(t) h$$
(2.3)

#### 3. PRINCIPLE OF VIBRATION ABSORPTION AND ISOLATION

A foundation on absorbers is usually made of two parts: a lower slab or a sole plate on which the absorbers are placed and an upper foundation block resting on the absorbers. The machine is anchored to the upper foundation block. A schematic sketch of a machine foundation on absorbers is shown in Fig. 1a, and the commonly used model for analyzing this system is shown in Fig. 2b.



Figure 2 a) Schematik diagram of a foundation on absorbers (supported type), b) Equivalent two-spring-mass model of foundation absorber system, c) Free-body diagram.

Each of the rigid masses  $m_1$  due to the foundation and  $m_2$  for the concrete slabs will have six degrees freedom. The total number of degrees of fredom for the whole system are thus 12. Vibration



absorbers are generally used for machines undergoing vertical vibrations and having vertical unbalanced forces. Vertical vibrations are independent of vibrations in other modes. The problem of machine foundations on absorbers may thus be analyzed by treating the system as a two-degrees-of-freedom system (Fig. 2b). Assuming the masses of the system to be concentrated at their centers of gravity and located on the same vertical line, the differential equation of motion may be written as in Eqn. 31; where,  $F_0 \sin?t$ : exciting force, ? : frequency of machine operation (rad/sec),  $z_1$ ,  $z_2$ : vertical displacements of centers of gravity of masses  $m_1$  and  $m_2$ , respectively,  $k_1$ : equivalent stiffness of vertical soil spring below the base and  $k_2$  is the total equivalent stiffness of all springs in the absorber system. The maximum amplitudes  $Z_1$  and  $Z_2$  are given by Eqn. 3.2.

$$m_1 \ddot{z_1} + k_1 z_1 + k_2 (z_1 - z_2) = 0$$
,  $m_2 \ddot{z_2} + k_2 (z_2 - z_1) = F_0 \sin 2t$  (3.1)

$$Z_{1} = ?^{2}{}_{n2}F_{0}/[m_{1}?(?^{2})] = Sr^{2}{}_{2}/m_{1}[1-(1+\mu)(r^{2}{}_{1}+r^{2}{}_{2}-r^{2}{}_{1}r^{2}{}_{2})] , Z_{2} = (1+\mu)?^{2}{}_{n1}+\mu?^{2}{}_{n2}-?^{2})F_{0}/[m_{2}?(?^{2})]$$
(3.2)

In case no absorbers are used, the amplitude of vibration of the entire foundation resting on soil is given by Eqns. 3.3 and 3.4;  $?_{n1,2}$  are the natural frequencies of the system.  $?_{n1}$  is the limiting natural frequency of the entire system resting on soil (when no absorbers are used) and  $?_{n2}$  is the limiting natural frequency of the mass  $m_{P}$  resting on absorbers and calculated on the assumption that the system below the springs has large rigidity (Celep, 2001).

$$Z=F_0/[(m_1+m_2)(?_{n1}^2-?_{n1}^2)]=S/[m_1(1+\mu)(r_1^2-1)],?(?_{n1}^2)=[?_{-1}^4-(1+\mu)(?_{n1}^2+?_{n2}^2)?_{-1}^2+(1+\mu)(?_{n1}^2?_{n2}^2)] (3.3)$$

$$\mu = m_2/m_1 , ?_{n1}^2 = k_1/(m_1 + m_2) , ?_{n2}^2 = k_2/m_2 , F_0 = S?^2 , r_1 = ?_{n1}/? , r_2 = ?_{n2}/?$$
(3.4)

S is a constant, depending upon machine characteristics such as unbalanced mass and eccentricity. It is seen from Eqn. 3.2 that the amplitude with absorber will be small only if the ratio p is small. When  $r_2 = \frac{2}{n^2}$  is negligible, the amplitude of vibration  $Z_1$  is almost zero and the absorber efficiency is high. The effectiveness of the absorber is thus maximum when  $r_2=0$  and decreases as the ratio  $r_2$ increases. For very large values of  $r_2$  ( $r_2$ ? 8), the value of Z<sub>4</sub> approaches the value Z for the no absorber case. It may be concluded that for the absorbers to have a favorable effect on the amplitudes of foundation vibration, the natural frequency of the mass above the absorbers should be as small as possible in comparison with the frequency of machine operation. The required natural frequency of the foundation above the absorbers may be achieved by using absorbers of suitable stiffness and by appropriate selection of mass above the absorbers. For machines operating at high speeds, the required condition between  $2_{n2}$  and 2 can be easily satisfied without significant increase in the weight of the foundation above the absorbers. For machine operating at low frequency, the relationship is usually difficult to satisfy by just decreasing the rigidity of the absorber because this decrease beyond a certain limit is not practicable due to strength requirements. In such case, massive foundation above the springs is necessary. A proper choise of the type of absorber is very critical in such cases. The absorber system may be designed based upon the value of  $r_2$ , which depends upon the required degree of absorption ? defined by Eqn. 3.5. The principle of vibration absorber explained above will now be used for devoloping a procedure for the design of foundation on absorbers.

$$? = Z/Z_1 = [1 - (1 + \mu)(r_1^2 + r_2^2 - r_1^2 r_2^2)] / [r_2^2 (1 + \mu)(r_1^2 - 1)]$$
(3.5)

#### 3.1. Common Vibration Absorbers

Materials capable of undergoing elastic deformation can be used as vibration absorbers. Commonly used vibration absorbers are; steel or metal springs, cork pads, rubber pads, timber pads, neoprene pads and pneumatic absorbers. Helical springs made of steel are the most effective elastic supports for reducing amplitudes of vibration in machine foundations. This type of spring absorber will be suitable only for very low capacity machines. For machines of medium to high capacity, absorber units having several springs are used (Fig. 3). Two arrangements of mounting the spring absorbers

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are possible for supporting machine foundations. They are supported-type (Fig.2a) and suspended-type (Fig. 4). In a supported-type arrangement, the springs are placed directly under the machine or the foundation. In a suspended-type absorber system, the springs are located at or close to the floor level, and the main foundation is suspended from the springs. A typical suspended-type absorber is shown in Fig. 4. The choice of any arrangement depends on the balance of the machine and its operational speed. For high-speed machines that are relatively well balanced, a supported-type arrangement is used since in such cases a heavy foundation mass above the springs is not generally necessary. For low-frequency machines, a heavy mass above the springs becomes necessary and a suspended-type absorber arrangement is generally adopted. Suspended-type arrangement provides easy access to the casing housing the springs. Analysis of the absorber foundation system, irrespective of the supported or suspended type, can be made by treating it as a two-degrees-of-freedom system. Spring absorbers are commercially available in several sizes and capacities, and they are affacted by the environmental conditions and should be protected against corrosion.



## 3.2. Design Procedure for Foundations on Absorbers

Design procedure for a foundation on absorbers and supporting a reciprocating machine having its main unbalanced force component in vertical direction is described below:

1. Procure all design data about the machine and soil and limiting amplitudes.

2. First trial : Make a trial design of the foundation without absorber satisfying the limiting amplitudes. The foundation size may turn out to be too big for the size of the machine or for the space available.

3. Second trial : Depending upon the requirements of minimum foundation size for the machine and available space, select the area of the foundation in contact with the soil and the weight of the foundation part below the absorber  $W_1$ .

4. Determine the equivalent spring stiffness of the soil  $k_i$  beolw the base, and the limiting natural frequency of the whole system resting on soil  $?_{n1}$ , and the ratio of frequencies  $r_1$ .

5. Compute the amplitude Z for the system resting on soil (no absorber) and calculate the degree of absorption ?, and determine the frequency ratio  $r_2$ ,  $?_{n2}$ , and total vertical stiffness of absorber  $k_2$ .

6. Select the type of absorber. An absorber having total stiffness  $k_2$  may now be chosen. This selection can be easily made from the information given in manufacturers catalog.

7. Find the amplitude of vibration  $Z_2$  of the system above the absorbers.

8. Actual load per spring  $P_a=kZ_2$ . Check the safety of the spring. From the consideration of stresses in spring  $P_a < P$ . But the absorber system works within narrow ranges. So  $P_a \sim P$  will be more reasonable.

#### 3.3. Vibration Isolation with Wave Barriers

Effective protection from harmful effects of Rayleigh waves may be obtained by using concepts of vibration screening which is made possible by proper interception, scattering and diffraction of



surface waves with wave barriers. The wave barriers may consist of open trenches, trenches filled with bentonite slurry, sawdust, or sand, sheet piles, and piles. Screening problems may be classified into two groups as follows:



Figure 5 Vibration isolation using a circular trench surrounding the source of vibrations-active isolation

Figure 6 Schematic of vibration isolation using a straight trench-passive isolation

Active Isolation : The isolation is provided at the source of vibration. A wave barrier is provided close to or surrounding the source of disturbance as shown schematically in Fig. 5, in which a circular trench of radius  $R_0$  and depth H surrounds the foundation for the machine (the source of vibration). Passive Isolation : The isolation is provided near the location of the structure sought to be protected from the incoming waves. The wave barriers are thus provided remote from the source of vibration but near the site where reduction of vibration amplitudes is required. Figure 6 shows an example of passive isolation in which an open trench of length L and depth H is used to protect a sensetive instrument from the harmful effect of waves. The wave barriers are considered effective in reducing vibrations if the amplitude reduction factor (ARF) is 0.25 or less. The ARF is defined as

ARF=(amplitude of vertical vibration with trench) / (amplitude of ver. vib. without trench) (3.6)

#### 3.3.1 Trench barriers

There have been several successful and unsuccessful applications of the trench barriers for vibration isolation in the past. Woods and Richart (1967) conducted a comprehensive series of field tests to evaluate the screening effect of trenches. The cases of active as well as passive isolation were investigated. The following conclusions were drawn regarding the use of trenches for active isolation. 1) For full circle trenches (angular dimension  $?=360^{\circ}$ ), a minimum value for H/?<sub>R</sub> of 0.6 is required to achieve ARF equal to or less than 0.25 (?<sub>R</sub> is Rayleigh wave length). 2) The zone screened by a full circle trench extended to a distance of at least 10 wavelength (10?<sub>R</sub>) from the source of excitation. 3) For partial circle trenches  $(90^{\circ} < ? < 360^{\circ})$ , the screened zone was defined as the area outside the trench extending to at least 10?<sub>R</sub> from the source and bounded on the sides by radial lines from the center of the source through points  $45^{\circ}$  from ends of the trench. A minimum value for H/?<sub>R</sub> of 0.6 is required for the trench to be effective. 4) Partial circle trenches having angular lenght  $?<90^{\circ}$ , did not provide an effectively screened zone. 5) Trench width is not an important parameter. Amplification of vibratory energy occurred in the direction of open side of the trench. Woods also conducted passive isolation tests using open rectangular trenches and investigated the effect of trench length L, width B, depth H, and the distance from the source R<sub>0</sub>. Significant results of this study were as follows: 1) For effective passive isolation ( $R_0=2?_R$  to  $7?_R$ ), the depth of the trench H should be at least 1.33?<sub>R</sub>. 2) Larger trenches were required at greater distances from the source. To maintain the same ARF, the scaled area of the trench  $(H/?_R \times L/?_R = HL/?_R^2)$  should be increased with increasing scaled distance  $R_0/?_R$ . The least area of the trench in the vertical direction should be 2.5?<sup>2</sup><sub>R</sub> at  $R_0=2?_R$ 



and  $6?_{R}^{2}$  at  $R_{0}=7.3$ ) Trench width had practically no influence effectiveness of open trench (for B/?<sub>R</sub> =0.13 to 0.91). 4) Amplification of vertical motion occurred in zones in front of trenches and to the sides of the trenches. The open trenches are more effective as wave barriers but may present instability problems necessitating trenches backfilled with bentonite slurry, sawdust, concrete, or sand (Al-Hussaini, 1996; Hawwa, 1998).

3.3.2 Pile barriers

When vibrations are occasioned by a source operating at a ver low frequency, the Rayleigh wavelenght will be long and may range up to 50m or more. For a trench to be effective in such case, its depth will range from 30m (0.6?<sub>R</sub>) for active isolation to 66.5m (1.33?<sub>R</sub>) for passive isolation. When the Rayleigh wavelength is long, the trench depth often limits the application of trenches and open or slurry-filled trenches are impracticable. Pile may be used as barriers in such cases as they can be installed to any depth. This alternative of using rows of piles as passive isolation barriers has been investigated by Woods (1974). They used the principle of holography and observed vibrations in a model half-space to evaluate the effect of void cylindirical obstacles on reduction of vibration amplitudes. A schematic sketch of the test showing the geometry of the problem is given in Fig. 7. D is the diameter of the void cylindirical obstacle and S<sub>n</sub> is the net space between two consecutive void holes through which energy can pass through the barrier. The effectiveness of the barrier was found to be significantly affected by the material of the pile and void holes and acoustically soft piles were more efficient than acoustically hard piles. The relative hardness or softness was defined in terms of impedance ratio (IR) as follows (Prakash and Vijay, 1988):



Figure 7 Definition of parameters for cylindirical hole barriers

IR=(Rayleigh wave impedance of the pile) / (Rayleigh wave impedance of the soil medium) IR=  $?_p V_{R(p)} / [?_s V_{R(s)}]$  (3.7)

in which,  $?_p$  and  $?_s$ : density of pile material and soil medium,  $V_{R(p)}$  and  $V_{R(s)}$ : Rayleigh wave velocity in the pile material and in the soil medium. The piles are considered soft if IR<1 and hard if IR>1. The values of Rayleigh wave impedance for various materials are given.

#### 3.3.3 Design procedure for wave barriers

A step-by-step procedure will now be given for design of trench and pile isolation barriers. The information listed below should be procured before attempting the design of any type of vibration isolation barriers: 1. Source data and soil data are obtained. 2. Design of isolation barriers. - Active isolation-Trench barrier: Calculate H,  $?_{R}$  and ?. - Passive isolation-Trench barrier: Calculate H, location of the trench and length of the trench L. - Passive isolation-Pile barrier: Calculate H, D,  $S_{h}$ , IR,  $V_{s}$  and length of the pile barrier L. If one row of piles is inadequate, a second row of piles may be provided to increase the effectiveness of isolation.

#### 3.4. Methods of Reducing Vibration Amplitudes in Existing Machine Foundations

Excessive foundation vibrations may sometimes develop soon after the installation of the machine or sometimes thereafter due to an increase in the unbalanced loads arising out of wear and tear of the machine, change in the subsoil conditions, defective design or condition. It may be possible to reduce

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or limit these vibrations by appropriate selection of the following remedial measures. It must be emphasized that before any remedial measures are considered, the cause of excessive vibrations must be established by proper investigation, which will also help in choosing the most effective measure. Improper selection of the remedial measures may further worsen the situation rather improve it. The methods used to reduce vibrations in existing machine foundations are: *1*. Counterbalancing the unbalanced exciting loads, *2*. Chemical soil stabilization, *3*. Structural measures, *4*. Providing vibration dampers, (Srinivasulu, 1976).

## **4. CONCLUSIONS**

In this study, the principle of vibration absorption and isolation have been discussed. The absorber system is designed on the assumption that the operating speed is constant. Fluctuations in the operating speed will adversely affect the efficiency of the absorber system. The damping in the absorber system has not been included. Damping has a favorable effect on the performance of the absorber and takes care of he influence of minor fluctuations in the speed of the machine. Performance characteristics of commercial absorbers are supplied by their manufacturars and are helpful in selecting the appropriate absorber system for given operating conditions. Isolation procedures using trench barriers have been investigated both experimentally and analytically. Based on the available information, design procedures have been developed as described in this study. Standardized procedures are not currently available for design of pile barriers. Based on present recommendations, a procedure has been suggested for the design of pile barriers which may need modification as more information becomes available.

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